경사판을 흐르는 박막의 electrohydrodynamic 안정성

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Stability of the electrohydrodynamic film flowing down an inclined plane

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Introduction

The film flow systems have attracted much attentions along with the development of the efficiency-based industries like the current nano-technologies because they usually transfer the heat and mass more efficiently. Let alone the nano-sized operations, because the behaviors of the ocean water can be considered as a thin film supported by the surface of the earth, the information on the leading order waves of the oceanic water takes the essential role on the predictions of its behavior and characteristics such as kinetic energy, wave speed and so forth. In those contexts, our research will gives the basic skemes on analyzing the film flow characteristics.

Evolution equation

We use the evolution equation derived by Kim[1] such as

$$h_{\tau} = -3h^2 h_x + \xi \frac{\partial}{\partial x}\left\{ -\frac{6}{5} Re h^6 h_x + h^3 h_x \tan\beta - \frac{2h^3 h_{xxx}}{3Ca} + \frac{4}{3} K_E h^3 (\phi_{\zeta}\phi_{\zeta x} + \varepsilon_f \phi_x \phi_{xx}) \right\} \quad (1)$$

with the definitions of the Reynolds number, Re , the Capillary number, Ca , and electrostatic number, K_E , respectively. The subscription denotes the partial derivatives with respect to the corresponding letter. Also we can find the leading order function describing the electrostatic potential, ϕ , in the same article like

$$\phi = \frac{\varepsilon_f}{(1-\varepsilon_f)h + \varepsilon_f H}(\zeta - H) + 1 \qquad \text{for } h(x,\tau) \le \zeta \le H,$$
$$\phi = \frac{\zeta}{(1-\varepsilon_f)h + \varepsilon_f H} \qquad \text{for } 0 \le \zeta \le h(x,\tau). \quad (2)$$

Linear stability

To investigate the linear stability of our system, we suppose that the surface is initially disturbed from the basic flow in a small amount η, to say, $h = 1 + \eta$. And this small disturbance is plausibly assumed as a normal harmonic mode such as $\eta = \exp\{i\alpha(x - c\tau)\}$ with the wavenumber, α, and the complex wave speed, $c = c_R + ic_I$. If the imaginary part of the small disturbance, which belongs to the amplitude of the wave, is zero, the disturbance will have a neutral stable state. And the positive one makes the amplitude of the disturbance grow exponentially to become unstable and vice versa. At the neutral stability, the critical Reynolds number is denoted by a function of the wavenumber, α, such as

$$Re_c = \frac{5}{6}\tan\beta + \frac{5}{9}\frac{\alpha^2}{Ca} + \frac{10}{9}\frac{(\varepsilon_f - 1)K_E}{(\varepsilon_f(H-1)+1)^3} \quad . \tag{3}$$

Because the electrostatic number, K_E, is always negative, the effect of an electrostatic field makes the flowing system unstable. Without the effect of the electrostatic field, i.e., $K_E = 0$, the neutral curve has exactly same form as derived by Benjamin[2].

Stability in the supercritical region

To find a supercritical wave number, α_s, we suppose that the film height can be expressed by the Fourier series such as

$$h(x,\tau) = 1 + \sum_{n=1}^{\infty} A_n(\tau)\exp\{in\alpha x\} + \text{ complex conjugates } . \tag{4}$$

Substituting the equation (4) into (1) and rearrangement it with $A_n(\tau)$ will produce

$$\begin{aligned} \bar{A}_1 &= \alpha_1 A_1 + \beta_1 A_1^* A_2 + \gamma_1 |A_1|^2 A_1, \\ \bar{A}_2 &= \alpha_2 A_2 + \beta_2 A_1^2 \end{aligned} \tag{5}$$

where,

$$\begin{aligned} \alpha_1 &= -3\alpha i + \{\frac{6}{5}\alpha^2 Re - \alpha^2\tan\beta - \frac{2\alpha^4}{3Ca} + \frac{4K_E\alpha^2\Gamma}{3(\Gamma+\Theta)^2}\}\xi, \\ \alpha_2 &= -6\alpha i + \{\frac{24}{5}\alpha^2 Re - 4\alpha^2\tan\beta - \frac{32\alpha^4}{3Ca} + \frac{16K_E\alpha^2\Gamma}{3(\Gamma+\Theta)^2}\}\xi, \\ \beta_1 &= -6\alpha i + \{\frac{36}{5}\alpha^2 Re - 3\alpha^2\tan\beta - \frac{14\alpha^4}{Ca} + \frac{4K_E\alpha^2\Gamma\Theta(5(\Gamma+\Theta)-4)}{(\Gamma+\Theta)^4}\}\xi, \\ \beta_2 &= -6\alpha i + \{\frac{72}{5}\alpha^2 Re - 6\alpha^2\tan\beta - \frac{4\alpha^4}{Ca} + \frac{4K_E\alpha^2\Gamma\Theta(\Gamma+\Theta+1)}{(\Gamma+\Theta)^4}\}\xi, \end{aligned}$$

$$\gamma_1 = -3\alpha i + \{18\alpha^2 Re - 3\alpha^2 \tan\beta - \frac{2\alpha^4}{Ca} + \frac{4K_E \alpha^2 \Gamma\Theta(\Theta-\Gamma)(3(\Gamma+\Theta)-2)}{(\Gamma+\Theta)^5}\}\xi, \quad (6)$$

and,

$$\Gamma = 1-\varepsilon_f, \qquad \Theta = \varepsilon_f H. \quad (7)$$

Now let us set

$$A_1 = B_1 \exp\{i\theta_1\},$$

$$A_2 = B_2 \exp\{i\theta_2\},$$

and

$$\triangle_1 = \theta_2 - 2\theta_1. \quad (8)$$

Then, we can obtain the following set of ordinary differential equations

$$\dot{B}_1 = \alpha_{1r} B_1 + (\beta_{1r}\cos\triangle_1 - \beta_{1i}\sin\triangle_1) B_1 B_2 + \gamma_{1r} B_1^3$$

$$\dot{B}_2 = \alpha_{2r} B_2 + (\beta_{2r}\cos\triangle_1 + \beta_{2i}\sin\triangle_1) B_1^2 \quad (9)$$

$$\dot{\triangle}_1 = (\beta_{2i}\cos\triangle_1 - \beta_{2r}\sin\triangle_1) B_1^{2/B_2} + 6\alpha B_1^2$$

If a long wave is stable in a supercritical region, there exist a steady state, thus we can obtain the stationary solutions of equations

$$B_1 = \sqrt{\frac{\alpha_{1r}\alpha_{2r}}{\beta_{1r}\beta_{2r} - \beta_{1i}\beta_{2i} - \alpha_{2r}\gamma_{1r}}},$$

$$B_2 = \frac{\alpha_{1r}}{\beta_{1r}\beta_{2r} - \beta_{1i}\beta_{2i} - \alpha_{2r}\gamma_{1r}} (\beta_{2r}^{2} + \beta_{2i}^{2})^{1/2}, \quad (10)$$

$$\tan\triangle_1 = \beta_{2i}/\beta_{2r}$$

where the subscripts r and i denote the real and the imaginary part, respectively. These equations are the only meaningful solutions of equations (9) because all the B_j must be positive. The last equation in (10) can be derived by ignoring the terms of second or higher order in the equation (9). Also as stated above, since B_j take the real values, the condition that the waves have a supercritical stable motions is that

$$\alpha_{1r}\alpha_{2r} < 0, \quad (11)$$

which is equivalent to

$$\left\{\frac{6}{5}Re-\tan\beta-\frac{2\alpha^2}{3Ca}+\frac{4K_E\Gamma}{3(\Gamma+\Theta)^2}\right\}\times \left\{\frac{24}{5}Re-4\tan\beta-\frac{32\alpha^2}{3Ca}+\frac{16K_E\Gamma}{3(\Gamma+\Theta)^2}\right\}<0 \quad (12)$$

Using the fact that α is always real and positive in the equation (12), we can find the lower limit of the wave number, α_s , and thus over $\alpha_s<\alpha<\alpha_c$,the supercritical stability exists,

$$\alpha_s=\sqrt{Ca\left(\frac{9}{20}Re-\frac{3}{8}\tan\beta+\frac{1}{2}\frac{K_E\Gamma}{(\Gamma+\Theta)^2}\right)} \quad (13)$$

Conclusions

We have investigated the linear stability and the supercritical stability of the film flow down an inclined plane under an electrostatic field. And thus their stability is well defined explicitly. For further work, we must examine if these stability conditions are well applied to wave by using the numerical simulations. As well as, the weakly nonlinear stability must be examined. We leave them as later works.

References

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