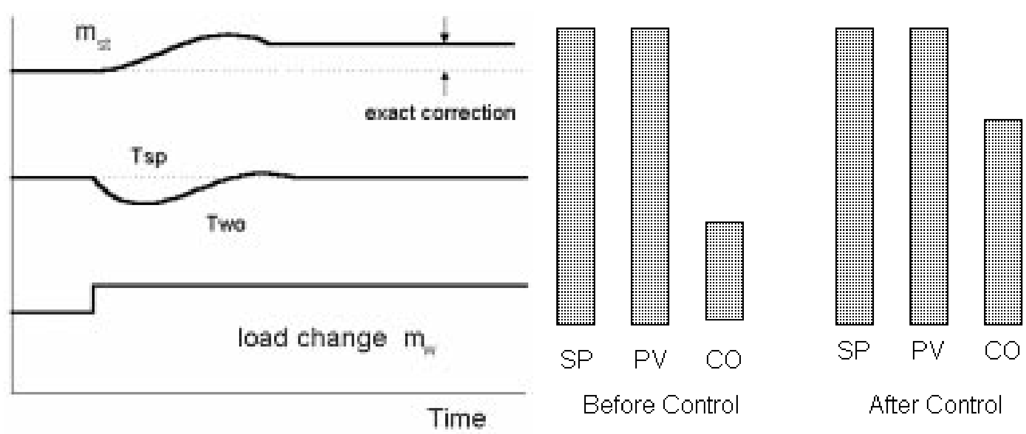
1.4 PID FORMULATIONS

1.4.1 INTEGRAL(I) CONTROL

$$u(t) = u_{bias} + \frac{1}{T_I}\int_0^t e(\tau)d\tau = u_{bias} + \frac{1}{T_I}\int_0^t (r(\tau) - y(\tau))\, d\tau$$

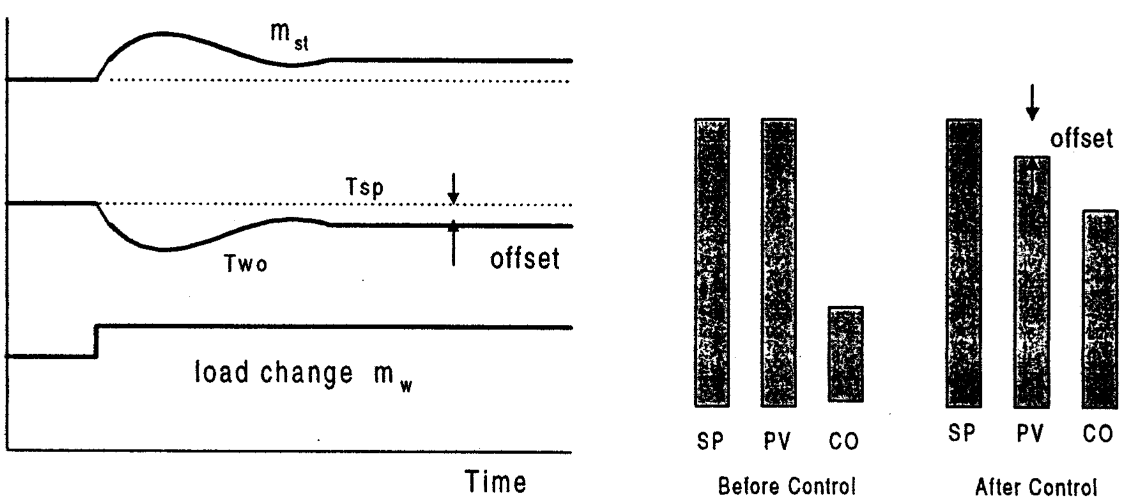
- u_{bias} is the manually set CO value before switching to AUTO mode.
- $t = 0$ represents the instant the AUTO mode is started.
- Corrective action based on accumulated control error.
- If control error persists, $u(t)$ continues to increase (decrease) until $e(t)$ is settled at zero.
- The I-mode ultimately finds a new u_{bias} for exact correction. In this sense, the I-mode is often called the RESET mode.
- When switching AUTO to MANUAL, $u(t)$ replaces u_{bias}. (Bumpless transfer)



1.4.2 PROPORTIONAL(P) CONTROL

$$u(t) = u_{bias} + K_c e(t) = u_{bias} + K_c\left(r(t) - y(t)\right)$$

- Corrective action proportional to the current control error. $\rightarrow$ Faster control action than I-mode



- For a load and/or set point change, exact correction cannot be made because u_{bias} is not reset. $\rightarrow$ OFFSET
- K_c : Proportional Gain (%/% or may have a Physical Unit depending on controllers)
- REVERSE mode $\leftrightarrow$ $K_C > 0$
  DIRECT mode $\leftrightarrow$ $K_C < 0$
- In industrial controllers, PB (Proportional Band) is dominantly used instead of K_c.

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$$\text{PB}(\%) = \frac{100}{K_c}, \qquad \Delta u(t) = \frac{100}{\text{PB}}\, e(t)$$

PB represents the percent change of control error which induces 100% movement of CO.

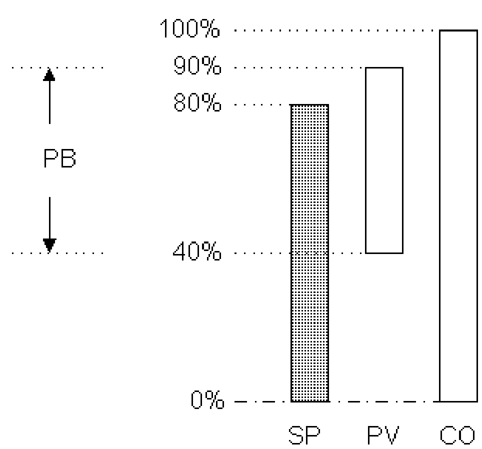
– Note that $0\% \leq u(t), y(t) \leq 100\%$.

Depending on r and u_{bias} settings, the controllable range of $y(t)$ which induces the maximum travel of CO may be different from PB.

The controllable range of $y(t)$ is not necessarily centered with respect to r.

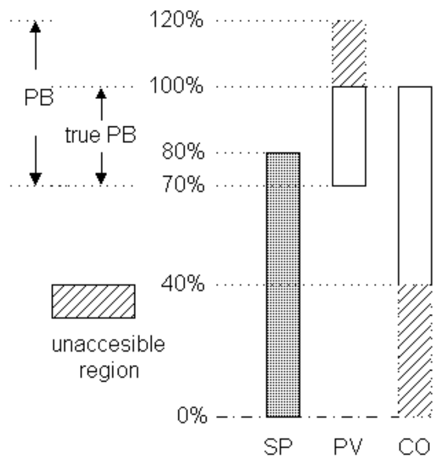
Example. 1 $\quad u(t) = 20 + (100/\text{PB})(80 - y(t)), \quad \text{PB} = 50\%$

$$\begin{aligned} \text{At } y(t) &= 90\%, u(t) = 0\% \\ \text{At } y(t) &= 40\%, u(t) = 100\% \end{aligned}$$



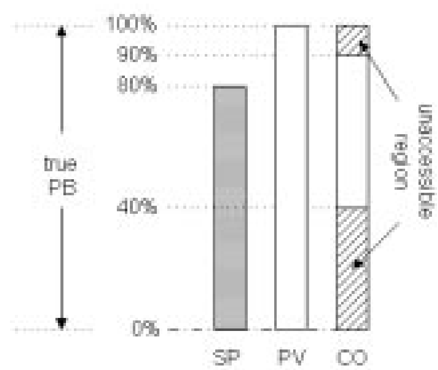
Example. 2 $u(t) = 80 + (100/\mathrm{PB})(80 - y(t)), \quad \mathrm{PB} = 50\%$

$$\begin{aligned} \text{At } y(t) &= 70\%, u(t) = 100\% \\ \text{At } y(t) &= 100\%, u(t) = 40\% \end{aligned}$$



Example. 3 $u(t) = 80 + (100/\mathrm{PB})(20 - y(t)), \quad \mathrm{PB} = 200\%$

$$\begin{aligned} \text{At } y(t) &= 0\%, u(t) = 90\% \\ \text{At } y(t) &= 100\%, u(t) = 40\% \end{aligned}$$

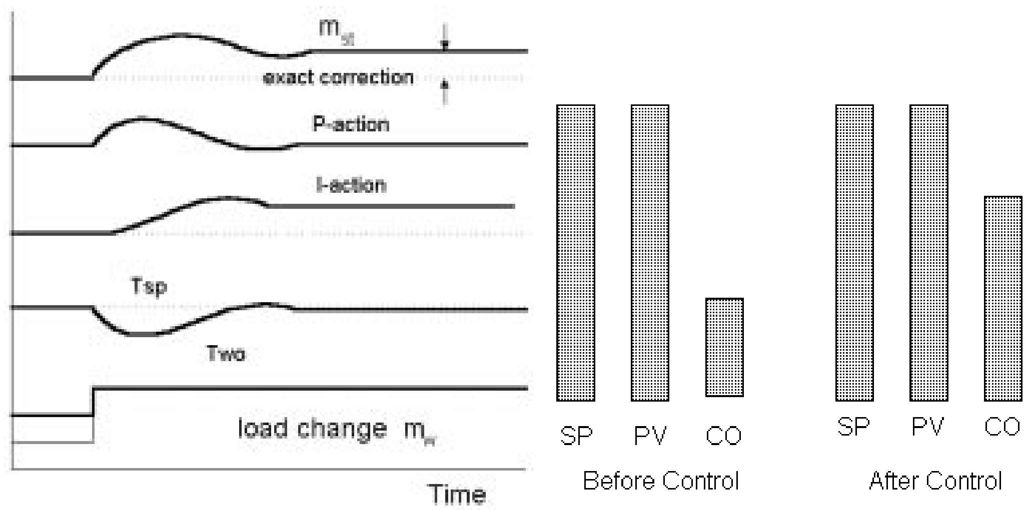


- The process is in control state only when PV is within the true PB. Outside the PB, CO is staturated (or PV itself is saturated.)

1.4.3 PROPORTIONAL INTEGRAL(PI) CONTROL

$$u(t) = u_{bias} + K_c \left[e(t) + \frac{1}{T_I} \int_0^t e(\tau) d\tau \right]$$

- Immediate action by P-mode PLUS offset elimination by I-mode



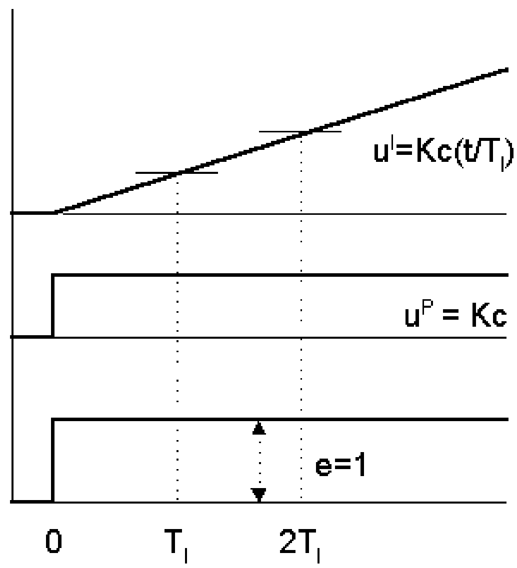
- T_I : Integral time (min/repeat)

Consider the situation where $e(t) = 1$ persists.

$$\begin{aligned} u^P(t) &= K_c e(t) = K_c \\ u^I(t) &= \frac{K_c}{T_I} \int_0^t e(\tau) d\tau = K_c \frac{t}{T_I} \end{aligned}$$

$$\Rightarrow \qquad u^I(T_I) = K_c = u^P, \quad u^I(2T_I) = 2K_c = 2u^P, \cdots$$

I-mode *repeats* u^P every T_I minutes.



- Sometimes, Reset Rate R_I defined by $R_I(\text{repeat/min}) = 1/T_I$ is used in industrial controllers.

1.4.4 PROPORTIONAL INTEGRAL DERIVATIVE(PID) CONTROL

$$u(t) = u_{bias} + K_c \left[e(t) + \frac{1}{T_I}\int_0^t e(\tau)d\tau + T_D \frac{de(t)}{dt} \right]$$

- $de(t)/dt$ detects how fast the error increases(or decreases).
- D-mode can much imporve the control performance by anticipating the future trend of control error (in a linear fashion) and taking a compensating action in advance.

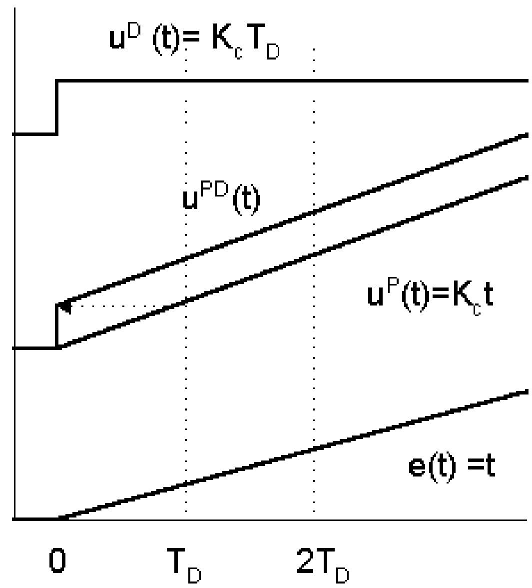
- T_D: Derivative time (min)

  Consider the situation where $e(t) = t$ (linearly increases).

$$
\begin{aligned}
u^P(t) &= K_c e(t) = K_c t \\
u^D(t) &= K_c T_D \frac{de(t)}{dt} = K_c T_D
\end{aligned}
$$

$$\Rightarrow \quad u^D(t) = u^P(T_D)$$

  or $u^{PD}(t)$ acts T_D(min) ahead of $u^P(t)$.



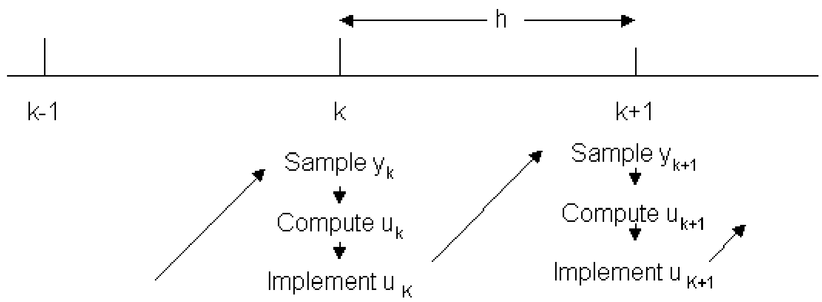
  In this sense, D-mode is sometimes called a *preact mode* and T_D is called the preact time.

- D-mode can also serve to reduce oscillation, thereby allowing use of higher gains.

- D-mode is sensitive to high frequency noise in the control error. Because of this trouble, not employed for flow rate, pressure, and level control where the measurements usually have significant process noise. Mostly used for temperature control.

1.4.5 DIGITAL IMPLEMENTATION

Digital control is run according to the following sequence:



Position Form Digital PID Algorithm

At the instance $t = kh$,

$$e(t) = e_k$$
$$\int_0^t e(\tau)d\tau \approx \sum_{i=1}^{k} e_i h$$
$$\frac{de(t)}{dt} \approx \frac{e_k - e_{k-1}}{h}$$

Using the above approximations, we can derive a digital PID control algorithm

$$u_k = u_{bias} + K_c \left(e_k + \frac{h}{T_I} \sum_{i=1}^{k} e_i + \frac{T_D}{h} \left[e_k - e_{k-1} \right] \right)$$

- The above is called the *position form* because the positional value of u_k is directly computed.
- Required to store e_k and $\sum_{i=1}^{k} e_i$ in the memory.
- Inconvenient to handle the *reset windup* problem.

Velocity Form Digital PID Algorithm

The position from algorithm at $t = (k-1)h$ is

$$u_{k-1} = u_{bias} + K_c \left(e_{k-1} + \frac{h}{T_I} \sum_{i=1}^{k-1} e_i + \frac{T_D}{h} \left[e_{k-1} - e_{k-2} \right] \right)$$

Subtracting this from u_k yields

$$u_k = u_{k-1} + K_c \left(\left[e_k - e_{k-1} \right] + \frac{h}{T_I} e_k + \frac{T_D}{h} \left[e_k - 2e_{k-1} + e_{k-2} \right] \right)$$

- It is called the *velocity form* algorithm since only $\Delta u_k = u_k - u_{k-1}$ is computed at each sampling instant.
- Can effectively and conveniently handle the *reset windup* problem.

- As $h \rightarrow 0$, digital PID $\rightarrow$ continuous PID.

In single loop controllers or local control units of DCS, 100-500 msec of h is usually used ($\approx$ continuous controller)

In computer implementations, $h \approx$ (time constant)/(10∼20) is sufficient for most applications.