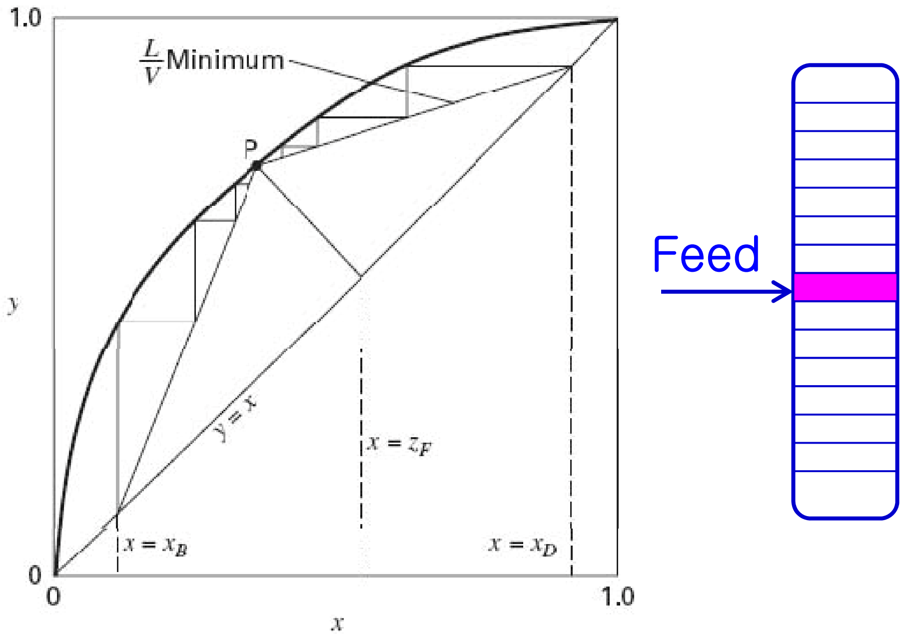
Lecture 14.
Approximate Multicomponent Methods (2)
[Ch. 9]

- Minimum Reflux
  - In binary systems
  - In multicomponent systems
  - Underwood equation
- Gilliland Correlation for Actual Reflux Ratio and Theoretical Stages
- Feed-Stage Location
- Distribution of Nonkey Components at Actual Reflux

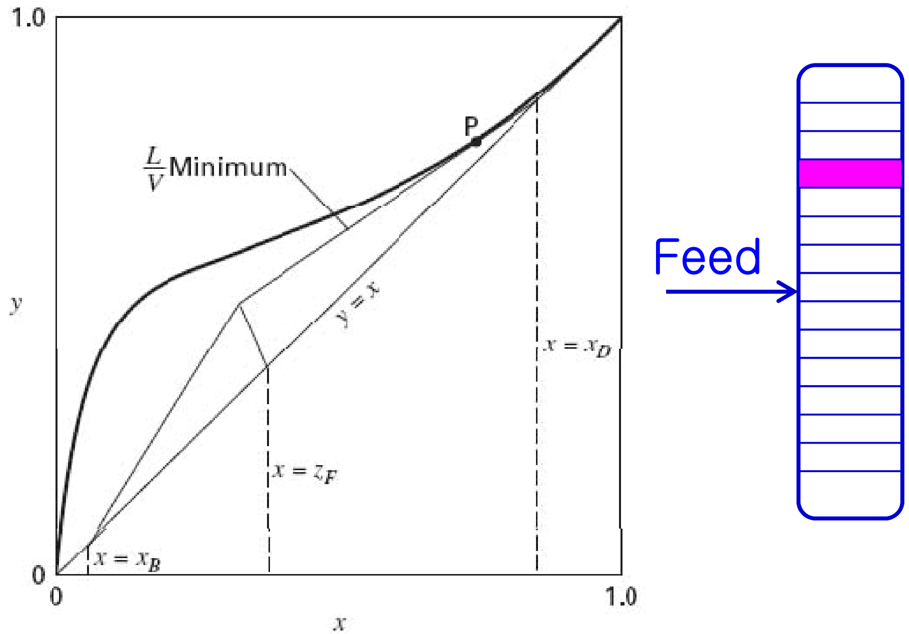
Minimum Reflux in Binary Systems

- Minimum reflux
  - Infinite stages
  - Pinch point (or point of infinitude)
  - Most of the stages are crowded into a constant-composition zone

Binary system, ideal conditions



Binary system, nondeal conditions

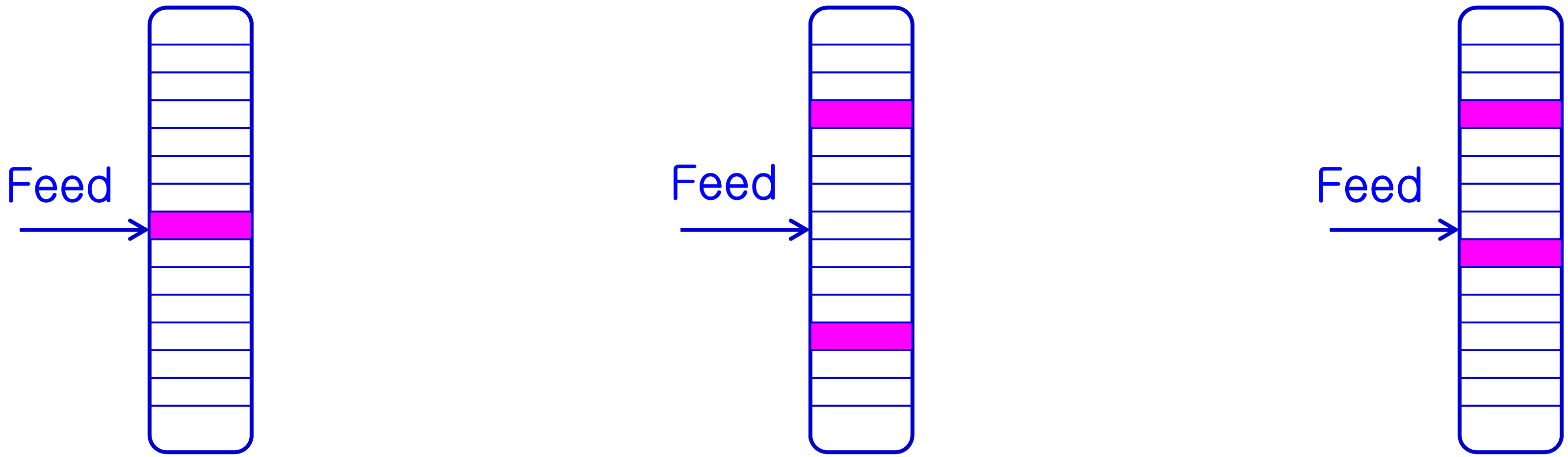


: location of pinch-point zone

Minimum Reflux in Multicomponent Systems

- Class 1 (one pinch point): all components in the feed distribute to both the distillate and bottoms products (narrow-boiling-range mixtures or the degree of separation between key components is not sharp)
- Class 2 (two pinch points): one or more of the components appear in only one product

All components distributed (Class 1)	Not all LLK and HHK distributing (Class 2)	All LLK, if any, distributing, but not all HHK distributing (Class 2)



LLK: lighter than light key; HHK: heavier than heavy key

: location of pinch-point zones

Underwood Equation for Minimum Reflux (1)

Material balance over all stages

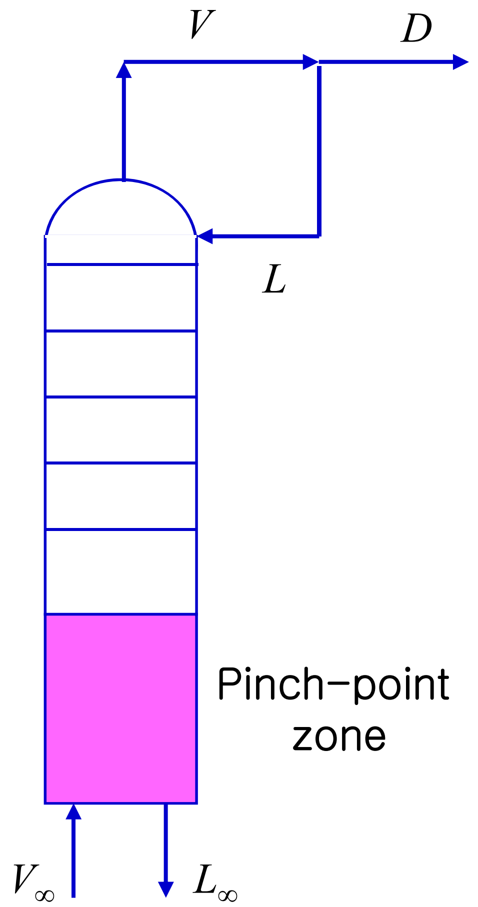
$$y_{i,\infty}V_\infty = x_{i,\infty}L_\infty + x_{i,D}D \qquad y_{j,\infty}V_\infty = x_{j,\infty}L_\infty + x_{j,D}D$$

$$V_\infty = L_\infty + D$$

Phase equilibrium relation
(phase compositions do not change in the pinch zone)

$$y_{i,\infty} = K_{i,\infty}x_{i,\infty} \qquad y_{j,\infty} = K_{j,\infty}x_{j,\infty}$$

$$\frac{L_\infty}{D} = \frac{\left[(x_{i,D}/x_{i,\infty}) - (\alpha_{i,j})_\infty (x_{j,D}/x_{j,\infty})\right]}{(\alpha_{i,j})_\infty - 1}$$



For Class 1 separation, flashed feed- and pinch-zone compositions are identical $(x_{i,\infty} = x_{i,F})$

$$\frac{(L_\infty)_{\min}}{F} = \frac{(L_F/F)\left[(Dx_{LK,D}/L_F x_{LK,F}) - (\alpha_{LK,HK})_F (Dx_{HK,D}/L_F x_{HK,F})\right]}{(\alpha_{LK,HK})_F - 1}$$

Underwood Equation for Minimum Reflux (2)

- Distribution of nonkey components

$$\frac{(L_\infty)_{\min}}{F} = \frac{(L_F/F)\left[(Dx_{LK,D}/L_F x_{LK,F}) - (\alpha_{LK,HK})_F (Dx_{HK,D}/L_F x_{HK,F})\right]}{(\alpha_{LK,HK})_F - 1}$$

Replace ***LK*** with component ***i***

$$\frac{(L_\infty)_{\min}}{F} = \frac{(L_F/F)\left[(Dx_{i,D}/L_F x_{i,F}) - (\alpha_{i,HK})_F (Dx_{HK,D}/L_F x_{HK,F})\right]}{(\alpha_{i,HK})_F - 1}$$

$$\Rightarrow \frac{Dx_{i,D}}{L_F x_{i,F}} = \left[\frac{(\alpha_{i,HK})_F - 1}{(\alpha_{LK,HK})_F - 1}\right]\left(\frac{Dx_{LK,D}}{L_F x_{LK,F}}\right) + \left[\frac{(\alpha_{LK,HK})_F - (\alpha_{i,HK})_F}{(\alpha_{LK,HK})_F - 1}\right]\left(\frac{Dx_{HK,D}}{L_F x_{HK,F}}\right)$$

$$0 < \left(\frac{Dx_{i,D}}{Fx_{i,F}}\right) < 1$$

For all nonkey components in a Class 1 separation

Gilliland Correlation for Actual Reflux Ratio and Theoretical Stages

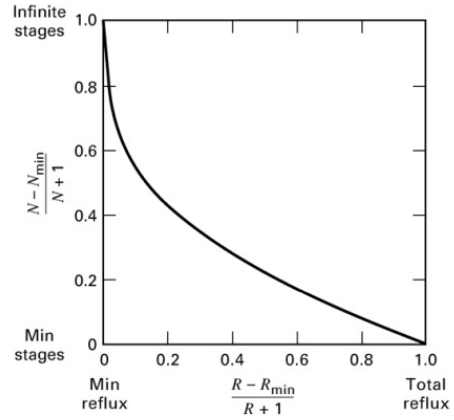
- Gilliland correlation : empirical correlation

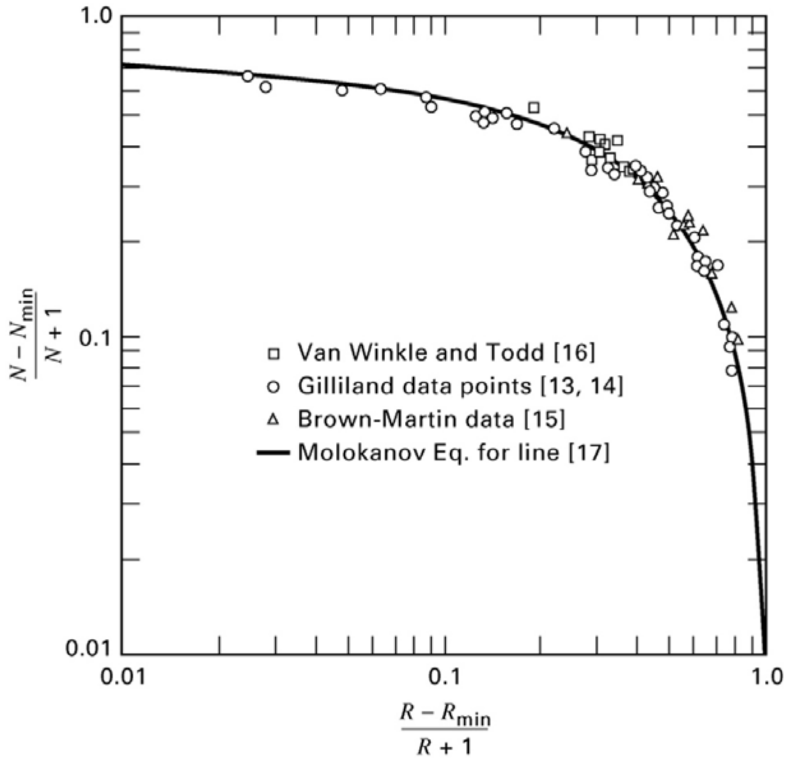
$$N = N\{N_{\min}(x_{i,D}, x_{i,B}, \alpha),\ R_{\min}(z_{i,F}, x_{i,D}, q, \alpha),\ R\}$$

$$Y = \frac{N - N_{\min}}{N+1}$$

$$= 1 - \exp\left[\left(\frac{1+54.4X}{11+117.2X}\right)\left(\frac{X-1}{X^{0.5}}\right)\right]$$

$$X = \frac{R - R_{\min}}{R+1}$$





- The value of N includes one stage for a partial reboiler and one stage for a partial condenser, if used
- A small initial increase in R above $R_{\min}$ causes a large decrease in N, but further changes in R have a much smaller effect on N

Feed-Stage Location

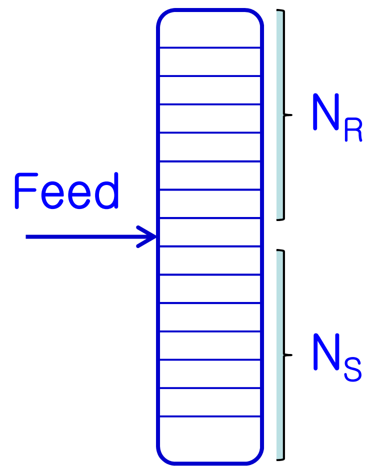
- Suggestion of Brown and Martin (1939)
  - Based on the Fenske equation (at total reflux conditions)
  - Not reliable except for fairly symmetrical feeds and separations

$$\frac{N_R}{N_S} \approx \frac{(N_R)_{\min}}{(N_S)_{\min}}$$

$$= \frac{\log\left[(x_{LK,D}/z_{LK,F})(z_{HK,F}/x_{HK,D})\right]\log[(\alpha_B\alpha_F)^{1/2}]}{\log\left[(z_{LK,F}/x_{LK,B})(x_{HK,B}/z_{HK,F})\right]\log[(\alpha_D\alpha_F)^{1/2}]}$$

- Empirical equation of Kirkbride

$$\frac{N_R}{N_S} = \left[\left(\frac{z_{HK,F}}{z_{LK,F}}\right)\left(\frac{x_{LK,B}}{x_{HK,D}}\right)^2\left(\frac{B}{D}\right)\right]^{0.206}$$



Distribution of Nonkey Components at Actual Reflux

- For multicomponent mixtures
  - Total reflux condition: all components distribute to some extent between distillate and bottoms
  - Minimum reflux condition: none or only a few of the nonkey components distribute
  - Reflux ratio near minimum: the product distribution lies between the two limits
  - High reflux ratio: the product distribution may lie outside the limits → inferior separation