$$\mathbf{A}(q^{-1})\mathbf{y}(k) = \mathbf{B}(q^{-1})\mathbf{u}(k) + \mathbf{C}(q^{-1})\mathbf{n}(k) \quad \rightarrow$$

$$\begin{bmatrix} A_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_n \end{bmatrix} \begin{bmatrix} y_1(k) \\ \vdots \\ y_n(k) \end{bmatrix} = \begin{bmatrix} B_{11} & \cdots & B_{1m} \\ \vdots & \ddots & \vdots \\ B_{n1} & \cdots & B_{nm} \end{bmatrix} \begin{bmatrix} u_1(k) \\ \vdots \\ u_m(k) \end{bmatrix} + \begin{bmatrix} C_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_n \end{bmatrix} \begin{bmatrix} n_1(k) \\ \vdots \\ n_n(k) \end{bmatrix}$$

$$\begin{array}{rcll} A_1(q^{-1})y_1(k) & = & B_{11}(q^{-1})u_1(k) + \cdots + B_{1m}(q^{-1})u_m(k) & +C_n(q^{-1})n_1(k) \\ \vdots & \vdots & \vdots & \vdots \\ A_n(q^{-1})y_n(k) & = & B_{n1}(q^{-1})u_1(k) + \cdots + B_{nm}(q^{-1})u_m(k) & +C_n(q^{-1})n_n(k) \end{array}$$

FIR (Finite Impulse Response) Model

$$y(k) = H_1 u(k-1) + \cdots + H_{n_b} u(k-n_b) + w(k)$$

where H_i is a impulse response coefficient (matrix) and $w(k)$ is a zero-mean random noise (not necessarily i.i.d.).

- Cannot be used for description of unstable systems.
- Requires (much) more parameters than the corresponding ARMAX or state space model does.
  For description of a SISO stable system, usually 40 or more pulse response coefficients are needed if the sampling interval is appropriately chosen (not too short and too long).
- Irrespective of the nature of $w(k)$ as far as it is of zero-mean, unbiased parameter estimates can be obtained using a simple least squares method.

State space model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k) \\ y(k) &= Cx(k) + v(k) \end{aligned}$$

where $\{w(k)\}$ and $\{v(k)\}$ are white noise sequences.

- Adequate to MIMO system description.
  Many useful canonical forms are well developed
- Powerful identification methods called the subspace method which directly finds a state space model in a balanced form has been recently developed.
  A version of the subspace method was commercialized by SETPOINT.

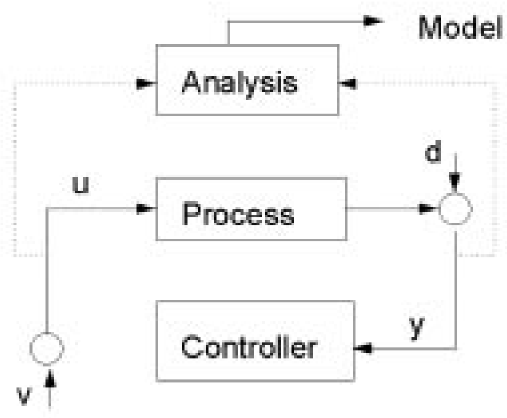
3.4 EXPERIMENTAL CONDITIONS

3.4.1 SAMPLING INTERVAL

- Too long sampling interval $\rightarrow$ too much loss of information
  Too short sampling interval $\rightarrow$ too much computation
- There are many different guidelines. $h \approx \tau/10$ is thought to be adequate for most applications.

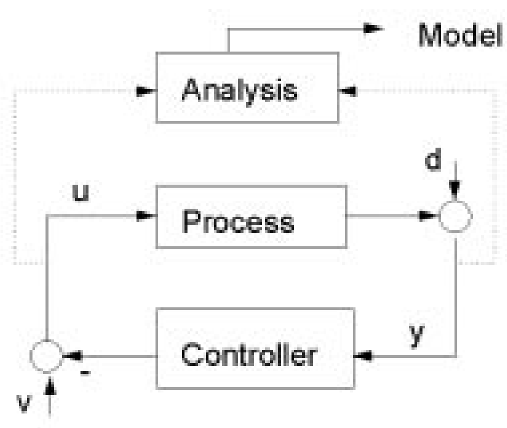
3.4.2 OPEN-LOOP VS. CLOSED-LOOP EXPERIMENTS

Open-loop experiment



$$u = \text{process input}$$
$$y = \text{process output}$$

Closed-loop experiment



$$u = \text{process input, controller output}$$
$$y = \text{process output, controller input}$$

$$\Rightarrow \quad \text{Identified Model} \approx \text{Process or } 1/\text{Controller}$$

- For nonparametric models (typically transfer functions),

$$\begin{aligned} \hat{G}_{model}(s) &\approx G_{process}(s) \text{ when } d = 0 \\ \hat{G}_{model}(s) &\approx G_{control}(s) \text{ when } v = 0 \end{aligned}$$

- For parametric models (FIR, ARMAX, State Space ..),

$$\hat{G}_{model}(s) \approx G_{process}(s)$$

if *identifiability* is satisfied.

Identifiability is *in most case* satisfied if

1. a FIR model is used and/or
2. a high-order controller is used and/or
3. independent excitation is given on v.

3.4.3 INPUT DESIGN

- Remember that the excitation input has limited energy with finite magnitudes over a finite duration. Hence, it is inevitable that the identified model has biased information of the process.
- Depending on the way how to distribute the input energy over different frequencies and also over different input principal directions (for MIMO cases), the identified model may have different characteristics.
- The input should preferrably be designed to sufficiently excite the system modes which are associated with the desired closed-loop performance.

  For a SISO process, process information near the crossover frequency is most important. The Ziegler-Nichols tuning method (*i.e.*, continuous cycling method) is justified in this sense.

- In general, the PRBS(Pseudo-Random Binary Sequence) is used as an excitation signal. By adjusting the minimum step length, we can change the frequency contents in the PRBS.