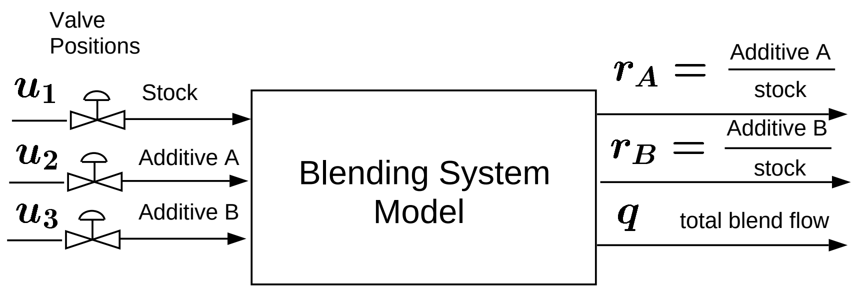
- constraints
- competing optimization requirements

MPC provides a systematic, unified solution to problems with these characteristics.

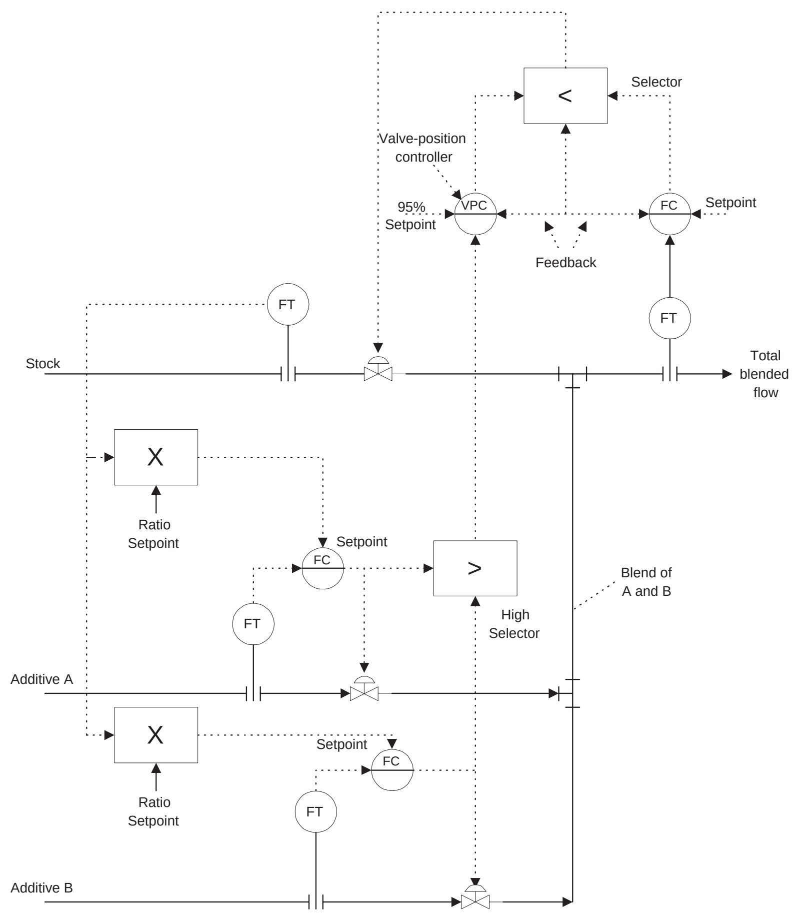
1.3.1 SOME EXAMPLES

Example I : Blending systems (input constraints)



- control r_A & r_B (first priority).
- control q if possible (second priority).
- possibility of valve saturation must be taken into account.

Classical Solution :



MPC Solution :

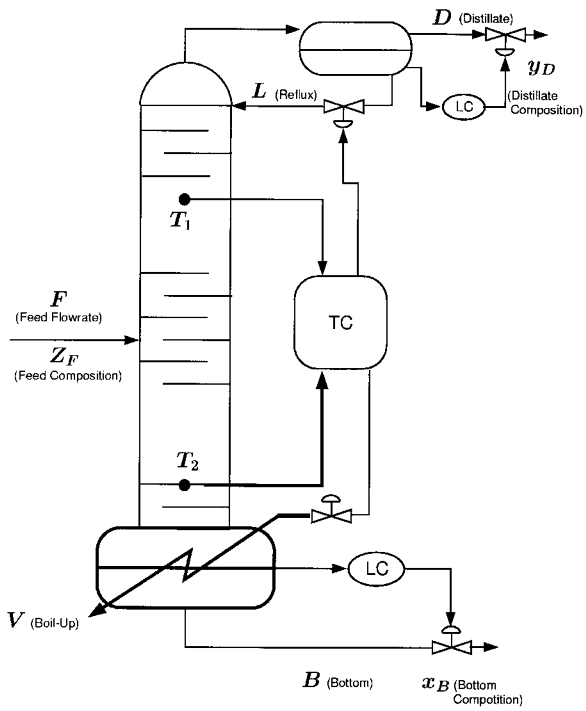
At t=k, solve

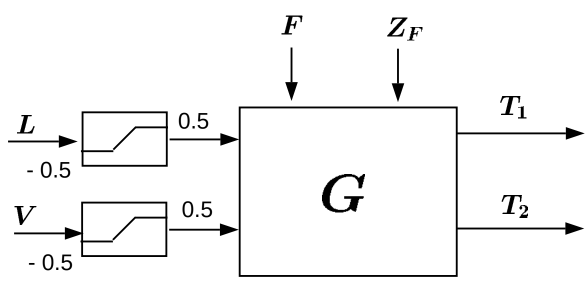
$$\min_{u_i} \quad \sum_{i=1}^{p} \left\| \begin{bmatrix} (r_A)_{k+i|k} \\ (r_B)_{k+i|k} \end{bmatrix} - \begin{bmatrix} (r_A)_{ref} \\ (r_B)_{ref} \end{bmatrix} \right\|_Q^2 + \|q_{k+i|k} - q_{ref}\|_R^2$$

$$Q \gg R$$

$$\begin{bmatrix} (u_1)_{min} \\ (u_2)_{min} \\ (u_3)_{min} \end{bmatrix} \leq \begin{bmatrix} (u_1)_j \\ (u_2)_j \\ (u_3)_j \end{bmatrix} \leq \begin{bmatrix} (u_1)_{max} \\ (u_3)_{max} \\ (u_2)_{max} \end{bmatrix}, \quad j = 0, \cdots, p-1$$

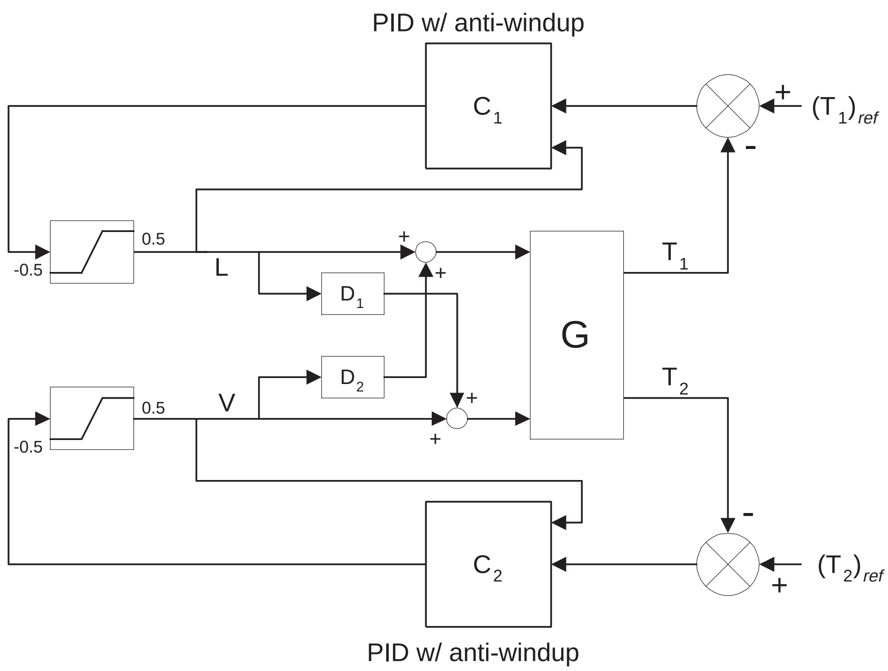
Example II : Two-point control in a distillation column (input constraints, interaction)





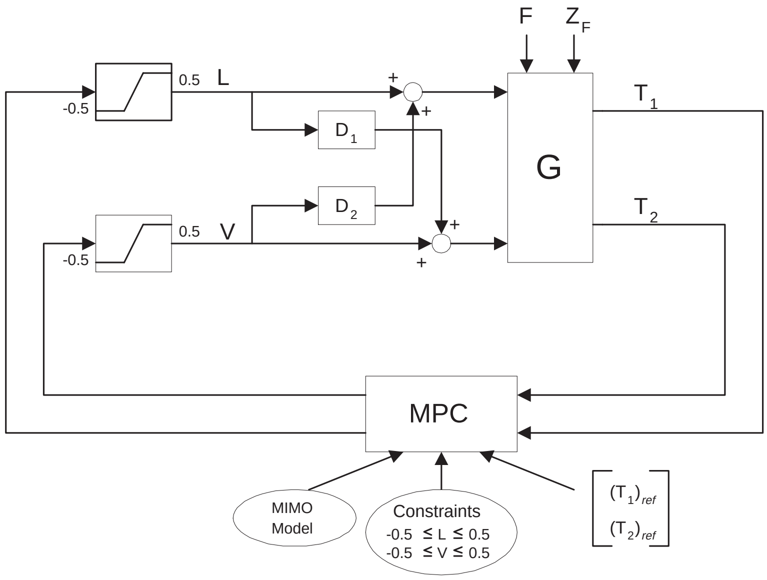
- strong interaction
- "wind-up" during saturation
- saturation of an input requires recoordination of the other input

Clasical Solution: Two single-loop controllers with anti-windup scheme (decouplers not shown)



- T_1 controller does not know that V has saturated and vice versa $\Rightarrow$ coordination of the other input during the saturation of one input is impossible.
- mode-switching logic is difficult to design / debug (can you do it?) and causes "bumps", etc.

MPC Solution:



At $t = k$, solve

$$\min_{\Delta\mathcal{U}_k} \sum_{i=1}^{p} \left\| \begin{bmatrix} (T_1)_{k+i|k} \\ (T_2)_{k+i|k} \end{bmatrix} - \begin{bmatrix} (T_1)_{\text{ref}} \\ (T_2)_{\text{ref}} \end{bmatrix} \right\|_Q^2 + \sum_{i=0}^{m-1} \left\| \begin{bmatrix} \Delta L_{k+i|k} \\ \Delta V_{k+i|k} \end{bmatrix} \right\|_R^2$$

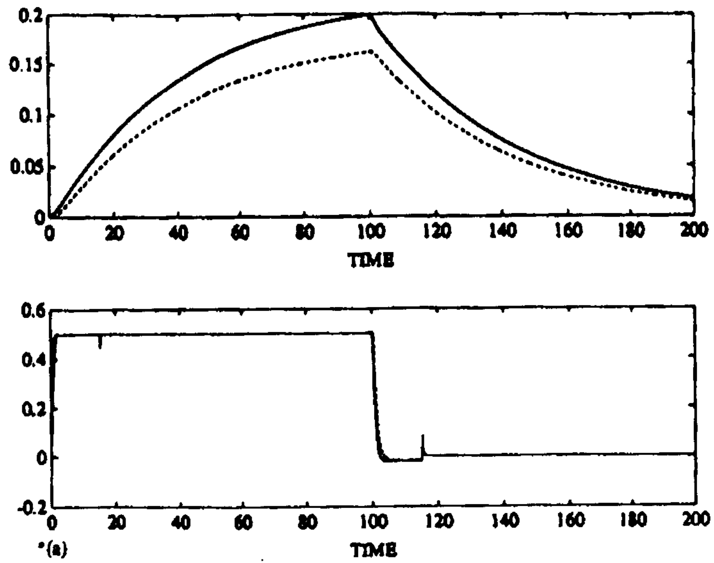
with

$$\begin{bmatrix} L_{\min} \\ V_{\min} \end{bmatrix} \leq \begin{bmatrix} L_{k+i|k} \\ V_{k+i|k} \end{bmatrix} \leq \begin{bmatrix} L_{\max} \\ V_{\max} \end{bmatrix} \qquad \text{for } i = 0, \cdots, m-1$$

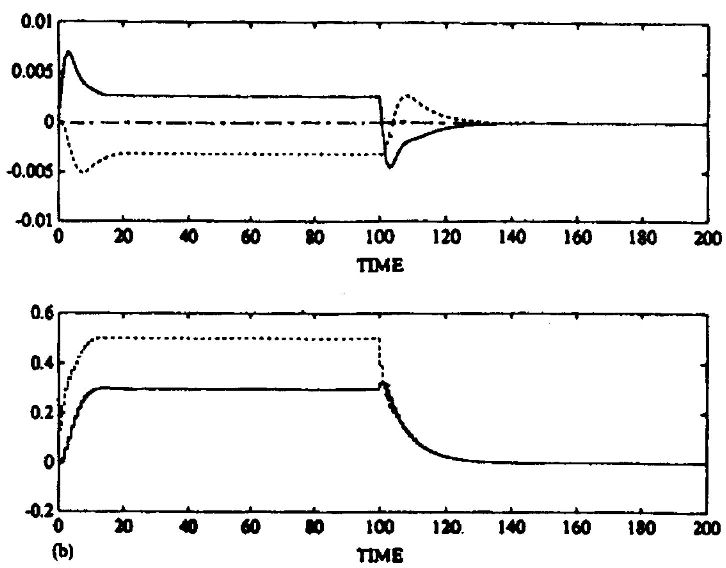
- easy to design / debug / reconfigure.
- anti-windup is automatic.
- optimal coordination of the inputs is automatic.

Performance of classical solution vs. MPC

SISO loops w/ anti-windup & decoupler (no mode switching):



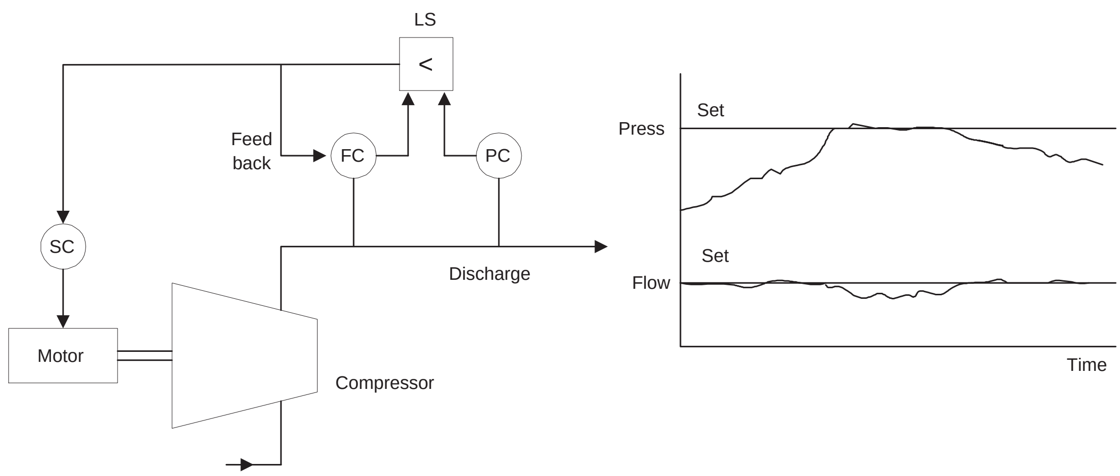
MPC:



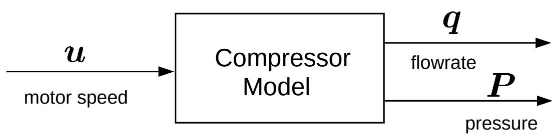
Example III : Override control in compressor(output constraint)

- control the flowrate
- but maintain $P \leq P_{max}$

Classical Solution :



MPC Solution:



At $t = k$, solve

$$\min_{\Delta \mathcal{U}_k} \sum_{i=1}^{p} \left\| q_{k+i|k} - q_{ref} \right\|_Q^2 + \sum_{i=0}^{m-1} \left\| \Delta u_{k+i|k} \right\|_R^2$$

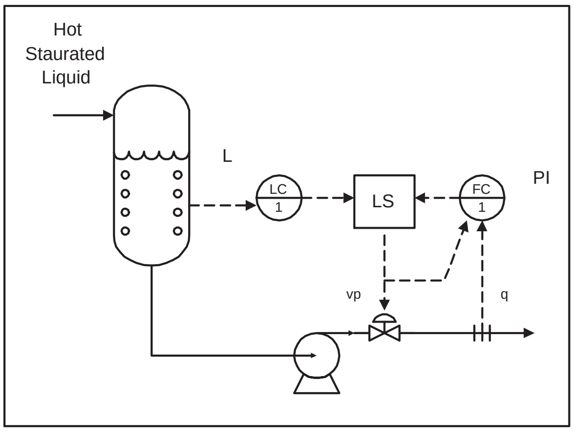
with

$$P_{k+i|k} \leq P_{max} \quad \text{for } i = 1, \cdots, p$$

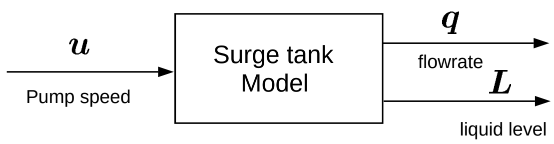
Example IV : Override control in surge tank(output constraints)

- control the outlet flowrate
- but maintain $L \geq L_{min}$

Classical Solution :



MPC Solution:



At $t = k$, solve

$$\min_{\Delta \mathcal{U}_k} \sum_{i=1}^{p} \left\| q_{k+i|k} - q_{ref} \right\|_Q^2 + \sum_{i=0}^{m-1} \left\| \Delta u_{k+i|k} \right\|_R^2$$

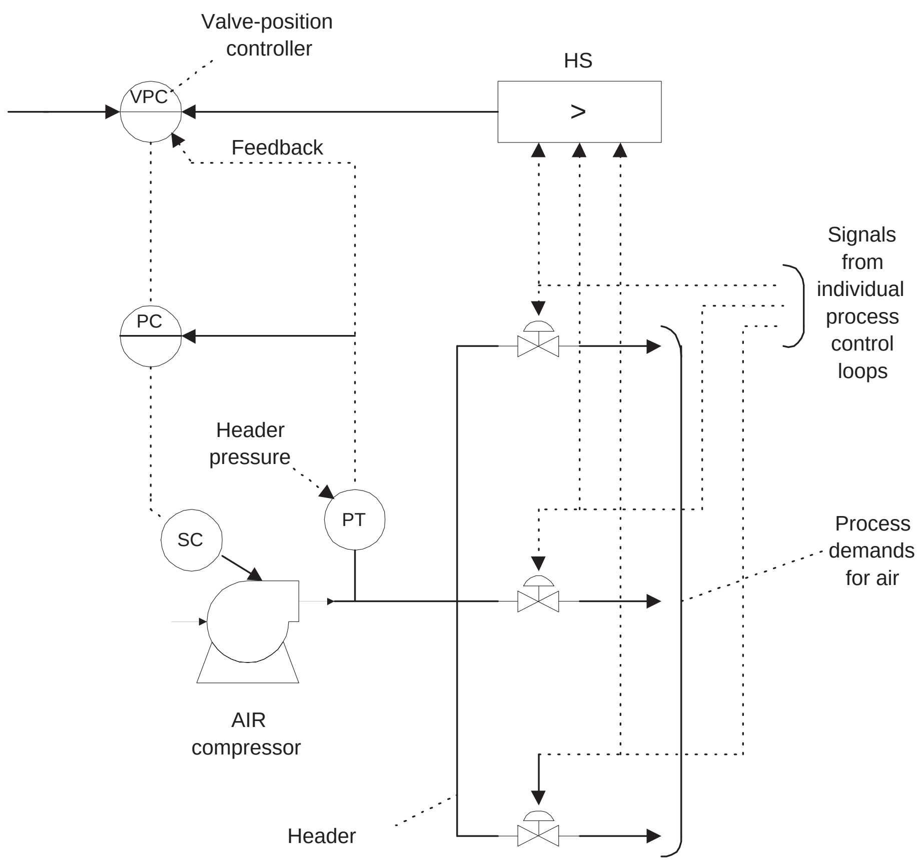
with

$$L_{k+i|k} \geq L_{min} \quad \text{for } i = 1, \cdots, p$$

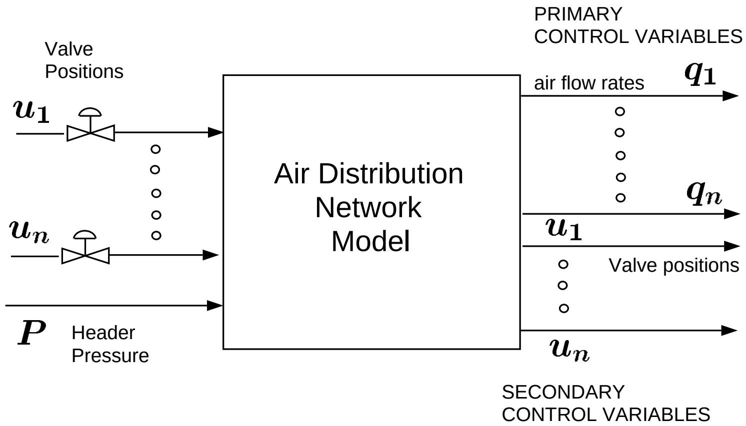
Example V : Valve position control in air distribution network (optimization requirement)

- control the flowrates of individual channels
- minimize the air compression

Classical Solution :



MPC Solution :



At $t = k$, solve

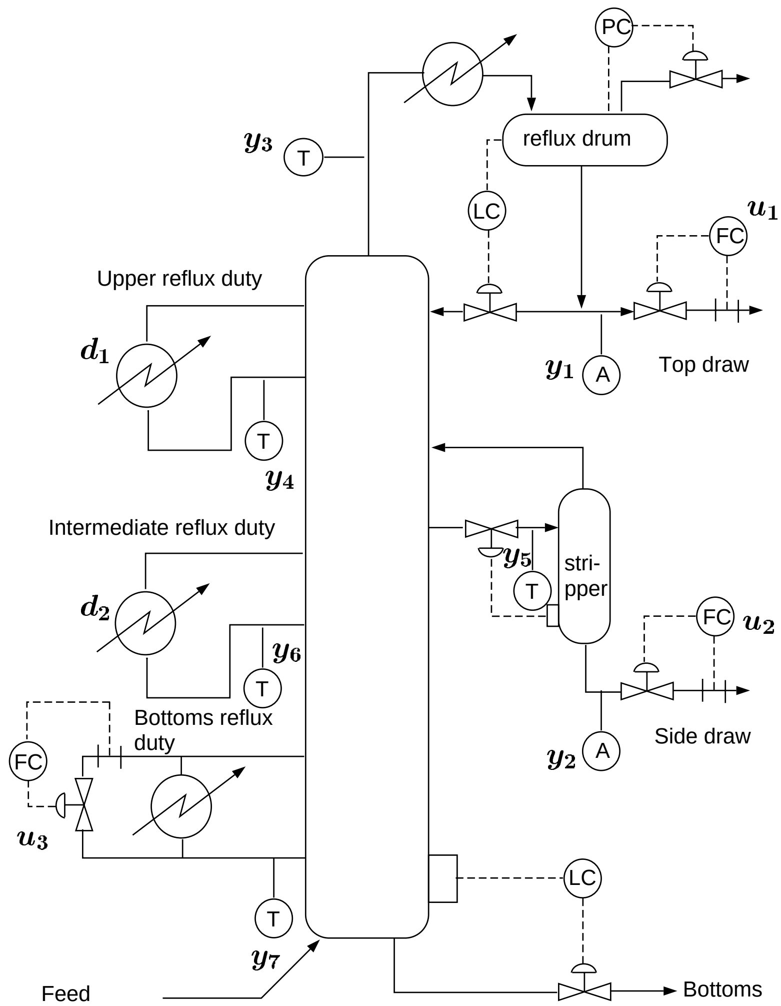
$$\min_{\Delta\mathcal{U}_k} \sum_{i=1}^{p} \left\| \begin{bmatrix} (q_1)_{k+i|k} \\ \vdots \\ (q_n)_{k+i|k} \end{bmatrix} - \begin{bmatrix} (q_1)_{ref} \\ \vdots \\ (q_n)_{ref} \end{bmatrix} \right\|_Q^2 + \sum_{i=1}^{m-1} \left\| P_{k+i|k} - P_{min} \right\|_R^2$$

with $Q \gg R$ and

$$\begin{bmatrix} P_{min} \\ (u_1)_{min} \\ \vdots \\ (u_n)_{min} \end{bmatrix} \leq \begin{bmatrix} P_{k+i|k} \\ (u_1)_{k+i|k} \\ \vdots \\ (u_n)_{k+i|k} \end{bmatrix} \leq \begin{bmatrix} P_{max} \\ (u_1)_{max} \\ \vdots \\ (u_n)_{max} \end{bmatrix} \qquad \text{for } i = 0, \cdots, m-1$$

Example VI : Heavy oil fractionator (all of the above)

- y_7 must be kept above T_{min}.
- y_1 and y_2 is to be kept at setpoint(measurements delayed).
- BRD must be minimized to maximize the heat recovery.

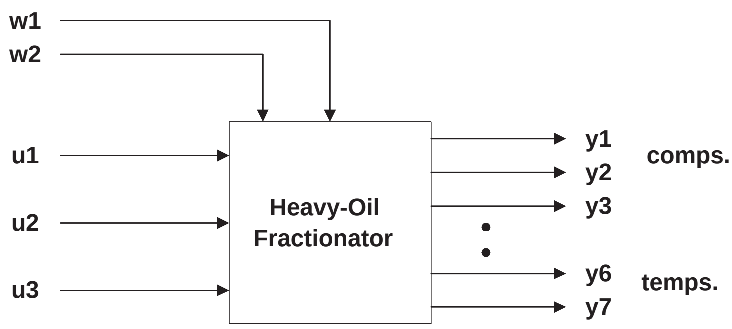


Classical Solution:

Not clear

- how to use temperature measurements to fight the effect of delays, unreliability, etc. of analyzers.
- how to accommodate the optimization requirement.

MPC Solution :

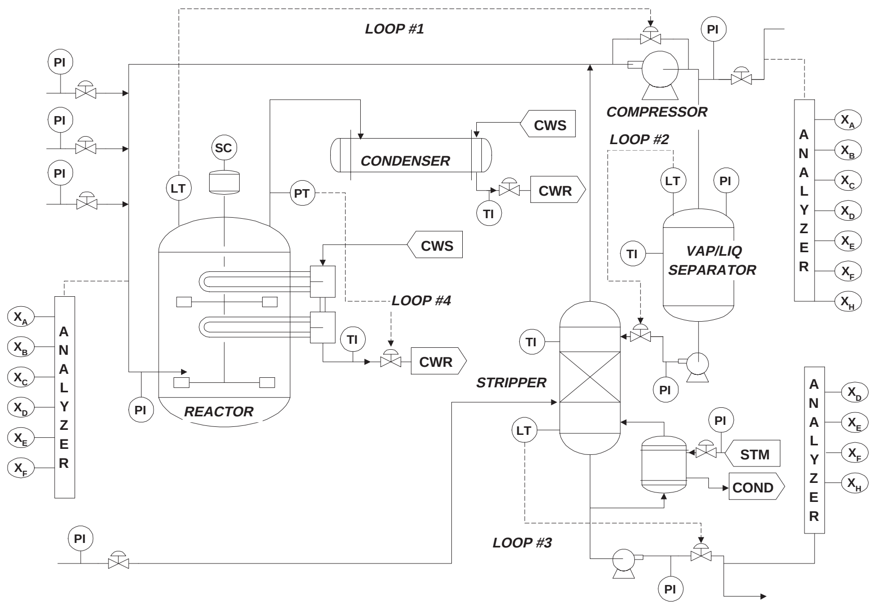


$$\min_{\Delta \mathcal{U}_k} \sum_{l=1}^{p} \left\| \begin{bmatrix} y_1 \\ y_2 \\ u_3 \end{bmatrix}_{k+l|k} - \begin{bmatrix} y_1 \\ y_2 \\ u_3 \end{bmatrix}_{ref} \right\|_Q^2 + \sum_{i=1}^{m} \left\| \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \\ \Delta u_3 \end{bmatrix}_{k+i|k} \right\|_R^2$$

$$y_7 \geq T_{min}$$

plus other input constraints.

Example VII : Tennessee Eastman process(supervisory control requirements)



Tier	Loop #	Controlled Variables	Manipulated Variables
	1	Reactor Level	Compressor recycle valve
I	2	Separator Level	Separator liquid flow
	3	Stripper Level	Stripper liquid flow
	4	Reactor Pressure	Reactor cooling water flow

$$\min_{\Delta \mathcal{U}_k} \sum_{l=1}^{p} \left\| \begin{bmatrix} Q \\ G/H \end{bmatrix}_{k+l|k} - \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}_{k+l|k} \right\|_Q^2 + \sum_{i=0}^{m-1} \left\| \Delta u_{k+i|k} \right\|_R^2$$

$$P_r \leq (P_r)_{max}$$

$$(H_r)_{min} \leq H_r \leq (H_r)_{max}$$

where
P_r: reactor pressure, $(P_r)_s$: setpoint to reactor pressure loop
H_r: reactor level, $(H_r)_s$: setpoint to reactor level loop
Q: total product flow G/H: mass ratio between products G and H
F_D: D feed flow F_E: E feed flow

1.3.2 SUMMARY

Advantages of MPC over Traditional APC

- control of processes with complex dynamics
- decoupling and feedforward control are "built in" (traditional approaches are difficult for systems larger than 2×2).
- constraint handling
- utilizing degrees of freedom
- consistent methodology
- realized benefits: higher on-line times and cheaper implementation / maintenance