15.드 브로이-물질파

화공과 김영훈 교수

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드브로이 정상파

- Standing wave: 보어 모델 해석의 근간 마련
  - Bohr’s assumptions in his hydrogen atom model
    - Angular momentum of the electron in a stationary state is nħ

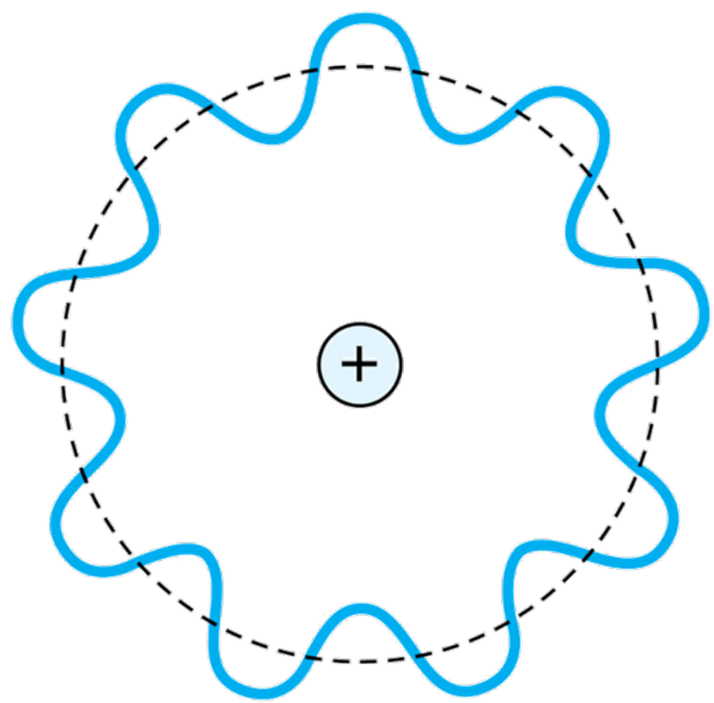
electron de Broglie wavelength

$$2\pi r = n\lambda = n\frac{h}{p}$$

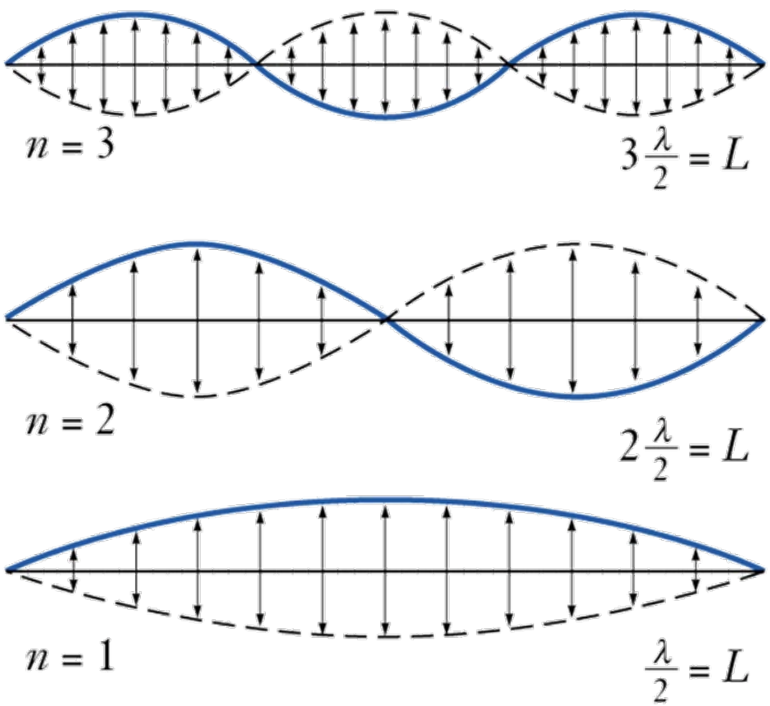
Circumference

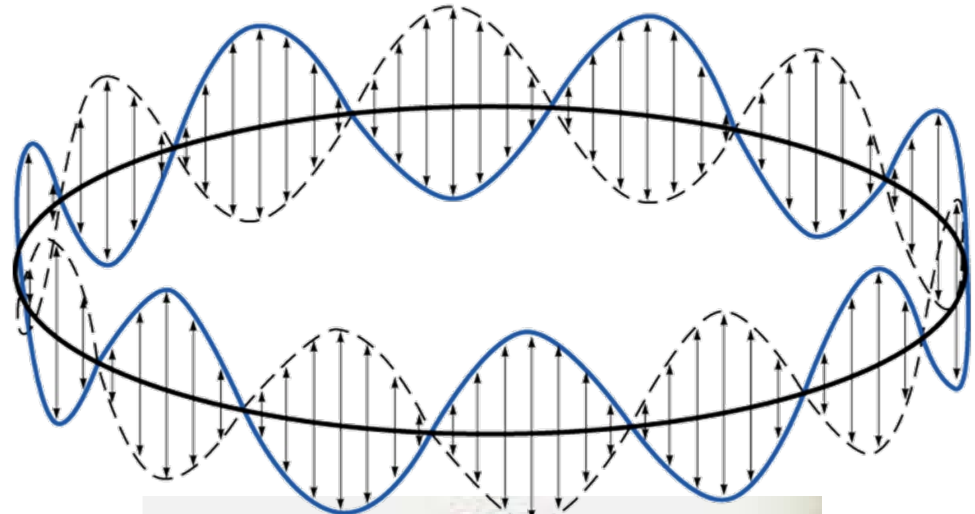
Multiplying by $p/2\pi$, we find the angular momentum:

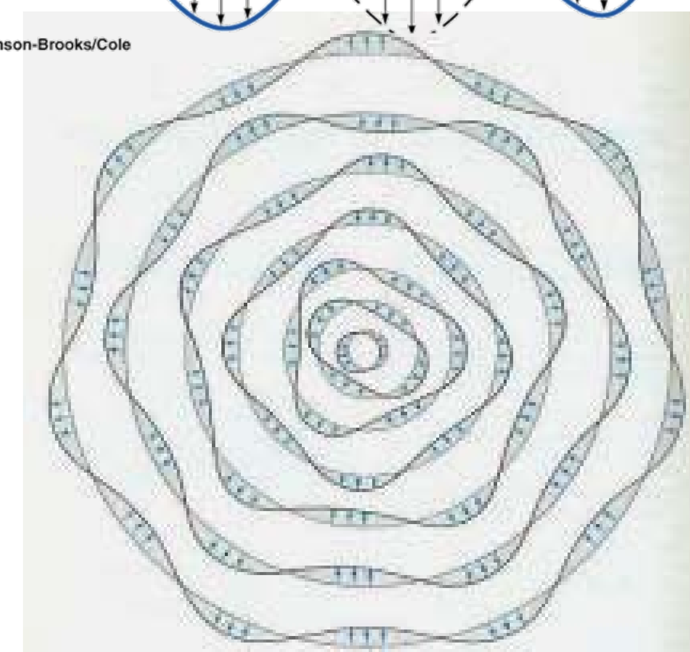
$$L = rp = \frac{nh}{2\pi} = n\hbar$$



원자궤도내 정상파





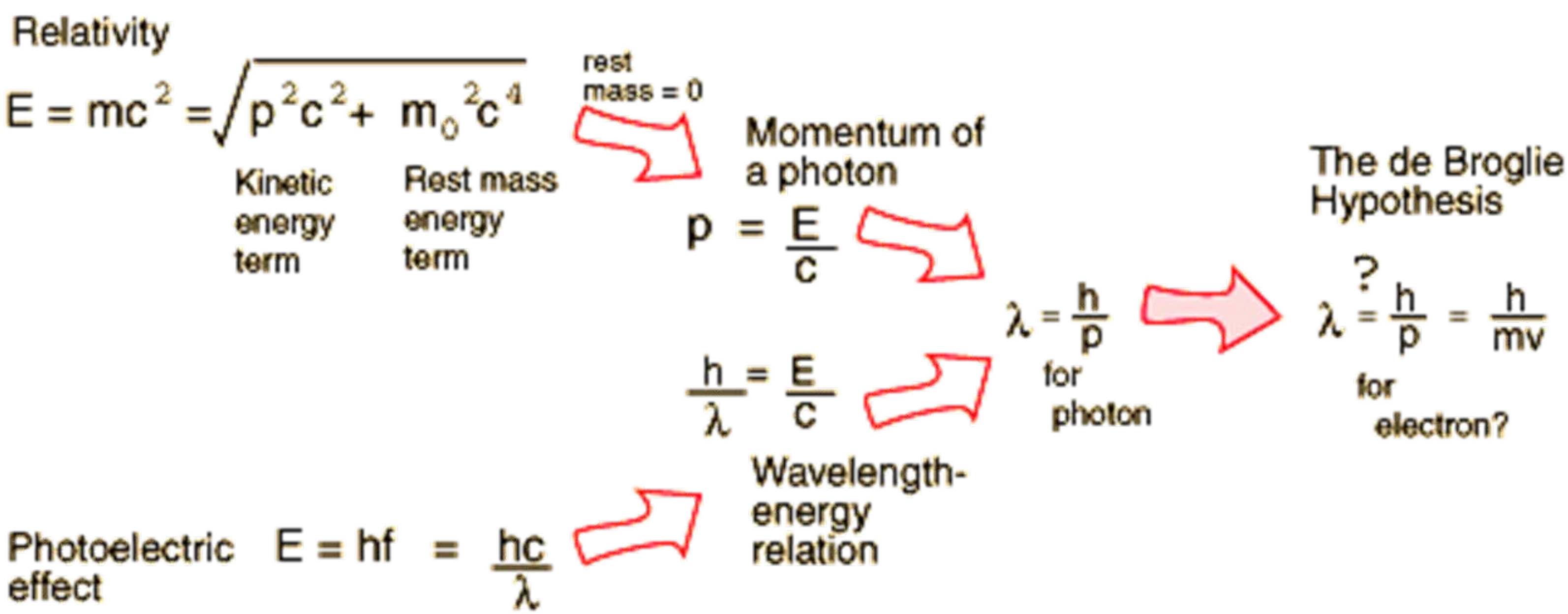


Louis-Victor de Broglie 왕자

- 드 브로이의 물질파(matter wave)
  - 아인슈타인의 광전효과
    - 빛의 파동성에서 입자성으로 전환 계기
  - 드 브로이의 제안
    - 입자성만 있는 것으로 믿어왔던 물질도 파동성을 가지고 있다
    - 입자가 어떤 속력로 운동할 때 나타나는 파장 λ는 1/(mv) 에 반비례한다
    - 즉, 어떤 입자가 속력을 가지고 운동을 하고 있다면, 파동적 성격을 가지게 된다
    - 박사 학위 논문으로 노벨상을 받은 드 브로이

물질파 공식 도출

- Relativistic energy 개념 이용



Special case: v<<c

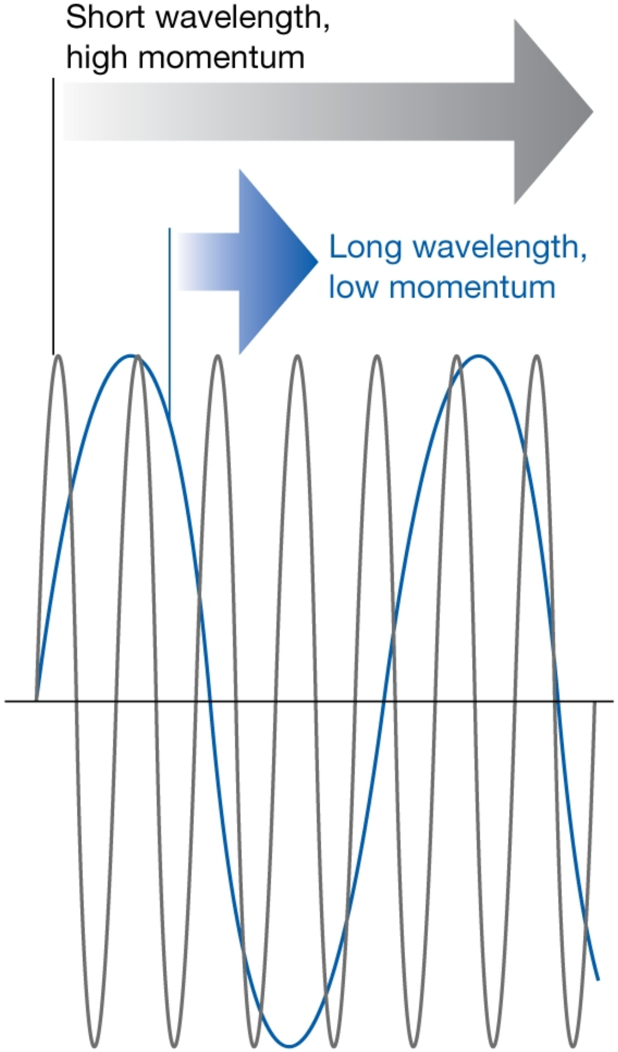
$$KE = m_0 c^2 \left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right]$$

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \ldots$$

$$\left(1-\frac{v^2}{c^2}\right)^{-1/2} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{\frac{-1}{2}\frac{-3}{2}}{2!}\frac{v^4}{c^4} + \ldots$$

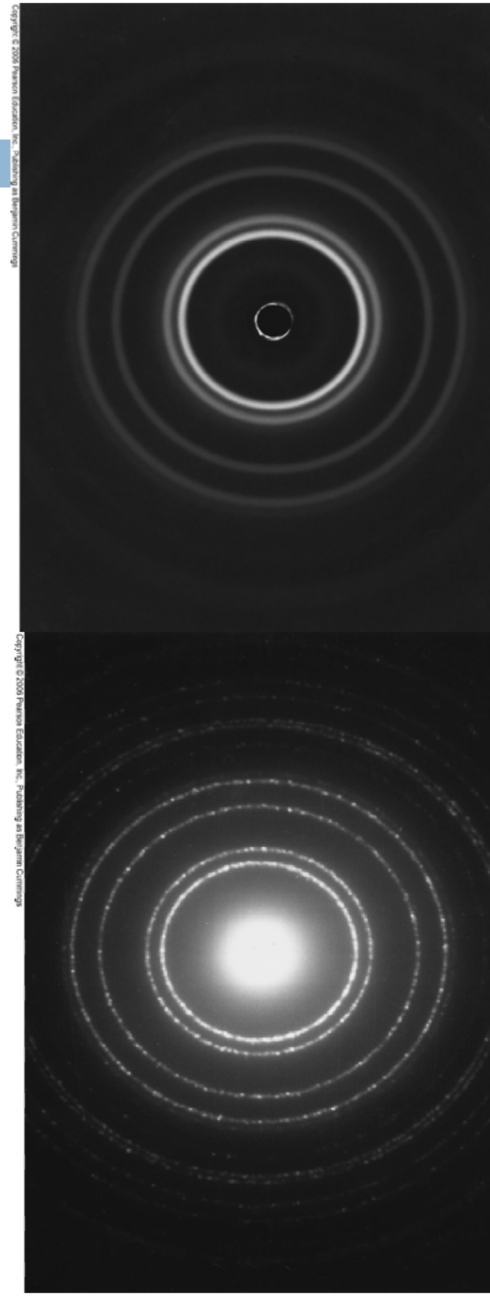
$$KE = \frac{1}{2}m_0 v^2 + \frac{3}{8}\frac{m_0 v^4}{c^2} + \frac{5}{16}\frac{m_0 v^6}{c^4} + \ldots$$

$$KE \approx \frac{1}{2}m_0 v^2 \quad for \quad v << c$$



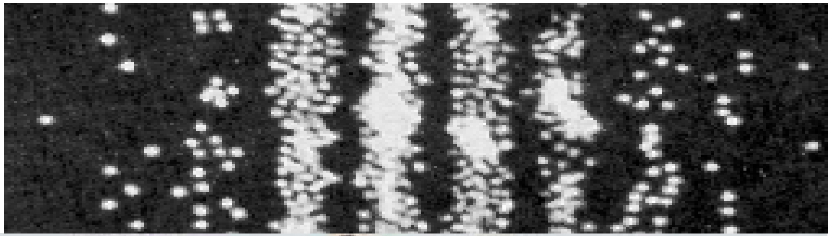
드 브로이 가설 입증 실험

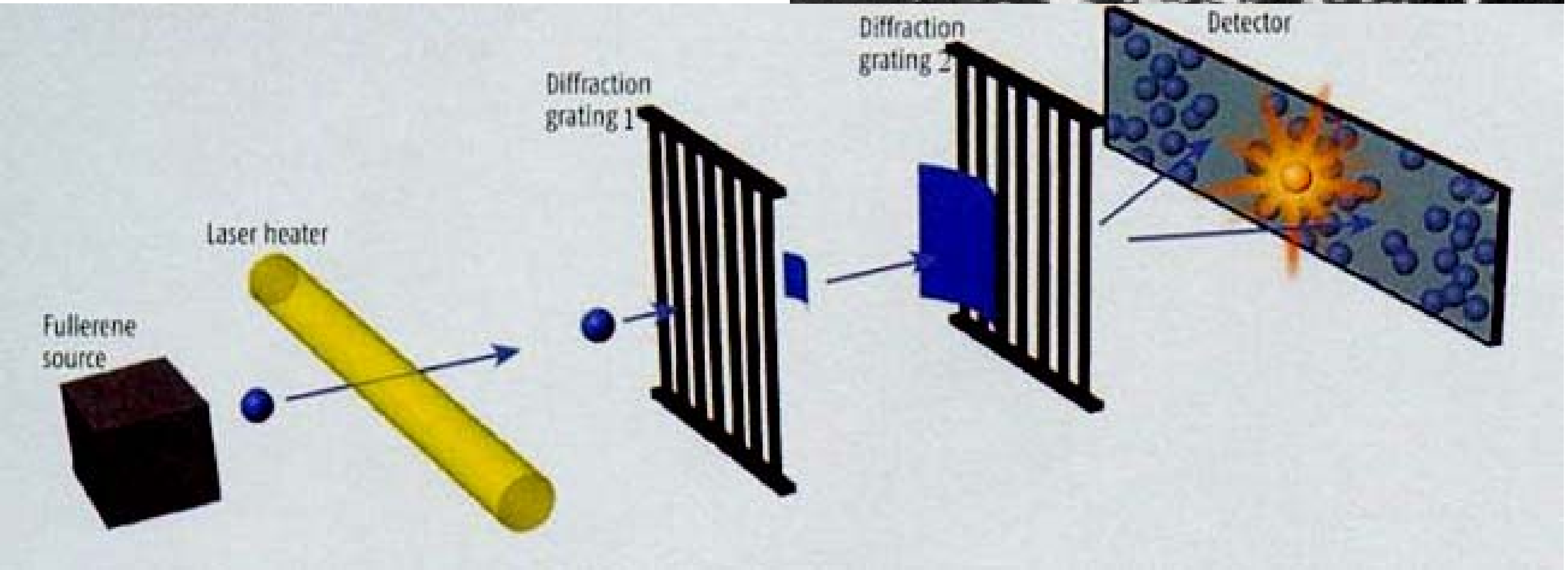
- Davisson-Germer의 실험(USA)
  - 파동에서만 나타나는 회절 현상이 전자에 의해서도 나타난다는 사실을 실험으로 우연하게 확인
  - 드브로이의 물질파 가설을 모르고 있는 상태에서 serendipity
  - 전자를 금속에 쏘여서 반사되는 전자가 특정한 방향으로 무척 강하다는 것을 관찰
  - 드 브로이의 가설이 입증, 노벨 물리학상
- G. P. Thompson의 실험(J. J. Thompson 아들)(UK)
  - 1년뒤, X선에 의한 회절(top)과 X선과 파장이 같은 전자에 의한 회절(bottom)이 거의 같음을 확인



물질파 회절

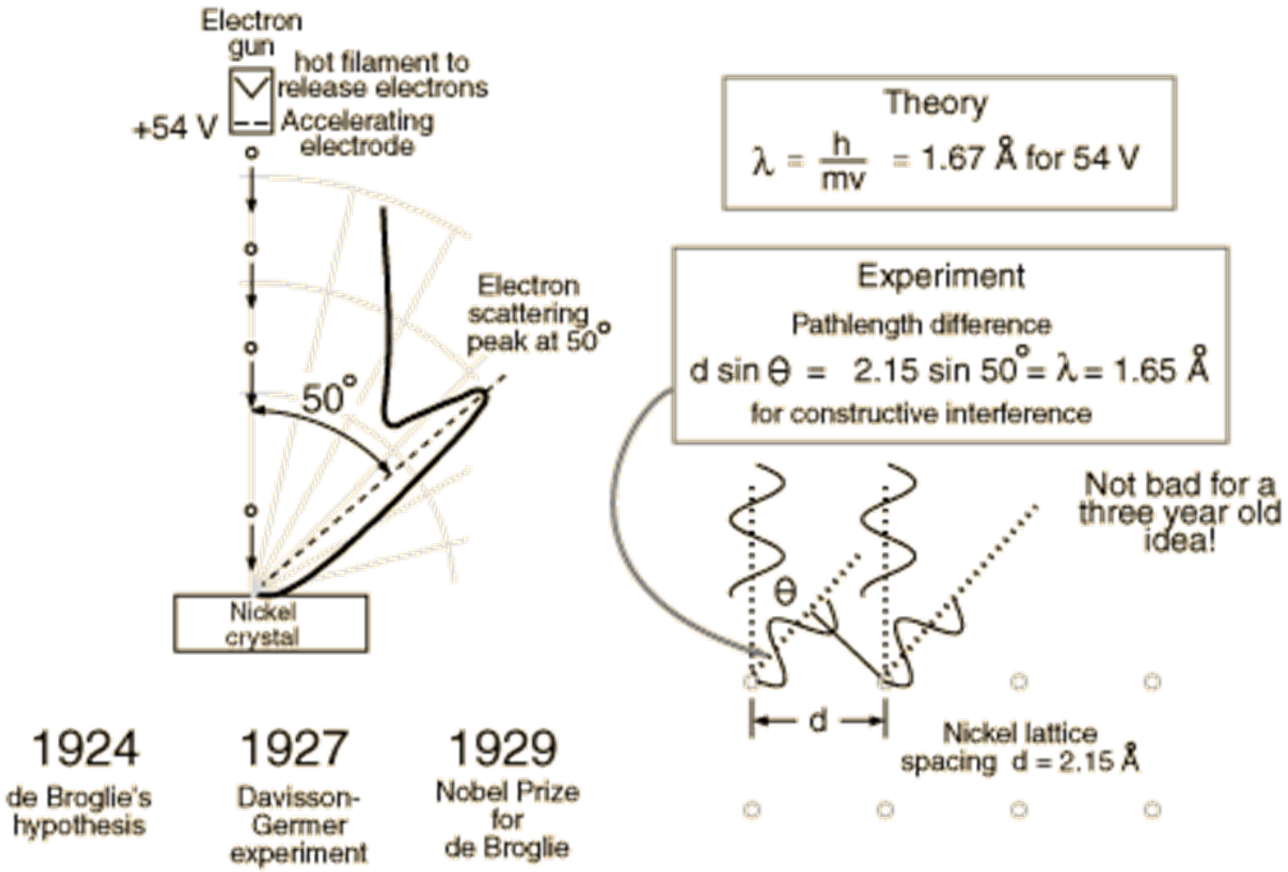
- C60을 이용한 물질파 확인
  - 1 nm 직경 입자를 레이저히터 통과하는 회절판에 조사시, 파동과 같은 회절 확인



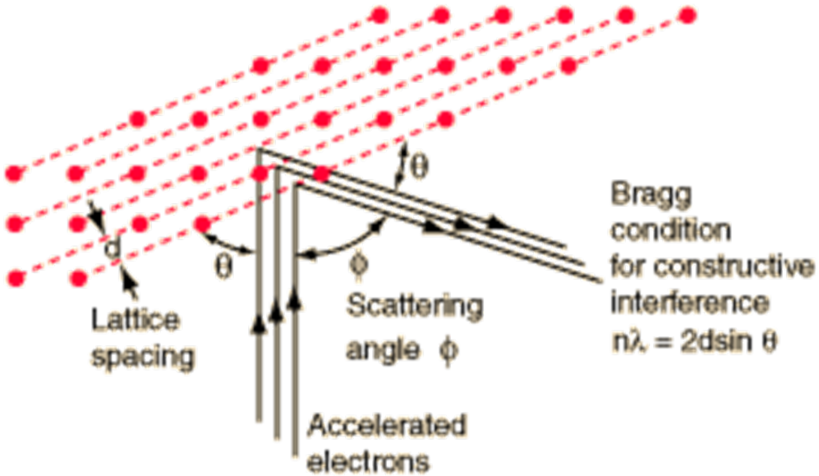


Davisson-Germer 실험

- 물질파 회절 피크 해석에 파동에서 적용되는 Bragg식이 적용됨



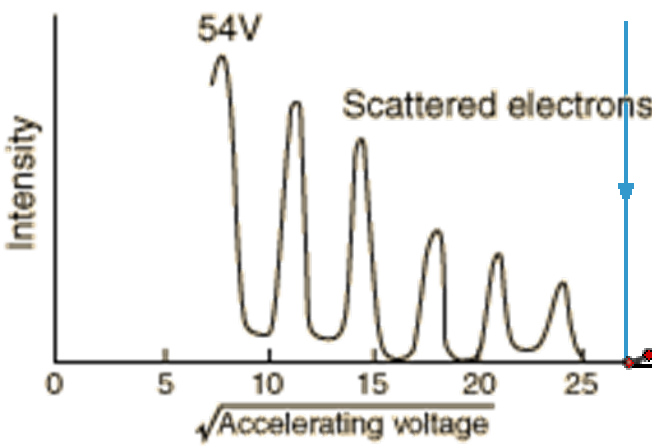
가속 전압에 따른 회절강도 변화

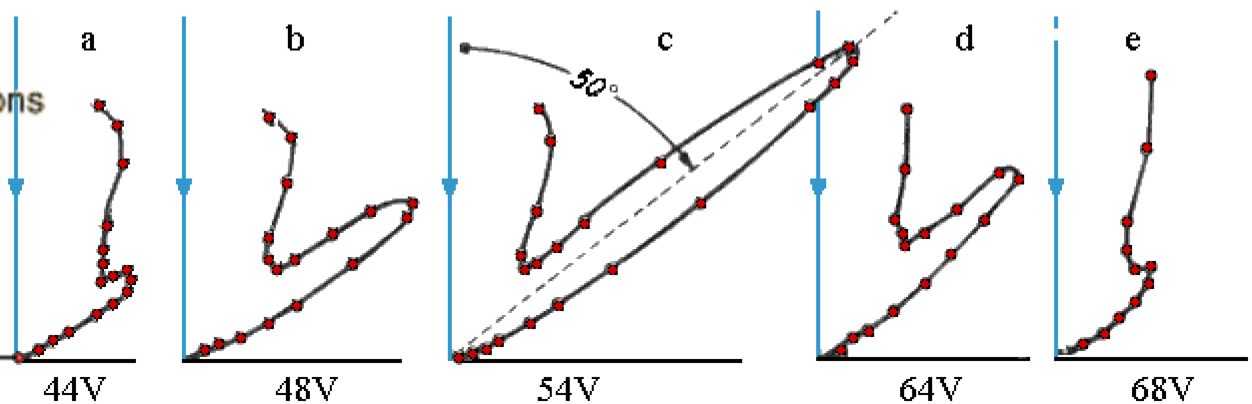


$$\frac{1}{\lambda} = \frac{n}{2d\sin\theta} = \frac{p}{h} = \frac{\sqrt{2mE}}{h} = \frac{\sqrt{2meV}}{h}$$

Electron wavelength | *Bragg law* | *deBroglie relationship* | *Acceleration through voltage V*

$$\frac{1}{\lambda(nm)} = \frac{n}{2d\sin\theta} = 0.815\sqrt{V}$$





야구공의 파장 계산

- 거시세계의 입자(야구공)은 파장이 너무 작아서 물질파 성질이 나타나지 않음
- 전자는 속도가 빨라서 빛 파장영역의 파장을 지님 → 물질파 특성 보임

Does this relationship apply to all particles? Consider a pitched baseball:

$\longrightarrow$ $v = 40\ \text{m/s} = 90\ \text{mi/hr}$

m=0.15 kg

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \text{J s}}{(0.15\ \text{kg})(40\ \text{m/s})} = 1.1 \times 10^{-34}\ \text{m}$$

10^{-10}m Atomic diameter

10^{-14}m Nuclear Diameter

For an electron accelerated through 100 Volts: $v = 5.9 \times 10^{6}$ m/s

$$\lambda = \frac{6.626 \times 10^{-34}\ \text{Js}}{(9.11 \times 10^{-31}\ \text{kg})(5.9 \times 10^{6}\ \text{m/s})} = 1.2 \times 10^{-10} = 0.12\ \text{nm}$$

This is on the order of atomic dimensions and is much shorter than the shortest visible light wavelength of about 390 nm.

다양한 물질파

- 광자와 입자의 파장

$$\lambda_{photon} = \frac{h}{mc}$$

$$\lambda_{particle} = \frac{h}{mv}$$

de Broglie wavelengths for various moving objects

Object	Mass (g)	Wavelength (Å)
1-Volt electron	9.11×10^{-28}	12.3
10-Volt electron	9.11×10^{-28}	3.88
100-Volt electron	9.11×10^{-28}	1.23
Helium atom at room temperature	6.65×10^{-24}	0.73
α particle from radium	6.65×10^{-24}	0.000066
Average protein molecule	6.64×10^{-20}	0.0073
Floating chalk dust	$\sim 10^{-6}$	$\sim 6.6 \times 10^{-13}$
Driven golf ball	45	4.9×10^{-24}
Pitched baseball	140	1.9×10^{-24}
Chemistry professor, pacing	8×10^{4}	8.3×10^{-26}
Car at 30 mph	9×10^{5}	5.5×10^{-28}
Earth in orbit around sun	6×10^{27}	3.7×10^{-53}

물질파로부터 파동방정식 도출

- Schrödinger Equation

$$-\frac{h^2}{8\pi^2 m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+V(x,y,z)\psi = E\,\psi$$

  - Ψ – wave function
  - E – total energy
  - V – potential energy

Observation and theory from CM to QM

CLASSICAL THEORY

Matter	Energy
particulate, massive	continuous, wavelike

↓

Since matter is discontinuous and particulate perhaps energy is discontinuous and particulate

Observation		*Theory*
Blackbody radiation	→	Planck: Energy is quantized; only certain values allowed
Photoelectric effect	→	Einstein: Light has particulate behavior (photons)
Atomic line spectra	→	Bohr: Energy of atoms is quantized; photon emitted when electron changes orbit

Since energy is wavelike perhaps matter is wavelike

Observation		*Theory*
Davisson/Germer: electron diffraction by metal crystal	←	de Broglie: All matter travels in waves: energy of atom is quantized due to wave motion of electrons

Since matter has mass perhaps energy has mass

Observation		*Theory*
Compton: photon wavelength increases (momentum decreases) after colliding with electron	←	Einstein/de Broglie: Mass and energy are equivalent: particles have wavelength and photons have momentum

↓

QUANTUM THEORY

Energy *same as* Matter

particulate, massive, wavelike