유한한 불규칙적인 공간에서 점탄성 유체의 자연대류

이원민, 임재영, 최영진, 박홍목*
서강대학교 화공생명공학과
(leewm99@naver.com*)

Rayleigh-Bènard convection of viscoelastic fluids in arbitrary finite domains

W.M. Lee, J.Y. Lim, Y. J. Choi, H. M. Park*
Department of Chemical and Biomolecular Engineering, Sogang University
(leewm99@naver.com*)

1. Introduction

In the present work, we study the Rayleigh-Bènard convection of viscoelastic fluids in arbitrary shaped domains. Our purpose is to investigate how the shape of the domain changes the critical Rayleigh number and the bifurcation sequences. Special interest is in the shape where the Hopf bifurcation occurs at a lower values of Deborah number. When we want to adopt the natural convection system as a tool of rheometry, it is more useful to invoke various phenomena peculiar to the viscoelasticity in the convection box. But it is shown that the Hopf bifurcation in the Rayleigh-Bènard convection of viscoelastic fluids in the horizontally unbounded domain occurs at unrealistically high values of Deborah number[4]. Thus, if a specific shape of the convection box induces the Hopf bifurcation at a lower Deborah number, that shaped box may be considered as a better equipment to estimate the rheological parameter values of a specific viscoelastic fluid. Until now, there have been no appropriate analysis tools for this interesting problems of hydrodynamic stability in arbitrary shaped domains. But recently we proposed a method of linear and nonlinear hydrodynamic stability analysis in conned rectangular domains with no-slip walls [1,2,3] by exploiting the Chebyshev pseudospectral method [5]. In the present investigation, we employ the same Chebyshev pseudospectral method to solve the linear Rayleigh-Bènard convection problems in two-dimensional arbitrary finite domains. After transforming the arbitrary shaped physical domains to a square computational domain, we reformulate the Boussinesq equation using the stream function so that the incompressibility condition is imposed exactly. The discretization through the Chebyshev pseudospectral method yields algebraic eigenvalue problems which can be solved to find the eigenvalues and eigenvectors needed in the linear stability analysis. A very general constitutive equation is employed in the present work that encompasses the Maxwell model, Oldroyd model and Phan-Thien-Tanner model. The effects of the shape of the domain and rheological parameter values on the critical Rayleigh number and convection pattern are examined.

2. Governing equations

We consider a Boussinesq fluid in a two-dimensional arbitrary finite domain whose bottom is maintained at a higher temperature than the top. The shapes of the domain are such that the top and bottom are at, whereas the sidewalls are of arbitrary shapes. Governing equations in dimensionless variables may be written as :

$$\nabla \cdot \mathrm{v} = 0$$

$$\frac{\partial \mathrm{v}}{\partial t} + \mathrm{v} \cdot \nabla \mathrm{v} = -\mathrm{Pr}\nabla \mathrm{P} + \mathrm{Pr}\nabla \cdot \tau + \mathrm{RPrTj}$$

$$\frac{\partial T}{\partial t} + \mathrm{v} \cdot \nabla \mathrm{T} = \nabla^2 \mathrm{T}$$

$$\tau + \lambda\left\{\kappa(tr\tau)\tau + D\tau - \frac{1}{2}a(\tau \cdot \dot{\gamma} + \dot{\gamma} \cdot \tau)\right\} = \dot{\gamma} + \varepsilon\lambda\{D\dot{\gamma} - a\dot{\gamma} \cdot \dot{\gamma}\}$$

3. Results

Fig.1 shows the critical Rayleigh number and the boundary separating the region of exchange of stabilities and that of Hopf bifurcation for the case of isothermal sidewalls in the $\varepsilon-\lambda$ plane for a rectangular domain with the aspect ratio dx/dy=2:0, where dx is the width and dy is the depth of the domain. We find that the critical Rayleigh number Rc remains the same regardless of (ε, λ) values when the exchange of stabilities is valid. This value is actually the critical Rayleigh number for the Newtonian fluids [1]. As ε decreases or λ increases, the overstability occurs, and Rc decreases rapidly as ε decreases or λ increases. The Deborah number λ indicates the elasticity of the fluid, which is an important mechanism that induces overstability.

4. Conclusion

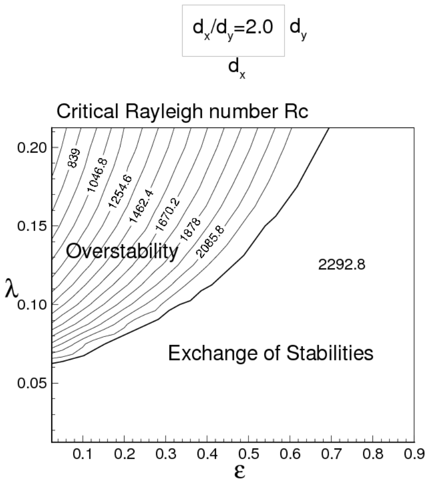

Fig.1. The critical Rayleigh number and the boundary separating exchange of stabilities and Hopf bifurcation in the ε- λ plane for the rectangular domain of dx/dy=2.0.

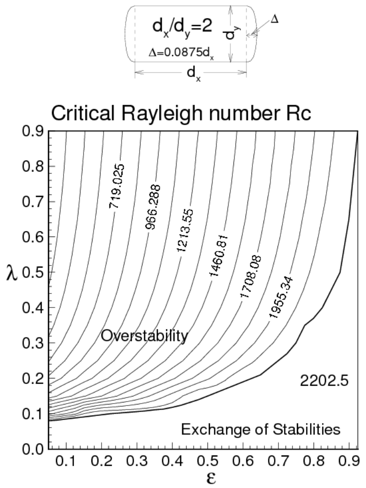

Fig.2. The critical Rayleigh number and the boundary separating exchange of stabilities and Hopf bifurcation in the ε- λplane for the irregular domain depicted in the figure.

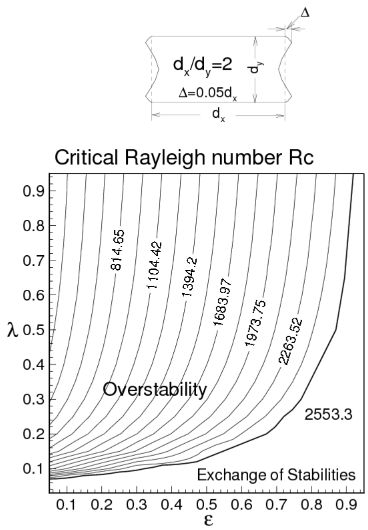

Fig.3. The critical Rayleigh number and the boundary separating exchange of stabilities and Hopf bifurcation in the ε- λplane for the irregular domain depicted in the figure.

We have investigated the linear hydrodynamic stability problems of viscoelastic fluids in arbitrary finite domains. Special interest is to find a shape where the Hopf bifurcation occurs at a small Deborah number. It has been found that the geometric shape affects the onset of convection appreciably. Also investigated is the difference in the switching pattern of cell numbers as the aspect ratio varies, between the isothermal sidewalls and the adiabatic sidewalls. In general, the critical Rayleigh number decreases as the aspect ratio increases or the domain's sidewall distorts in such a way that its virtual aspect ratio increases as in the case of Fig.2. The sidewall distortion in Fig.3 has the effect of decreased aspect ratio. It is found that smaller aspect ratio tends to induce Hopf bifurcation. It is also found that the minimum Deborah number where Hopf bifurcation can be induced takes a lower value if instability sets in at a higher Rayleigh number. The results of the present investigation may be exploited to design shapes of convection box where Hopf bifurcation occurs at realistic low values of Deborah number, which will enhance the usefulness of the natural convection system as a rheometry tool.

5. References

1. H. M. Park, D. H. Ryu, Rayleigh-Benard convection of viscoelastic fluids in nite domains, J. Non-Newtonian Fluid Mech. 98 (2001) 169-184.
2. H. M. Park, D. H. Ryu, Nonlinear convective stability problems of viscoelastic fluids in finite domains, Rheol. Acta, 41 (2002) 427-440.
3. H. M. Park, D. H. Ryu, Hopf bifurcation in thermal convection of viscoelastic fluids within finite domains, J. Non-Newtonian Fluid Mech. 101 (2001) 1-19.
4. R. G. Larson, Instabilities in viscoelastic flows, Rheol. Acta, 31 (1992) 213-263.
5. D. Haziavramidis, H. C. Ku, A pseudospectral method for the solution of the two-dimensional Navier-Stokes equations in the primitive variable formulation, J. Comput. Phys. 67 (1986) 361-371.
6. J. F. Thompson, Z. U. A. Warsi, Numerical Grid Generation, North-Holland, The Netherlands, 1985.
7. S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford Univ. Press, London, 1961.