Mass transfer

Lecture 17: *Mass transfer coefficients*

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Learning objectives

- **Be able to apply an appropriate (empirical) relation(s) in analyzing mass transfer along different types of fluid flow.**

Today's outline

- **Mass transfer coefficients**
  - ✓ experimental measures
  - ✓ Dimensional analysis
  - ✓ Flow inside pipes, and Ex. 17.2
  - ✓ Flow past single spheres
  - ✓ Penetration theory, and flow of drops & bubbles

17.3 Dimensional analysis

- **One can expect that the mass transfer coefficient k will depend on diffusivity and other variables affecting flow.**

$$k = f(D_v, D, u, \mu, \rho)$$

where units are k [cm/s], D_v [cm^2/s], D [cm], u [cm/s], μ [g/m/s], ρ [g/m^3]

✓ Among these, ***Sherwood number*** (*Sh*) is used to characterize *k*.

$Sh = \frac{k\ L}{D_{AB}}$ where k, L, and D_{AB} are mass transfer coefficient (m/s), characteristic length (m), and diffusivity (m^2/s)

✓ j_M **factor** is sometimes used instead to estimate *k*:

$$j_M \equiv \frac{k_c}{u}\left(\frac{\mu}{\rho\ D_v}\right)^{\frac{2}{3}}$$

17.3 Flow inside pipes

- **For turbulent-flow mass transfer to pipe walls,**
  - ✓ the following, simple relation gives a fairly good estimation of *k*:

$$Sh = 0.023\ Re^{0.8}\ Sc^{1/3}(\frac{\mu}{\mu_w})^{0.14}$$

  - ✓ Often, $\frac{\mu}{\mu_w}$~1.0. For gas-phase mass transfer,

$$j_M = \frac{k_y\ R\ T}{P\ u}\ Sc^{2/3}$$

  - ✓ For Schmidt numbers between 0.6 and 2.5, the eqn. in below is slightly more accurate:

$$Sh = 0.023\ Re^{0.81}\ Sc^{0.44}$$

  - ✓ For higher Schmidt numbers (430 to 100,000),

$$Sh = 0.0096\ Re^{0.913}\ Sc^{0.346}$$

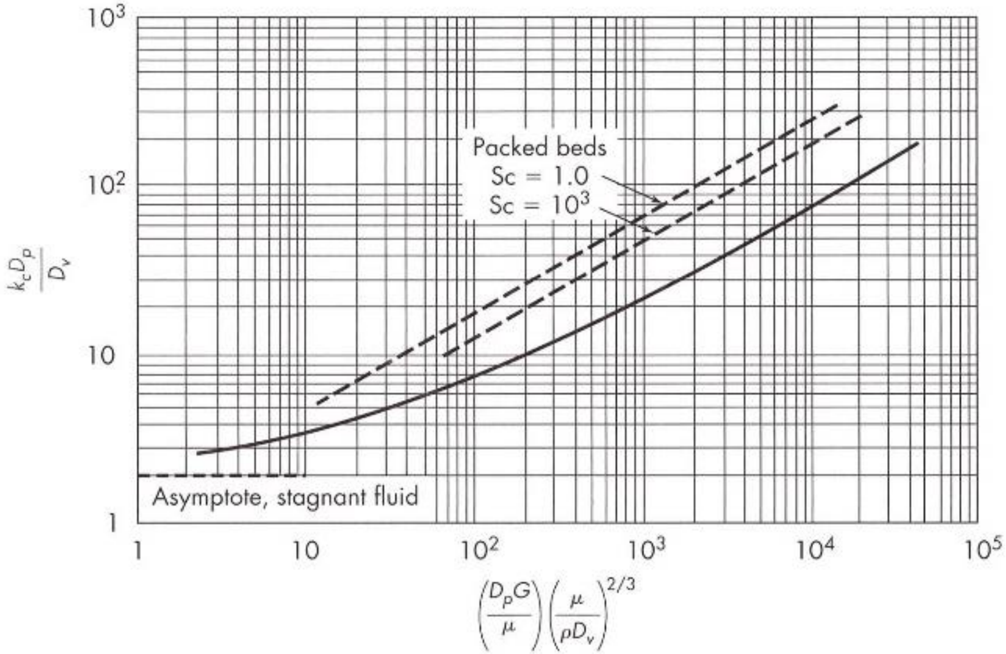
17.3 Two phase mass transfer

Ex. 17.5. (a) What is the effective thickness of gas film for the evaporation of water into air in a 50 mm diameter wetted-wall column at *Re* of 10,000 and a temperature of 40 ºC?

(b) Repeat the calculation for the evaporation of ethanol under the same conditions. At 1 atm the diffusivities are 0.288 cm^2/s for water in air and 0.145 cm^2/s for ethanol in air.

17.3 Flow past single spheres

- **The relationship between *Sh* and *Re* changes as *Re* increases from 0 to high numbers.**



- ✓ *Sh* approaches ? as *Re* approaches 0.
- ✓ For *Re* up to 1,000,

$$Sh = 2.0 + 0.6Re^{1/2}\ Sc^{1/3}$$

- ✓ The slope of the graph gradually increases for *Re* beyond 1,000.

17.3 Penetration theory

- **The change in concentration with distance and time is governed by Fick's second law:**

$$\frac{\partial c_A}{\partial t} = D_v \frac{\partial^2 c_A}{\partial b^2}$$

✓ The boundary conditions dictate $c_A = c_{A0}$ (t =0), $c_A = c_{Ai}$ (b=0; t >0)

✓ The instantaneous flux at any given time t is governed by

$$J_A = \sqrt{\frac{D_v}{\pi\, t}}\left(c_{Ai} - c_A\right)$$

✓ The average flux over the time interval 0 to t_T is

$$\overline{J_A} = \frac{1}{t_T}\int_0^{t_T} J_A \,\mathrm{d}t = \left(c_{Ai} - c_A\right)\int_0^{t_T}\sqrt{\frac{D_v}{\pi\, t}}\,\mathrm{d}t = 2\sqrt{\frac{D_v}{\pi\, t_T}}\left(c_{Ai} - c_A\right)$$

✓ Combination w/ k_c equation gives:

$$\overline{k_c} = 2\sqrt{\frac{D_v}{\pi\, t_T}}$$

✓ *Depth of penetration*, defined as the distance at which Δc_A is 1%, becomes 3.6 $\sqrt{D_v t_T}$.

17.3 Drops and bubbles

- **For a low-μ-drop falling through a viscous liquid w/out surfactants,**
  - ✓ Mass transfer between fluid and drop is governed by ? theory:

$$\overline{k_c} = 2\sqrt{\frac{D_v\, u_0}{\pi\, D_p}}$$

where D_p and u_0 are the drop-diameter, and velocity of the drop; the effective time for mass transfer is ??

  - ✓ Sherwood number becomes as follows:

$$Sh = \frac{kD_v}{D} = 2\sqrt{\frac{D_v\, u_0}{\pi\, D_p}}\frac{D_p}{D_v} = \frac{2}{\sqrt{\pi}}\sqrt{\frac{\rho\, u_0 D_p}{\mu}}\sqrt{\frac{\mu}{\rho\, D_v}} = 1.13 Re^{1/2}\, Sc^{1/2}$$

  - ✓ It is difficult to predict $\overline{k_c}$ for a practical application; volumetric mass-transfer coefficient $k_c a$ [s^{-1}], which is obtained experimentally, is often used instead