Chapter 2. Fluid Statics

Fluid mechanics (유체역학) – 단위조작의 기본 분야

관, 펌프에서의 유체이동, 교반, 반응

열전달, 확산, 물질전달 등 분리조작의 기초
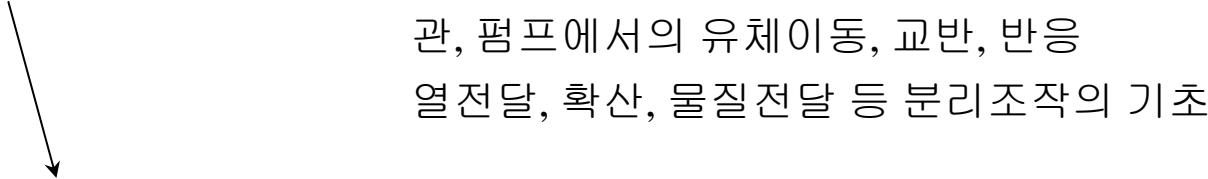
"Engineering science dealing with the behavior of <u>fluids</u>"

↓

liquid, gas, vapor

Fluid Statics (유체 정역학)

: treats fluids in the equilibrium state of no shear stress

Fluid Dynamics (유체 동역학)

: treats fluids when portions of the fluid are in motion

Incompressible fluids (비압축성 유체) – density changes are neglected (ρ = const.)

Compressible fluids (압축성 유체) – density changes are significant

Hydrostatic equilibrium

정지 유체의 압력: 높이에 따라 다름.

지면과 평행한 단면에서는 동일
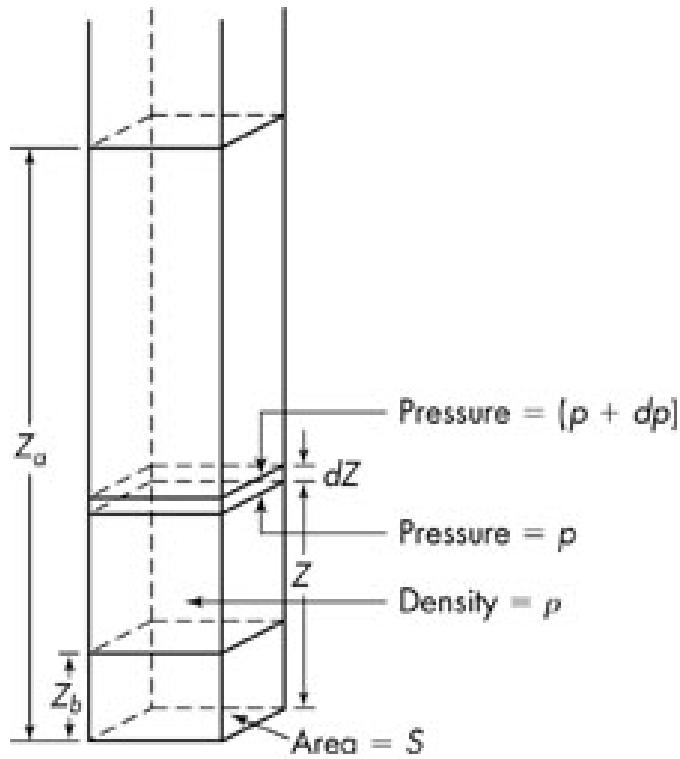
All forces on the small volume of fluid → 0

(그렇지 않은 경우엔 "유동"이 존재)

$$\therefore \ \sum F_i = 0$$

F_1 (the force acting in an upward direction)

: pS

F_2 (the force acting in a downward direction)

: $(p+dp)S$

F_3 (gravitational force): $g\rho S \cdot dZ$

Pressure = $[p + dp]$

dZ

Pressure = p

Density = ρ

Area = S

Z_a

Z

Z_b

$$\therefore \ pS - (p + dp)S - g\rho SdZ = 0$$

$\leftarrow$ upward direction: positive (+)

$$\boxed{dp + g\rho dZ = 0} \ \text{------ (2.2)}$$

For incompressible fluids (ρ = const.)

$$\boxed{p_b - p_a = \rho g(Z_a - Z_b)} \ \text{------ (2.4)}$$

즉, $\Delta p = \rho g \Delta Z$

- Head of a liquid (액체의 높이압)

: the relationship between pressure and the height of a liquid column

$$Z = p / \rho g \qquad (Z: \text{head})$$

- Barometric equation (기압 방정식)

For an ideal gas, $(\rho \neq 0)$

$$\rho = \frac{pM}{RT} \quad \longleftarrow \quad pV = nRT = \frac{m}{M}RT \quad (\rho = m/V)$$

Eq. (2.2) becomes

$$\frac{dp}{p} + \frac{gM}{RT}dZ = 0$$

Integrating between levels a & b, (assuming that T is constant)

$$\boxed{\frac{p_b}{p_a} = \exp\left(-\frac{gM(Z_b - Z_a)}{RT}\right)} \qquad : \text{barometric eq'n (2.7)}$$

$\longrightarrow$ 관련 문제: Prob. 2.2

- Rotating centrifuge (원심분리기)

Pressure drop over any ring of rotating liquid,

(회전하는 원심분리기 속 액체의 압력강하)

Δp

$$dF = r\omega^2 dm$$

$$\leftarrow F = ma, \quad \therefore dF = adm$$

$$a = r\omega^2$$

$$\leftarrow a = \frac{dv}{dt} = \frac{vd\theta}{dt} = v\omega = r\omega^2$$

$$dm = 2\pi\rho rbdr \qquad \leftarrow 2\pi rdr(\text{면적}), b(\text{높이})$$

$$\leftarrow dm = \rho dS \cdot b$$

$$dS = 2\pi rdr$$

$$\therefore \ dF = 2\pi\rho b\omega^2 r^2 dr$$
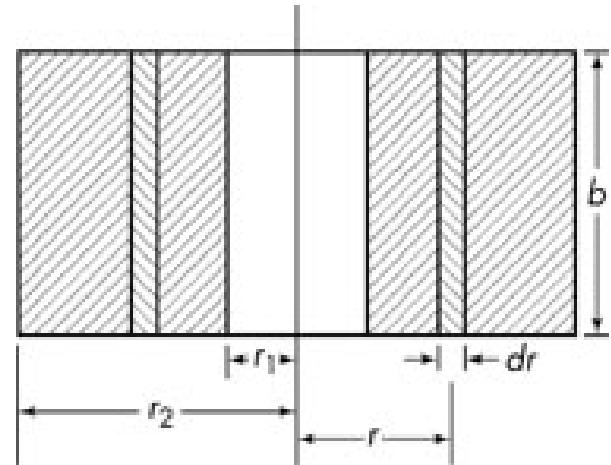
Since $dp = \dfrac{dF}{2\pi rb}$

$\qquad = \omega^2 \rho r dr$

r_1에서 r_2까지 적분하면,

$$p_2 - p_1 = \int_{r_1}^{r_2} \omega^2 \rho r dr$$

Assuming ρ is constant,

$$\therefore \quad p_2 - p_1 = \frac{\omega^2 \rho (r_2{}^2 - r_1{}^2)}{2}$$ ------ (2.8)

관련 문제: Prob. 2.6

- Manometer (압력차 측정기)   U관 마노미터

여기서는 U-tube manometer의 원리

p_1 (at pt. 1): p_a

p_2 (at pt. 2): $p_a + g(Z_m + R_m)\rho_B$

p_3 (at pt. 3) $= p_2$

p_4 (at pt. 4): $p_3 - gR_m\rho_A$

p_5 (at pt. 5): $p_4 - gZ_m\rho_B$

$\qquad\qquad\quad = p_b$

$$\therefore\ p_a + g\left[(Z_m + R_m)\rho_B - R_m\rho_A - Z_m\rho_B\right] = p_b$$

→ $\boxed{\therefore\ p_a - p_b = gR_m\left(\rho_A - \rho_B\right)}$ ----- (2.10)   관련 문제: Prob. 2.1
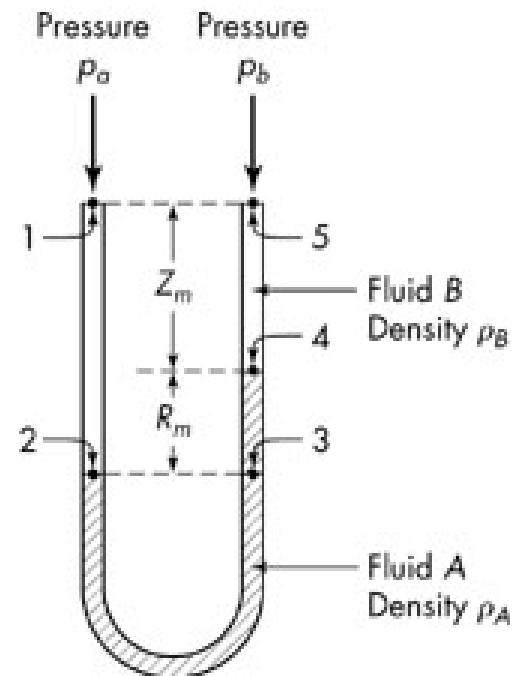
⟶ 만일 ρ_B를 무시할 수 있으면 $\Delta p = \rho g \Delta R_m$ 으로 됨

(Ex. 2.1) Pressure drop across an orifice (orifice meter)
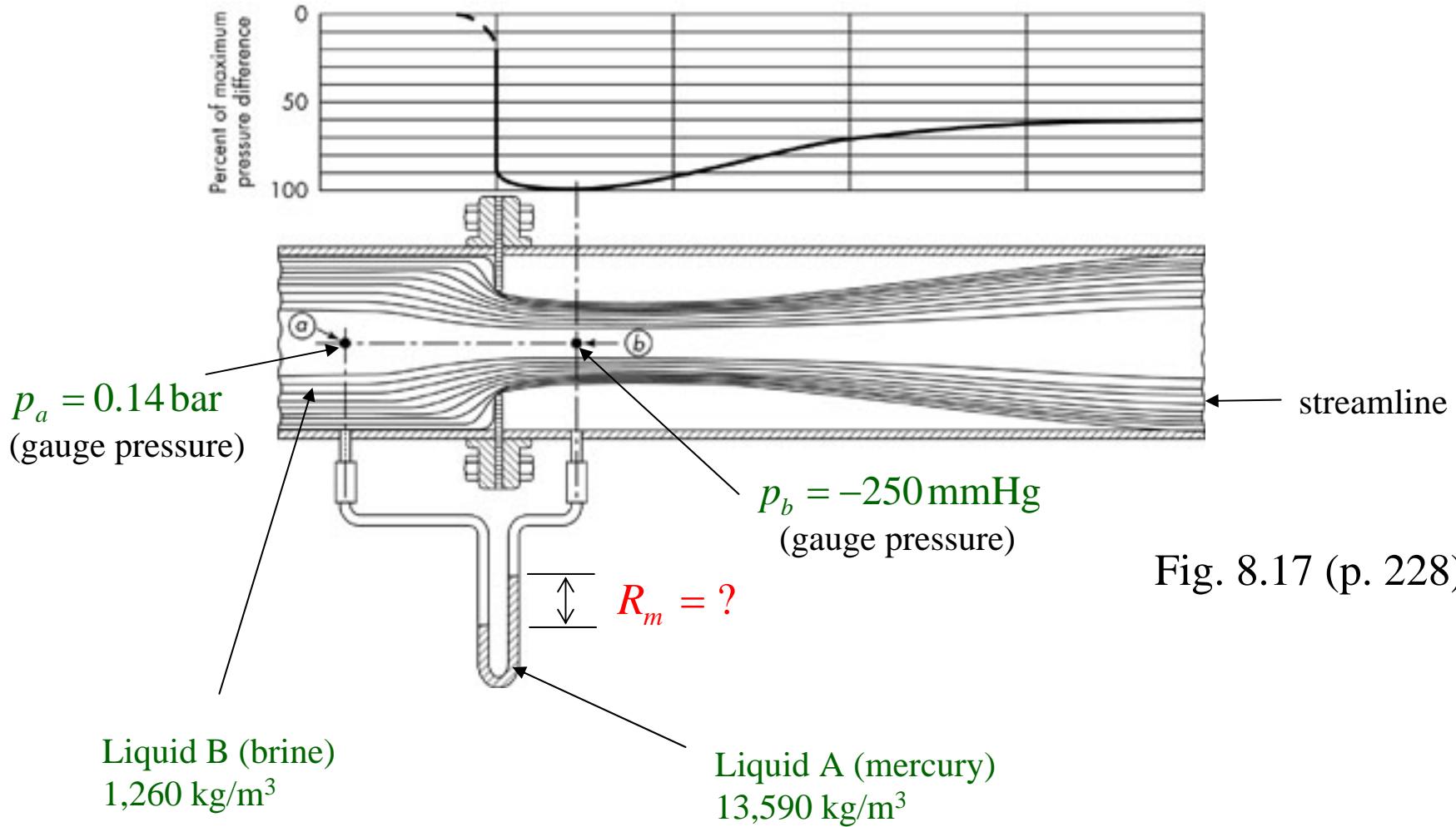
$p_a = 0.14 \, \text{bar}$
(gauge pressure)

$p_b = -250 \, \text{mmHg}$
(gauge pressure)

$R_m = \, ?$

streamline

Fig. 8.17 (p. 228)

Liquid B (brine)
1,260 kg/m^3

Liquid A (mercury)
13,590 kg/m^3

$$p_a = 0.14\,\text{bar} \qquad\qquad p_b = -250\,\text{mmHg}$$

$$= 0.14 \times 10^5\,\text{Pa} \qquad\qquad = \frac{-250}{1{,}000} \times g \times \rho$$

$$= 14{,}000\,\text{Pa} \qquad\qquad = -33{,}318\,\text{Pa}$$

From Eq. (2.10):

$$\boxed{p_a - p_b = gR_m\left(\rho_A - \rho_B\right)}$$

$$\therefore \quad R_m = 0.391\,\text{m} \ \ (391\,\text{mm})$$

cf.) Absolute p = Atmospheric p + Gauge p   (절대압 = 대기압 + 게이지압)

절대압으로는 $p_a = 1\,\text{atm} + 0.14\,\text{bar} = 1.153\,\text{bar}$

$$p_b = 1\,\text{atm} - 250\,\text{mmHg} = 510\,\text{mmHg}$$

Related Problems

(Probs.) 2.1, 2.2, 2.6, 2.11, 2.12 and 2.13