Chapter 2

DYNAMIC MATRIX CONTROL

Dynamic Matrix Control

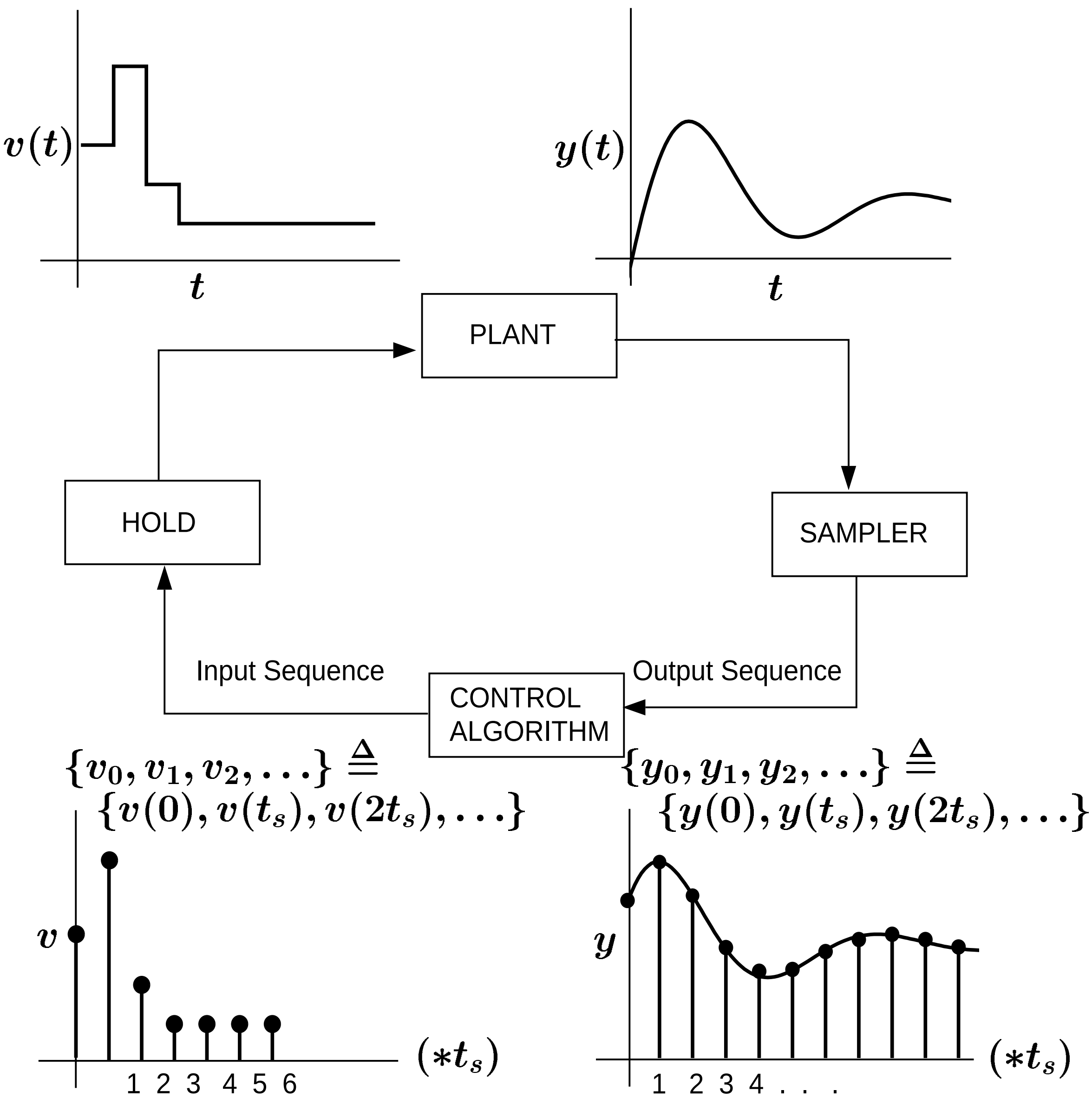
- Proposed by C. Cutler at Shell (later became the President of DMCC).
- Based on a system representation using step response coefficients.
- Currently being marketed by AspenTech (in the name of DMC-Plus).
- Prototypical of commercial MPC algorithms used in the process industries.

We will discuss the core features of the algorithm. There may be some differences in details.

2.1 FINITE IMPULSE AND STEP RESPONSE MODEL

2.1.1 OVERVIEW OF COMPUTER CONTROL

Computer Control System



Model for Computer Control

Should provide the following relation:

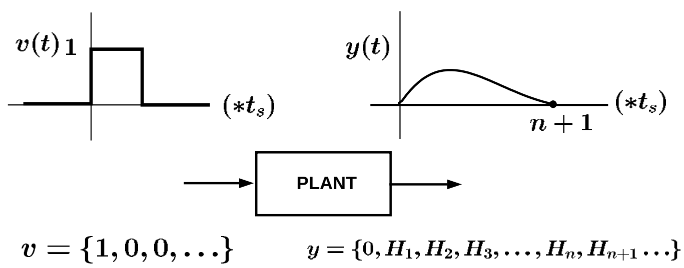
$$\{v(0), v(1), v(2), \cdots, v(\infty)\} \stackrel{model}{\rightarrow} \{y(1), y(2), \cdots, y(\infty)\}$$

We will concentrate on linear models. v and y are deviation variables, i.e., steady state is defined as

$$v^{'}(k) = 0 \quad \forall k \qquad \rightarrow \qquad y^{'}(0) = 0 \quad \forall k$$

2.1.2 IMPULSE RESPONSE AND IMPULSE RESPONSE MODEL

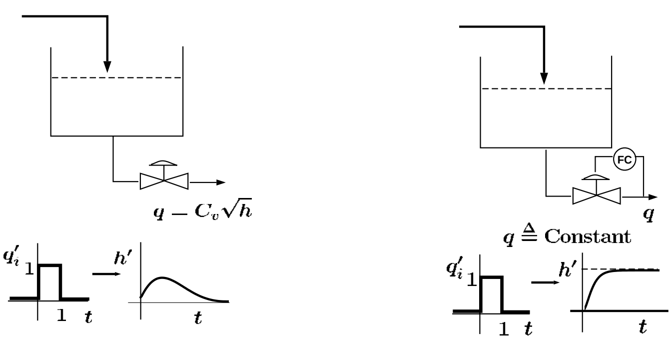
Impulse Response



Assumptions:

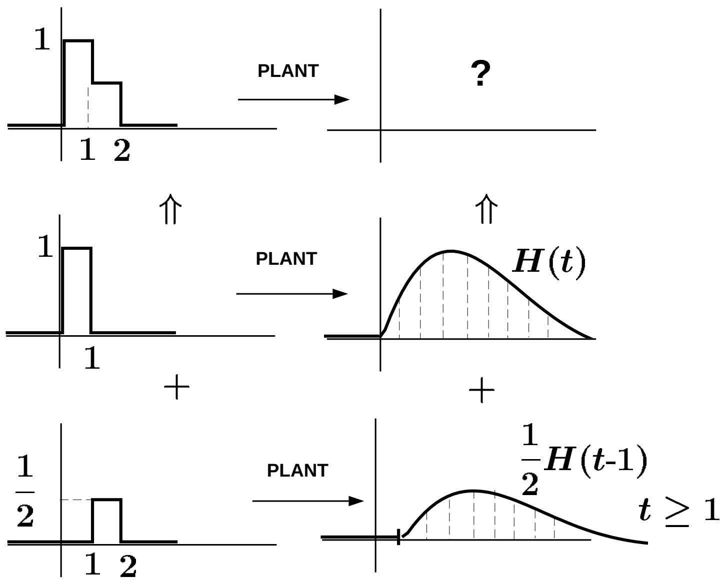
- $H_0 = 0$: no immediate effect of impulse response
- $\exists \;\; n$ s.t.$H_{n+1} = H_{n+2} = \cdots = 0$: "Finite Impulse Response" (reasonable for stable processes).

Examples:



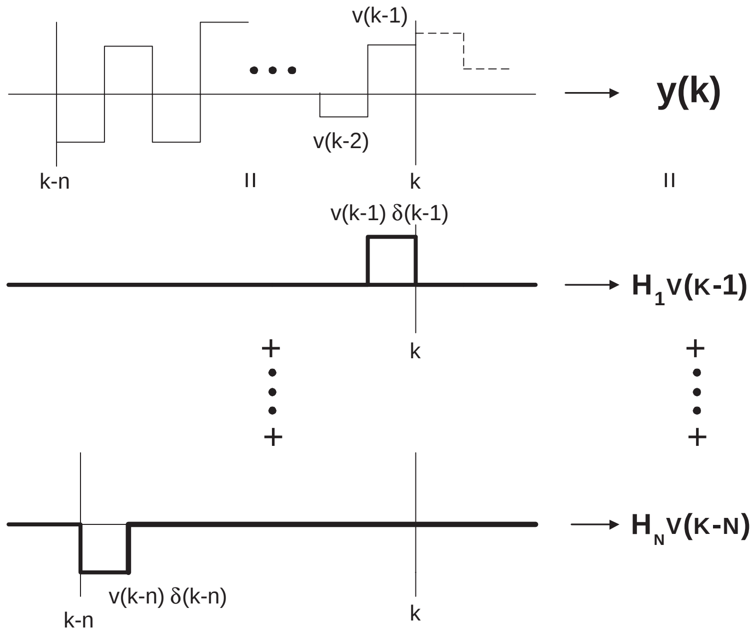
Finte Impulse Response Model

Superposition means $\Longrightarrow$ "Response adds and scales."



Using the superposition described above,

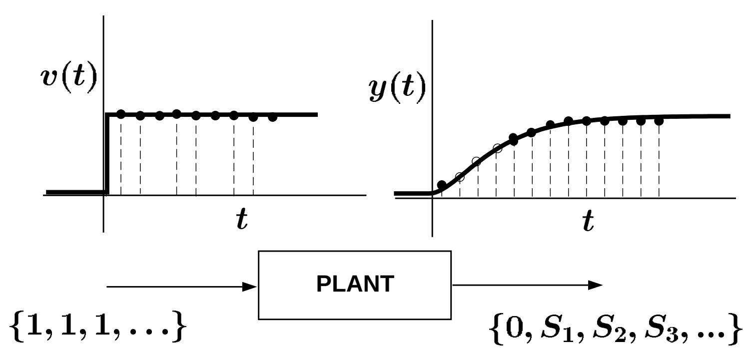
$$y(k) = H_1 v(k-1) + H_2 v(k-2) + \cdots\cdots + H_n v(k-n)$$



NOTE: need to have n-past inputs $(v(k-1), \cdots, v(k-n))$ in the memory.

2.1.3 STEP RESPONSE AND STEP RESPONSE MODEL

Step Response



Assumptions:

- $S_0 = 0$: no immediate effect of step input
- $S_{n+1} = S_{n+2} = \cdots = S_\infty$: equivalent "Finite Impulse Response"

(reasonable for stable processes)

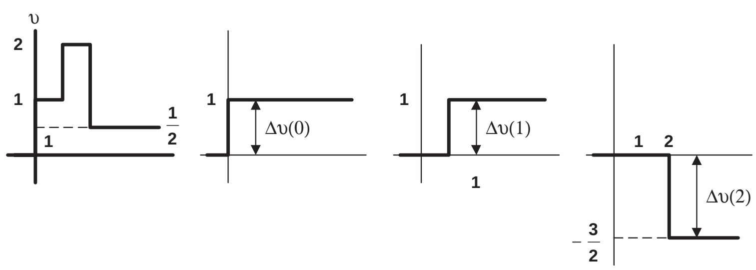
Relation between Impulse Response and Step Response:

$$S_k = \sum_{i=1}^{k} H_i v(k-i)$$

where $v(k-i) = 1$ for $1 \leq i \leq k$. Hence,

$$\begin{aligned} S_k &= \textstyle\sum_{i=1}^{k} H_i \\ H_k &= S_k - S_{k-1} \end{aligned}$$

Truncated Step Response Model



As shown above, any z.o.h. signal $v(t)$ can be represented as a sum of steps:

$$v(t) = \sum_{i=0}^{\infty} \Delta v(i) \mathcal{S}(t-i)$$

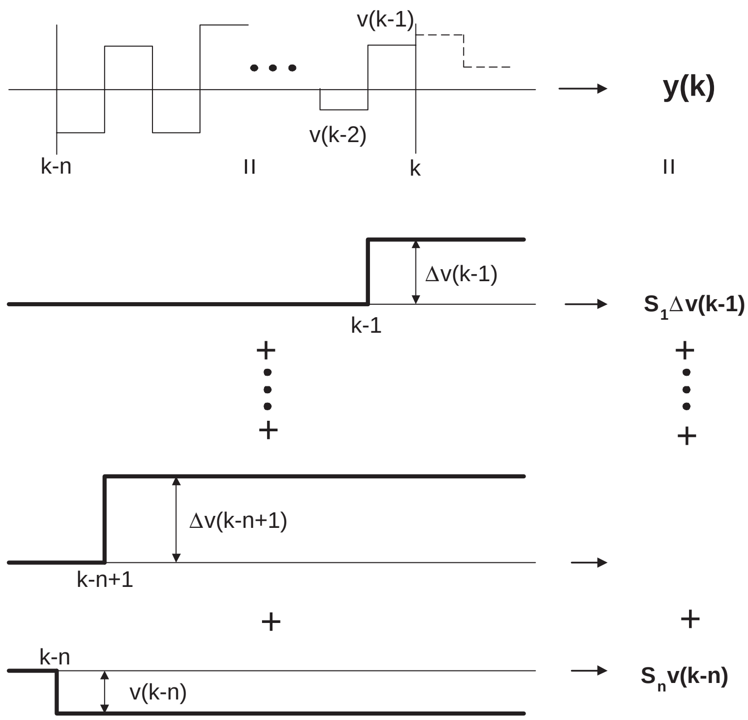
where $\Delta v(i) = v(i) - v(i-1)$ and $\mathcal{S}(t-i)$ is a unit step starting at the i_{th} time step.

Using this and superposition,

$$\begin{aligned} y(k) &= S_1 \Delta v(k-1) + S_2 \Delta v(k-2) + \cdots\cdots \\ &\quad + S_n \underbrace{(\Delta v(k-n) + \Delta v(k-n-1) + \cdots + \Delta v(0))}_{v(k-n)} \end{aligned}$$

More compactly,

$$y(k) = \sum_{i=1}^{n-1} S_i \Delta v(k-i) + S_n v(k-n)$$



Note:

1. $\Delta v(k-i)$ instead of $v(k-i)$ appears in the model.
2. $v(k-n), \Delta v(k-n+1), \ldots, \Delta v(k-2), \Delta v(k-1)$ must be stored in the memory (Same storage requirement as in the FIR model).