19.슈뢰딩거 고양이

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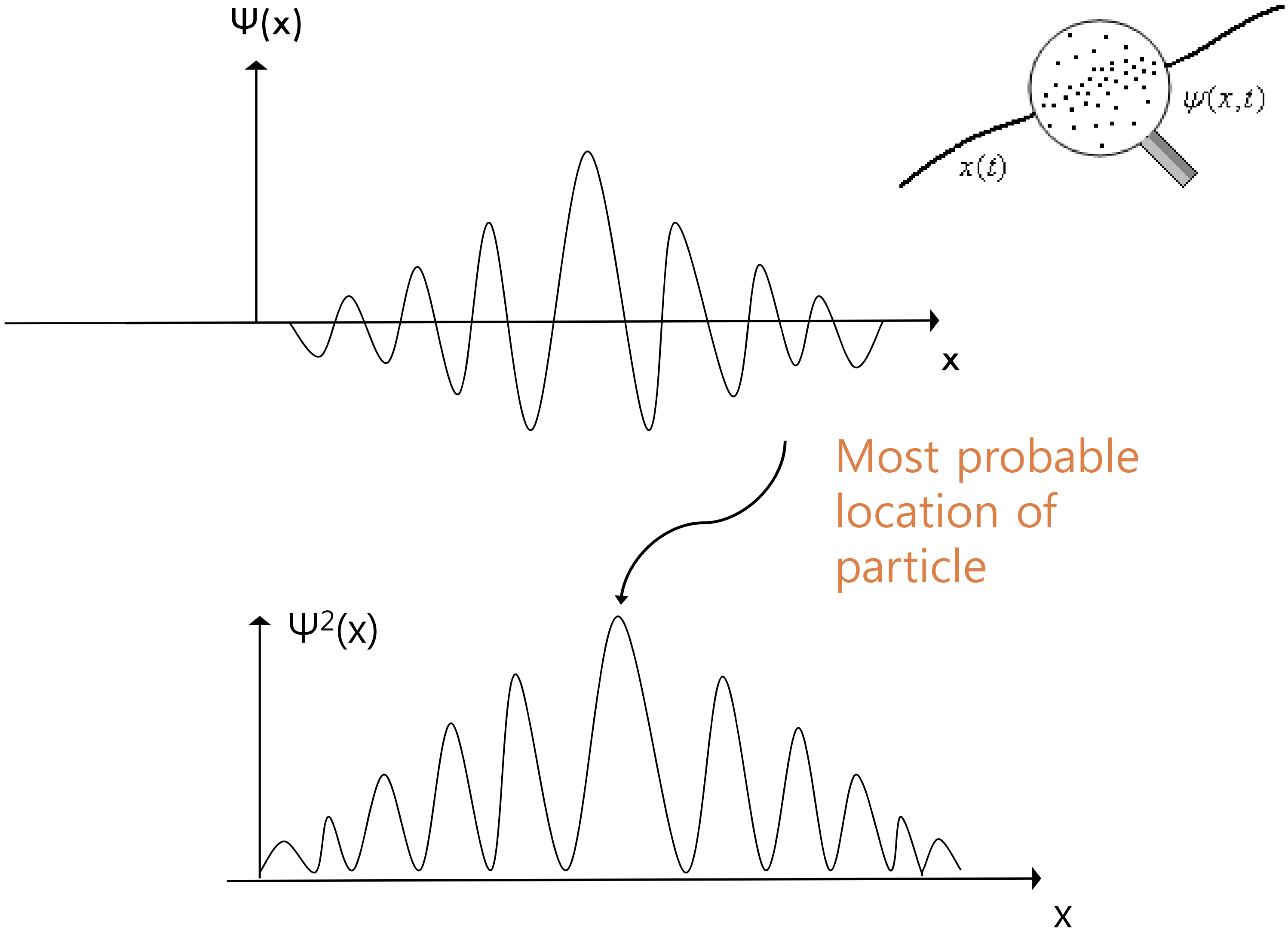
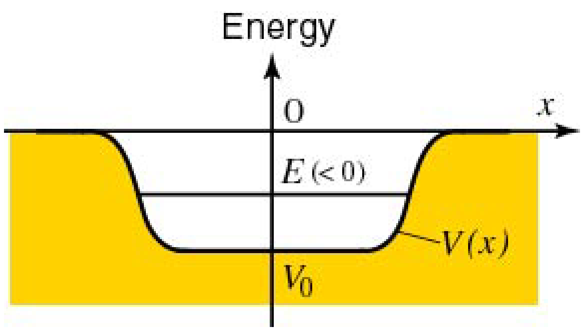
Wave fun’t & probability



Square-well potential

- Consideration
  - Particle is bound in square-well
  - Potential (V) is zero at long range



$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

$$0 > E > V_0, \quad (V_0 < 0).$$

$$\frac{d^2\psi}{dx^2} - K^2\psi = 0, \qquad K^2 = \frac{2m\,|E|}{\hbar^2}.$$

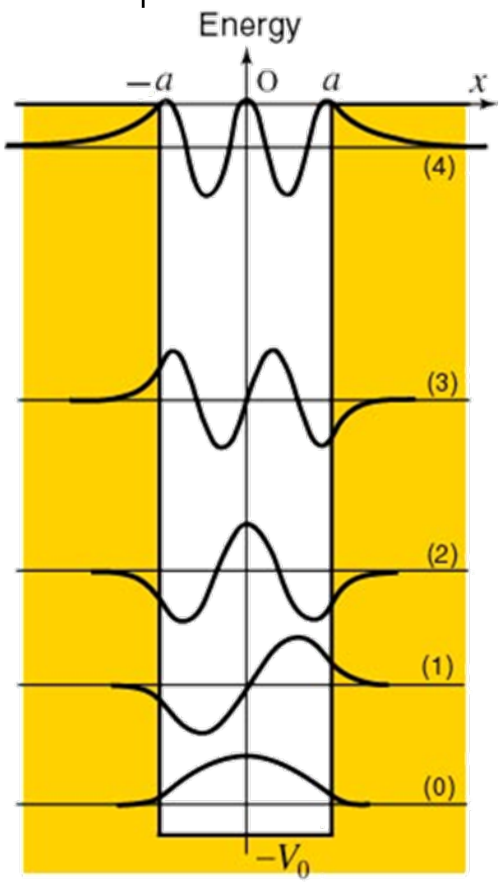
$$\psi(x) = A\,e^{Kx} + B\,e^{-Kx}, \qquad K = \frac{\sqrt{2m\,|E|}}{\hbar},$$

  - General solution:

$$\psi(x) = \begin{cases} B\,e^{-Kx} & \text{(in the region of } V(x) = 0 \text{ for } x > 0), \\ A\,e^{Kx} & \text{(in the region of } V(x) = 0 \text{ for } x < 0). \end{cases}$$

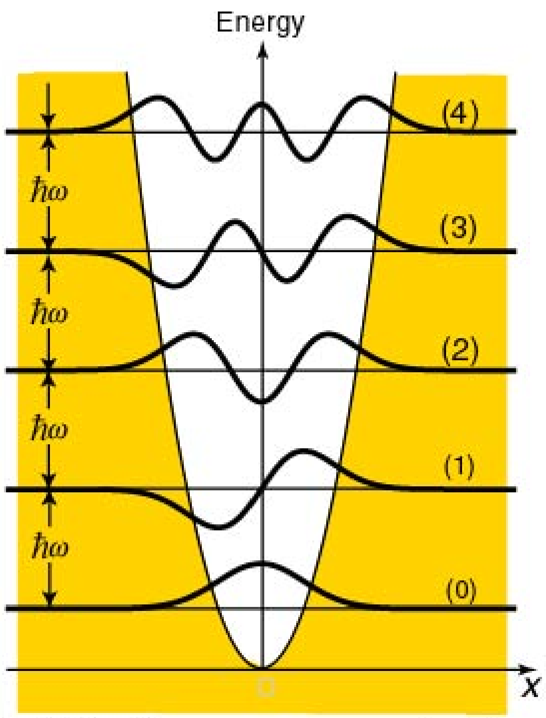
Eigenstate

- Square-well
  - The number of the possible bound states depends on the value of a^2V_0



The thick solid curves are the wave functions.
(0) : the ground state
(1) (2) (3) . . . : the excited states

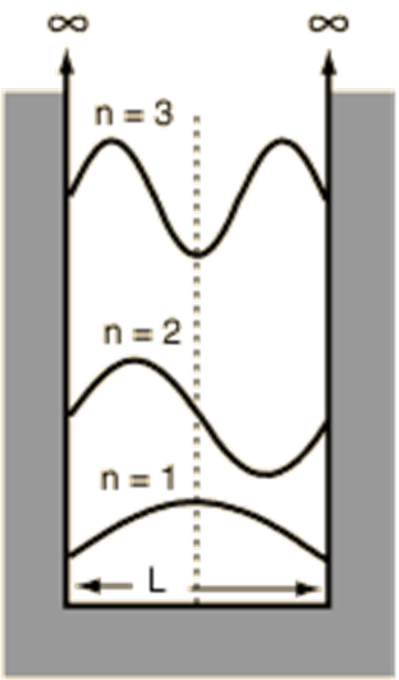
- Harmonic oscillator
  - Energy eigenvalue is $\hbar\omega$



The thick solid curves are the wave functions.
(0) : the ground state
(1) (2) (3) . . . : the excited states

Particle in box

- 박스 밖에서 입자 발견할 확률 0인 경우
  - 벽에서의 파동함수는 zero! : 정상파



x = 0 at left wall of box.

$$\Psi(x) = A\sin kx, \quad \Psi(0) = \Psi(L) = 0 \qquad k = \frac{n\pi}{L}$$

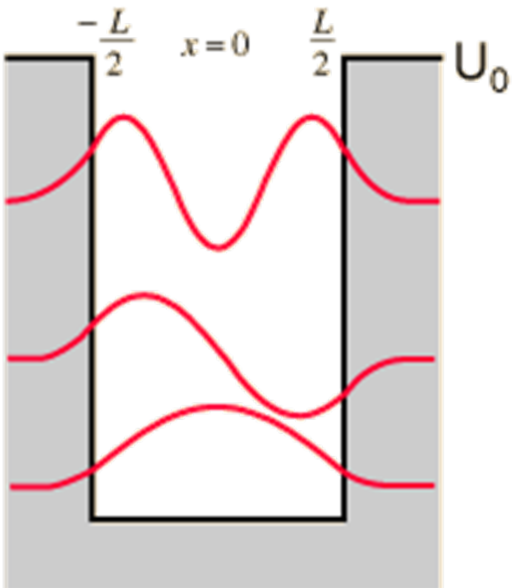
$$\int \Psi^*\Psi dx = \int_0^L A_n^2 \sin^2\left(\frac{n\pi}{L}x\right)dx = 1$$

$$= A_n^2\left[\frac{x}{2} - \frac{\sin\left(\frac{n\pi}{L}x\right)}{4n\pi/L}\right]_0^L \qquad A_n^2\frac{L}{2} = 1$$

$$\Psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x \qquad n = 1,2,3,\ldots$$

Finite potential well

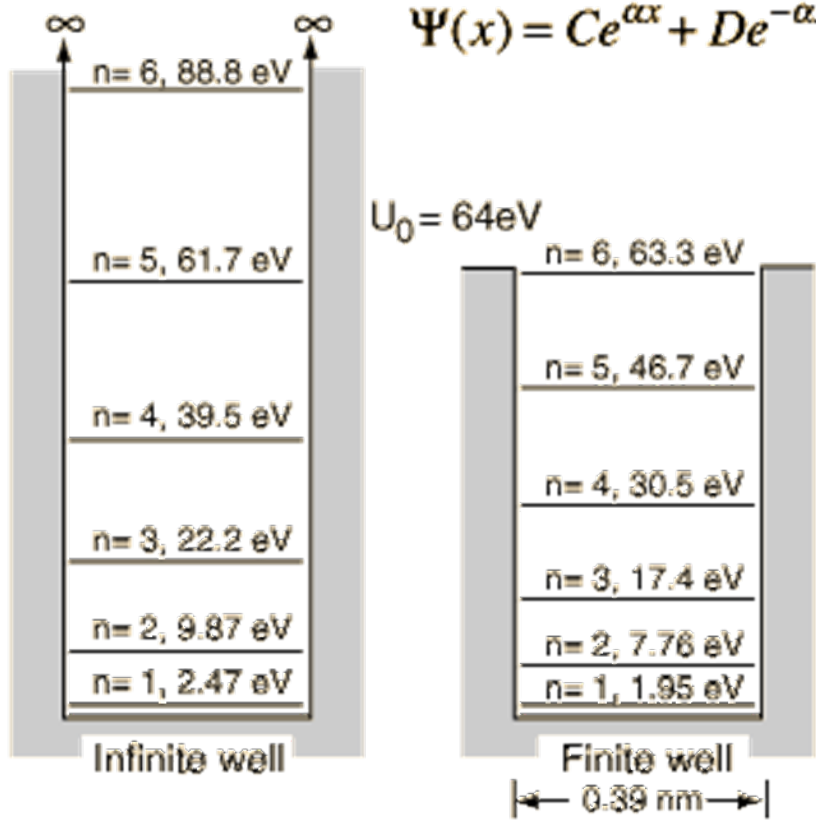
$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2} = (E - U_0)\Psi(x) \qquad \alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} = \sqrt{\beta^2 - k^2}$$



$$\frac{\partial^2\Psi(x)}{\partial x^2} = \alpha^2\Psi(x)$$

with general solution of form:

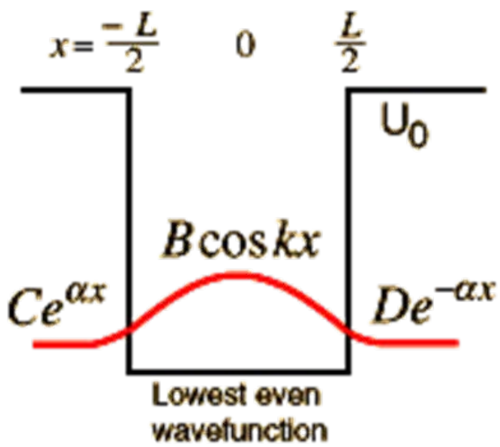
$$\Psi(x) = Ce^{\alpha x} + De^{-\alpha x}$$



$$\beta = \sqrt{\frac{2mU_0}{\hbar^2}}$$

$$E = \frac{\hbar^2 k^2}{2m}$$

Even solution of finite box



The condition of continuity for the wavefunction at the boundaries gives:

$$Ce^{-\alpha L/2} = B\cos(-kL/2)$$
$$De^{-\alpha L/2} = B\cos(kL/2)$$

so $C = D$

The condition of continuity for the derivative of the wavefunction gives:

$$\alpha Ce^{-\alpha L/2} = -kB\sin(-kL/2)$$
$$\alpha De^{-\alpha L/2} = -kB\sin(kL/2)$$

Dividing either of these two sets gives:

$$\alpha = k\tan\frac{kL}{2} = \sqrt{\beta^2 - k^2}$$

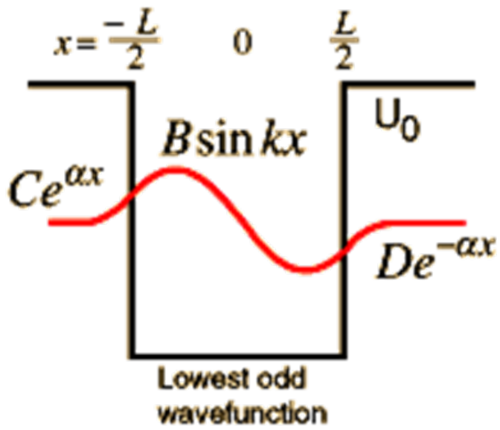
$$\alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} = \sqrt{\beta^2 - k^2}$$

Odd solution of finite box

The condition of continuity for the wavefunction at the boundaries gives:

$$Ce^{-\alpha L/2} = B\sin(-kL/2)$$
$$De^{-\alpha L/2} = B\sin(kL/2)$$

so $C = -D$



The condition of continuity for the derivative of the wavefunction gives:

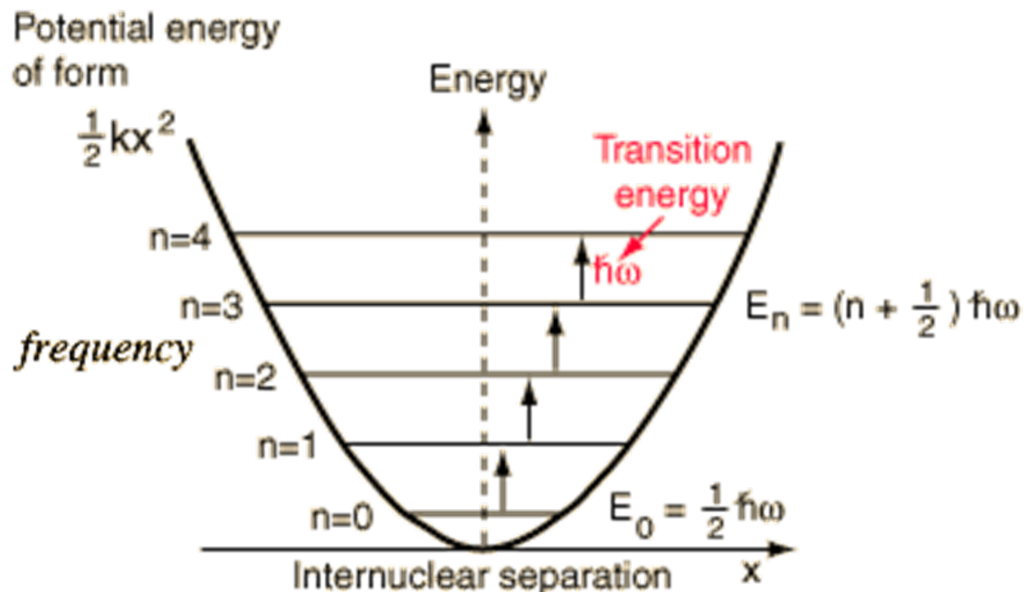
$$\alpha Ce^{-\alpha L/2} = kB\cos(-kL/2)$$
$$-\alpha De^{-\alpha L/2} = kB\cos(kL/2)$$

Dividing either of these two sets gives:

$$\alpha = \frac{-k}{\tan\frac{kL}{2}} = \sqrt{\beta^2 - k^2}$$

$$\alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} = \sqrt{\beta^2 - k^2}$$

Quantum harmonic oscillator



$$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$$

$$\omega = \sqrt{\frac{k}{m}} = \textit{angular frequency}$$

$$\omega = 2\pi \cdot \textit{frequency}$$

$$\Psi(x) = Ce^{-\alpha x^2/2}$$

$$\frac{d\Psi}{dx} = -C\frac{\alpha}{2}e^{-\alpha x^2/2}2x$$

$$\frac{d^2\Psi}{dx^2} = -C\alpha e^{-\alpha x^2/2} + C\alpha^2x^2e^{-\alpha x^2/2}$$

$$\frac{-\hbar^2}{2m}\left[-\alpha + \alpha^2x^2\right]\Psi + \frac{1}{2}m\omega^2x^2\Psi = E\Psi$$

$$\frac{-\hbar^2\alpha^2}{2m} + \frac{1}{2}m\omega^2 = 0 \qquad \alpha = \frac{m\omega}{\hbar}$$

Quantum harmonic oscillator

$$E = \frac{(\Delta p)^2}{2m} + \frac{1}{2} m\omega^2 (\Delta x)^2$$

Δx = position uncertainty

Δp = momentum uncertainty

$$\Delta x \Delta p = \frac{\hbar}{2}$$

$$E = \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2} m\omega^2 (\Delta x)^2$$

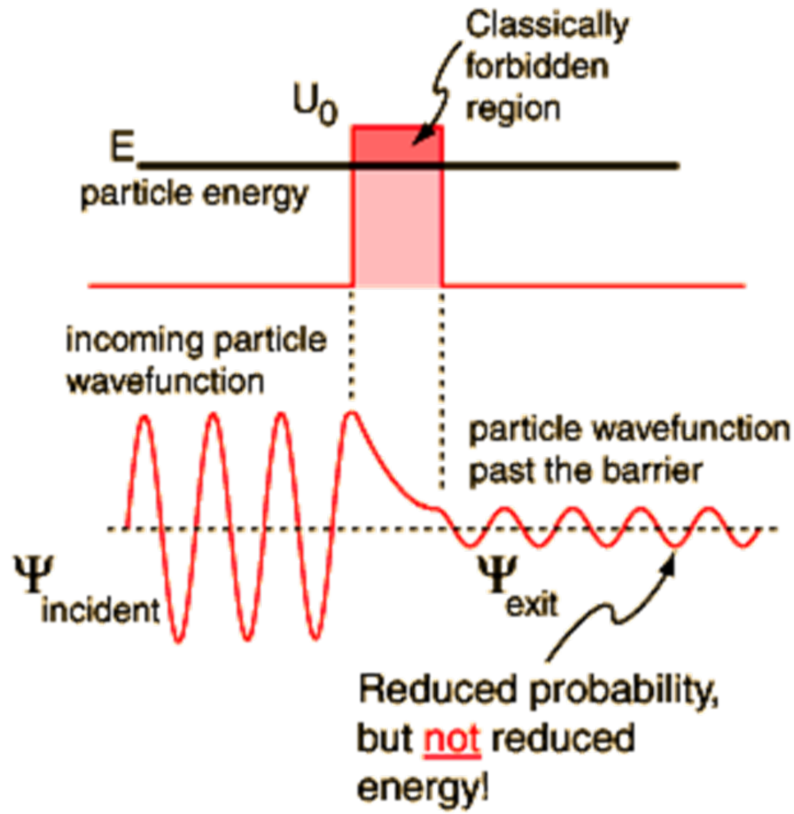
Minimizing this energy by taking the derivative with respect to the position energy and setting it equal to zero gives

$$-\frac{\hbar^2}{4m(\Delta x)^3} + m\omega^2 \Delta x = 0 \qquad \Delta x = \sqrt{\frac{\hbar}{2m\omega}}$$

$$E_0 = \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2} m\omega^2 (\Delta x)^2 = \frac{\hbar\omega}{4} + \frac{\hbar\omega}{4} = \frac{\hbar\omega}{2}$$

Barrier penetration

- Tunneling effect
  - According to classical physics, a particle of energy E less than the height U_o of a barrier could not penetrate
  - The wavefunction associated with a free particle must be continuous at the barrier and will show an exponential decay inside the barrier

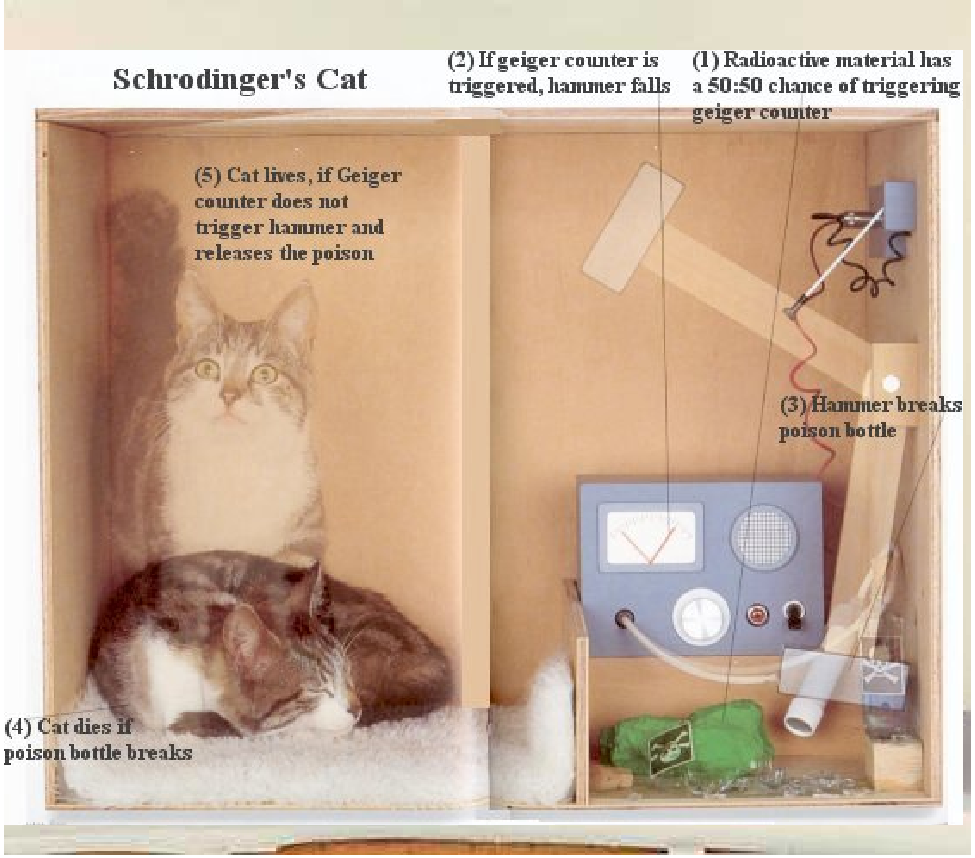


$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2} = (E - U_0)\Psi(x)$$

$$\Psi = Ae^{-\alpha x} \quad where \quad \alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$$

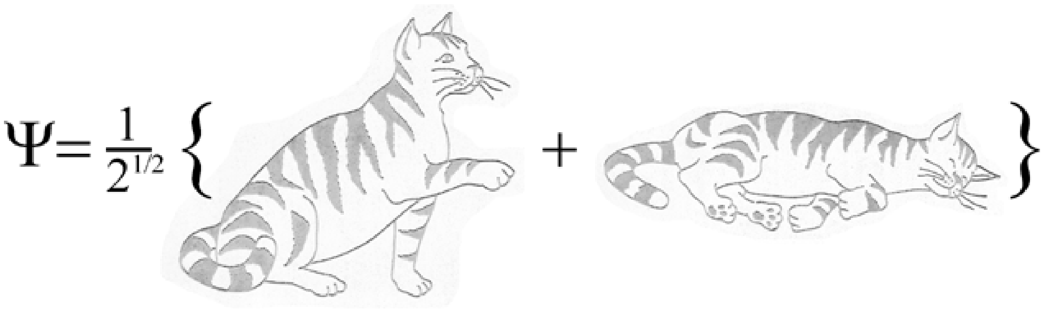
슈뢰딩거 고양이

- 미시세계의 거시세계로의 확장
  - 상자속 고양이 + ½확률로 분해되는 알파입자 가속기 + 청산가리 → 1시간 후 고양이의 생사?



고양이의 생과 사

- 생과 사는 ½ 확률이 아닌, 중첩 상태이다!!!



- 거시세계는 decoherence