

## 변형된 릴레이 피드백

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### Modified Relay Feedback

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#### Introduction

The PID controller has been widely used in industry because of its simple structure and robustness to the modeling error. Therefore, it was necessary to develop an identification method for its automatic tuning. Åström and Hägglund(1984) proposed a relay feedback identification method to obtain the ultimate information of the process. Their idea has been applied in many areas. The data obtained from this method can be used as initial tuning parameters of an adaptive control strategy. Lee et al.(1993) used the relay feedback method to automatically tune a nonlinear pH controller. Luyben(1987) and Li et al.(1991) proposed an identification method(ATV method) using relay feedback test for the autotuning. Lin and Yu(1993) presented relay feedback experiment to extract the process nonlinearity and the ultimate process information of the pH process. Lee and Sung(1993) used the relay feedback method to obtain the first order plus time delay model for the automatic tuning of the PID controller. Loh et al.(1993) and Shen and Yu(1994) proposed a good idea of combining the sequential loop closing and the relay feedback test to control the MIMO(Multi-Input, Multi-Output) processes.

The relay feedback identification uses a square signal to perturb the process. The theory for the ultimate information is based on the Fourier series of the square signal where only fundamental term of the series is considered. In general, the obtained ultimate frequency and gain have good accuracy for usual process(Li et al.(1991)). However, since the square signal approximates a sinusoidal signal, it is always possible for large harmonic terms to be dominant.

In this paper, we propose a new modified relay feedback identification method to get more accurate ultimate informations of the process. The proposed method uses a six-step signal instead of the two-step signal(original relay feedback signal) to reduce the harmonic terms when this signal is represented by the Fourier series. Since the harmonic terms are relatively smaller than the fundamental term, more accurate ultimate data can be obtained from the test.

#### Analysis of the Proposed Test Signal

The proposed relay feedback signal has three steps in the half-period as shown in Fig. 1. First of all, we determine the step size ( $d$ ) of the relay under the consideration of allowable deviation of the process output and the measurement noise. Next, the relay output of magnitude  $d$  enters until the process output deviates from zero, whose duration

time is  $P$ . Then, the relay output of the opposite sign enters the process. That is, the step size of  $d/2$  enters during  $P/4$  and the step size of  $d$  enters during the next  $P/2$ . However, the length of the remaining signal of  $d/2$  is determined from the feedback of the process output to guarantee that the period of the relay feedback signal ( $P_r$ ) converges to the ultimate period of the process. That means, the remaining step signal is cut when the process output crosses zero. If the process output signal crosses zero before the remaining signal activates, the present test signal is cut and the next signal of the opposite sign is generated. This procedure is repeated until the period  $P_r$  converges to some value. This signal can be represented by the Fourier series as follows.

$$u(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega t) \tag{1}$$

$$\omega = \frac{2\pi}{P_r} \tag{2}$$

$$b_1 = \frac{d(2 + 2^{0.5})}{\pi} \tag{3}$$

$$b_2 = 0 \qquad b_2/b_1 = 0$$

$$b_3 = \frac{d(2 - 2^{0.5})}{3\pi} \qquad b_3/b_1 = 0.05719$$

$$b_4 = 0 \qquad b_4/b_1 = 0$$

$$b_5 = \frac{d(2 - 2^{0.5})}{5\pi} \qquad b_5/b_1 = 0.03431$$

$$b_6 = 0 \qquad b_6/b_1 = 0 \tag{4}$$

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where  $P_r$  and  $d$  denote the period and the magnitude of the relay feedback signal, respectively. The effects of the high frequency terms are filtered out by the process dynamics.

The original relay feedback signal is as follows.

$$b_1 = \frac{4d}{\pi}$$

$$b_2 = 0 \qquad b_2/b_1 = 0$$

$$b_3 = \frac{4d}{3\pi} \qquad b_3/b_1 = 0.33333$$

$$b_4 = 0 \qquad b_4/b_1 = 0$$

$$b_5 = \frac{4d}{5\pi} \qquad b_5/b_1 = 0.22222$$

$$b_6 = 0 \qquad b_6/b_1 = 0$$

From the comparison, we notice that the proposed test signal produces much smaller third-order and fifth-order harmonic terms than the original relay feedback signal. It is possible to increase the number of steps to reduce the harmonic terms more. Since this kind of extension is straightforward, we confine our approach to this point. Using (2) and (3), we can estimate the ultimate information directly.

$$\omega_u = \frac{2\pi}{P_r}$$
$$k_{cu} = \frac{d(2 + 2^{0.5})}{\pi a}$$

where  $a$  is the peak value of the process output and  $\omega_u$  and  $k_{cu}$  represent the ultimate frequency and gain obtained from the modified relay feedback method, respectively.

### **Simulation Study**

We simulated the following process.

$$G_p(s) = \frac{\exp(-\theta s)}{(s + 1)^n}$$

Simulation results are shown in Table 1, where T, MR and R denote the true value, values obtained from modified and original relay feedback methods, respectively. For the first order ( $n = 1$ ) and time delay ( $\theta = 1$ ) process, as an example, the true values of the ultimate frequency and gain are 2.029 and 2.263, respectively. However, the original relay feedback method finds their values of 2.107 and 2.015, while the modified method gives 2.040 and 2.288. Comparing the absolute errors of the modified and the original relay feedback methods, we recognize that the proposed method provides more accurate ultimate informations than the original relay feedback method.

### **Conclusions**

We propose a new modified relay feedback identification method to provide more accurate ultimate informations of the process. The signal of the proposed method is more similar to the sinusoidal signal than the original relay feedback signal. Simulation results show that the proposed method is superior to the original one from the absolute error point of view.

### **Literatures Cited**

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Table 1. Simulation results of the proposed method and the original relay feedback method

n		$\theta = 0.1$			$\theta = 0.2$			$\theta = 1.0$			$\theta = 3.0$		
		MR	R	T	MR	R	T	MR	R	T	MR	R	T
1	$k_{cu}$	15.21	13.30	16.35	7.991	6.996	8.503	2.288	2.015	2.263	1.262	1.340	1.291
	$\omega_u$	16.32	16.36	16.32	8.468	8.537	8.444	2.040	2.107	2.029	0.821	0.856	0.819
2	$k_{cu}$	19.76	18.75	20.68	10.55	10.03	10.68	2.664	2.552	2.709	1.457	1.423	1.435
	$\omega_u$	4.304	4.189	4.436	3.080	3.006	3.111	1.304	1.314	1.307	0.661	0.677	0.659
3	$k_{cu}$	6.175	6.006	6.217	5.102	4.974	5.158	2.476	2.416	2.498	1.493	1.472	1.481
	$\omega_u$	1.532	1.518	1.543	1.402	1.387	1.409	0.915	0.919	0.917	0.549	0.558	0.547
4	$k_{cu}$	3.623	3.566	3.649	3.344	3.290	3.362	2.209	2.173	2.221	1.487	1.482	1.482
	$\omega_u$	0.951	0.946	0.954	0.911	0.907	0.913	0.700	0.702	0.700	0.467	0.473	0.466
5	$k_{cu}$	2.731	2.703	2.749	2.612	2.577	2.623	1.998	1.974	2.008	1.469	1.472	1.464
	$\omega_u$	0.704	0.704	0.706	0.684	0.684	0.686	0.566	0.569	0.567	0.406	0.411	0.406

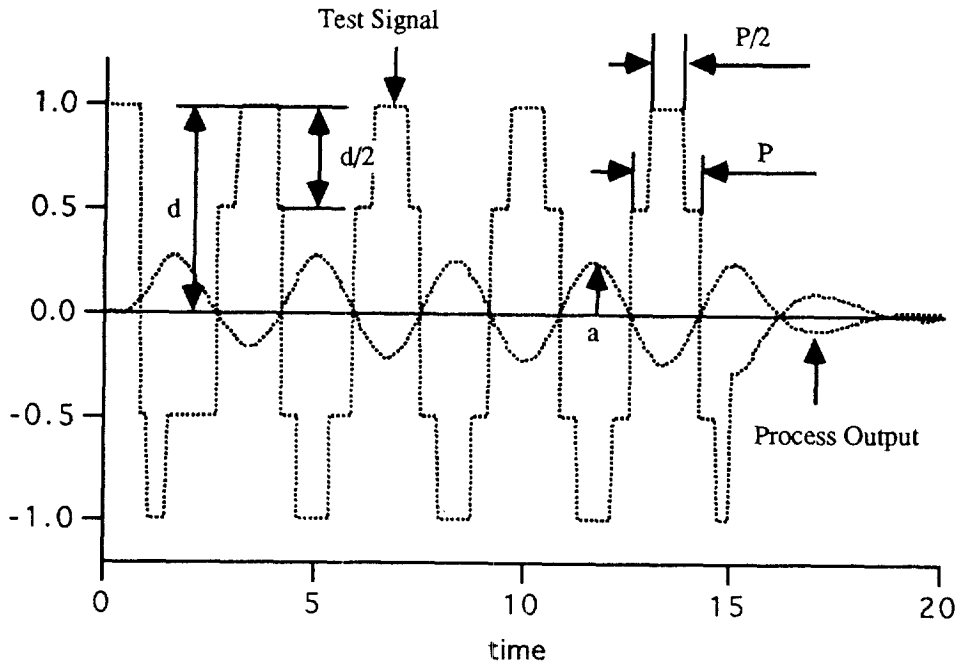


Fig.1 Test signal generation of the proposed strategy