Batch Fermentation: System

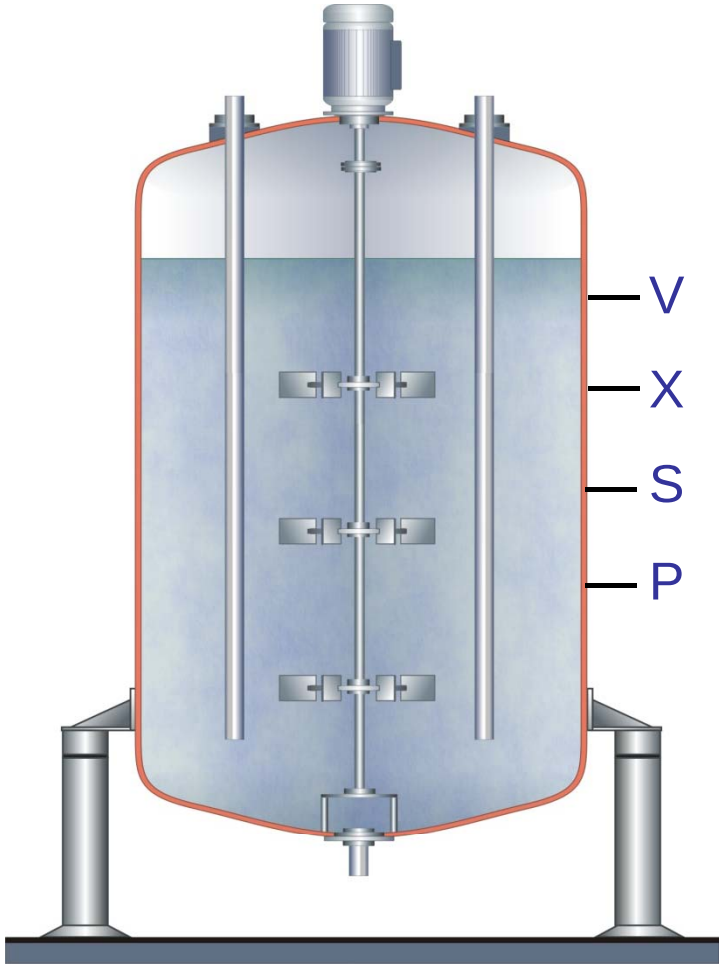
Important variables of the system are the biological dry mass or cell X, substrate concentration S and product concentration P. The reactor volume V is well mixed.

The growth is assumed to follow kinetics described by Monod equation.

Substrate consumption is related to cell growth by a constant yield $Y_{X/S}$.

Product formation is the result of both growth and non-growth associated rates of production.

The lag and decline phases of cell growth are not included in the model.



Stirred batch fermenter

Batch Fermentation: Model Mass Balance

Mass Balances:

$$\left(Rate\ of\ \ accumulation\ \right)=\left(Rate\ of\ \ production\ \right)$$

For Cells $V\frac{dX}{dt}=r_X\cdot V$ or $\frac{dX}{dt}=r_X$ (1)

For Substrate $V\frac{dS}{dt}=r_S\cdot S$ or $\frac{dS}{dt}=r_S$ (2)

For Production $V\frac{dP}{dt}=r_P\cdot V$ or $\frac{dP}{dt}=r_P$ (3)

Batch Fermentation: Model Kinetics

Reaction Rate:

For Cells $$r_X = \mu \cdot X \quad (4)$$

For Substrate $$r_S = -\frac{r_X}{Y_{X/S}} \quad (5)$$

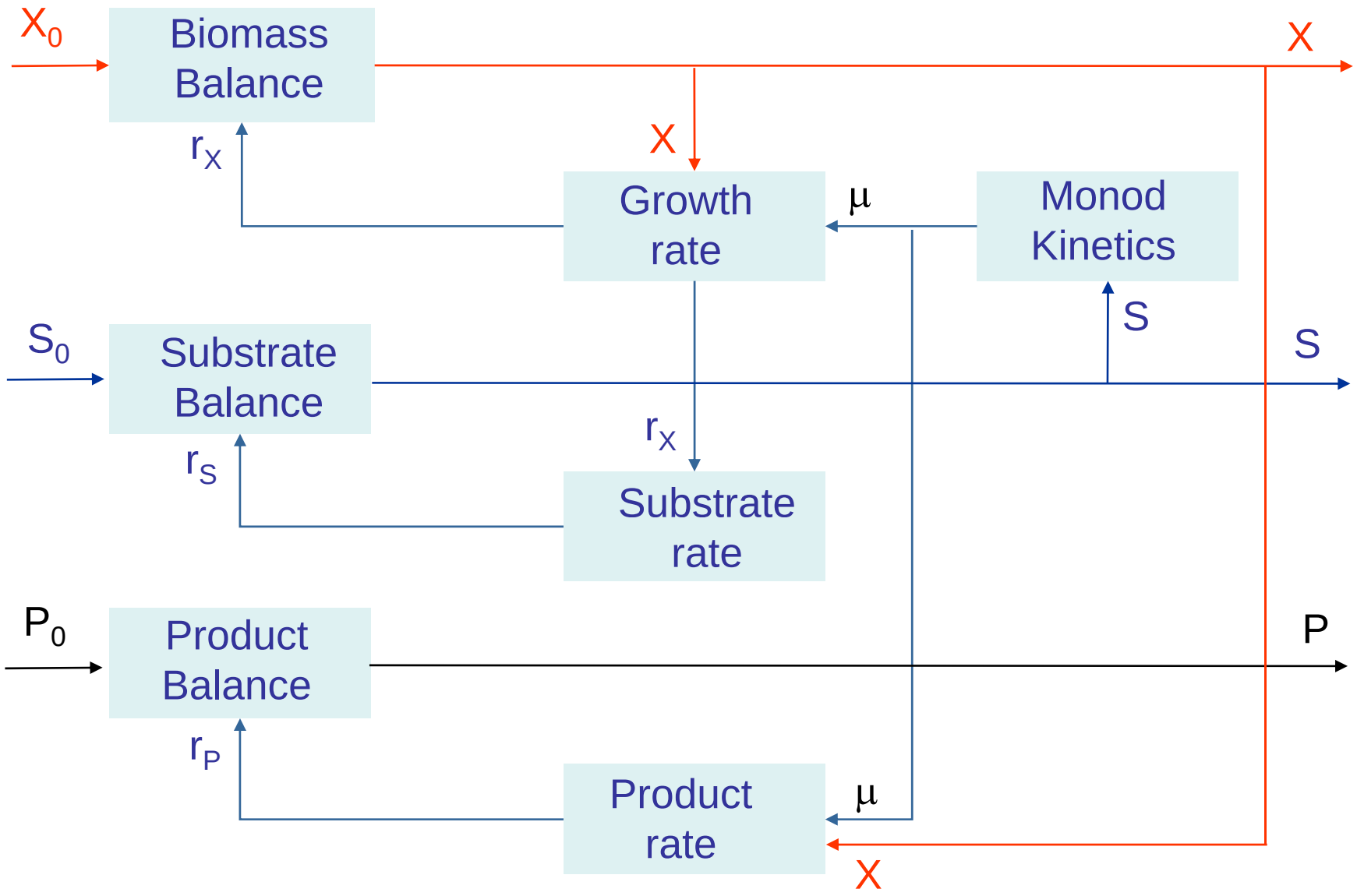
For Production $$r_P = (k_1 + k_2 \cdot \mu) \cdot X \quad (6)$$

Monod relation $$\mu = -\frac{\mu_m \cdot S}{(K_S + S)} \quad (7)$$

k_1 – is the non-growth associated coefficient and

k_2 – is the coefficient associated with growth.

Information Flow Diagram



Batch Fermentation: Program

As first step in solving the problem with MATLAB is to write the function file. This file will be saved in the folder bin and the name of the file will later be required for processing in the Command Window of MATLAB

```
1    function dy = batferm_func(t,y,UM,KS,K1,K2,Y)
2 -    dy = zeros(3,1);     % a column vector
3 -    if (y(2)<0) y(2)=0; end;
4 -    U = UM*y(2)/(KS+y(2));
5 -    dy(1) = U*y(1);           % RX
6 -    dy(2) = -dy(1)/Y;         % RS
7 -    dy(3) = (K1+K2*U)*y(1); % RP
8
```

Batch Fermentation: Program

In this section the ODEs with initial condition values are solved and plotted in diagram.

```
1  -    UM = 0.3;  KS = 0.1;  K1 = 0.1;                % Constant
2  -    K2 = 1;  Y = 0.8;
3  -    Tint = 1;  Tfin = 40;
4  -    X0 = 0.01;  S0 = 10;  P0 = 0;
5  -    Cinit = [X0 S0 P0];                             % Initial concentration
6  -    Tspan = linspace(0,Tfin,Tfin/Tint);             % Time span
7
8  -    [T C] = ode45(@(t,y) batferm_func(t,y,UM,KS,K1,K2,Y),Tspan,Cinit); % Solve ODE
9  -    plot(T,C(:,1),'-ro',T,C(:,2),'-k.',T,C(:,3),'-b+')
10   -    % C(:,1):X, C(:,2):S, C(:,3):P
11 -    title(['KS = ',num2str(KS)])
12 -    xlabel('Time','fontsize',12,'fontweight','b')
13 -    ylabel('X,S,P','fontsize',12,'fontweight','b')
14 -    h = legend('X','S','P',0);
15 -    set(h,'fontsize',8);
```

Batch Fermentation: Program

In this section the kinetic parameters are calculated.

```
17      %Calculating U, RX, RS, RP by definition
18 -    U = UM*C(:,2)./(KS+C(:,2));
19 -    RX = U.*C(:,1);
20 -    RS = -RX/Y;
21 -    RP = (K1+K2*U).*C(:,1);
22      %Plot of U, RX, RS, RP
23 -    figure
24 -    plot(C(:,2),U,'-ro',RS,RX,'-k.')
25 -    title('U vs S and RX vs RS')
26 -    xlabel('S, RS','fontsize',12,'fontweight','b')
27 -    ylabel('U, RX','fontsize',12,'fontweight','b')
28 -    h = legend('U','RX',0);
29 -    set(h,'fontsize',8);
30 -    pause
31 -    figure
```

Batch Fermentation: Program

In this section the range values for K_S are varied.

```
33      % Loop by changing the KS parameter
34 -    for KS = 0.1:0.2:1
35 -        [T C] = ode45(@(t,y) batferm_func(t,y,UM,KS,K1,K2,Y),Tspan,Cinit); % Solve ODE
36 -        plot(T,C(:,1),'-ro',T,C(:,2),'-k.',T,C(:,3),'-b+')
37          % C(:,1):X, C(:,2):S, C(:,3):P
38 -        title(['KS = ',num2str(KS)])
39 -        xlabel('Time','fontsize',12,'fontweight','b')
40 -        ylabel('X,S,P','fontsize',12,'fontweight','b')
41 -        h = legend('X','S','P',0);
42 -        set(h,'fontsize',8);
43 -        pause
44 -    end;
```

Batch Fermentation: Nomenclature

Symbol	Description	Unit
k1,k2	Product formation constants	L/h, kg/kg
K_S	Saturation constant	kg/m^3
P	Product concentration	kg/m^3
r	Reaction rate	$kg/(m^3\,h)$
S	Substrate concentration	kg/m^3
V	Reactor volume	m^3
X	Biomass concentration	kg/m^3
Y	Yield coefficient	kg/kg
m	Specific groth rate	L/h

Index	Description	Symbol	Description
1	Refers to non-growth association rate	P	Refers to Product
2	Refers to growth association rate	S	Refers to Substrate
m	Refers to maximum	X	Refers to Biomass

Batch Fermentation: Results

Kinetic Constants

```
UM = 0.3;
KS = 0.1;
K1 = 0.03;
K2 = 0.08;
Y = 0.8;
```

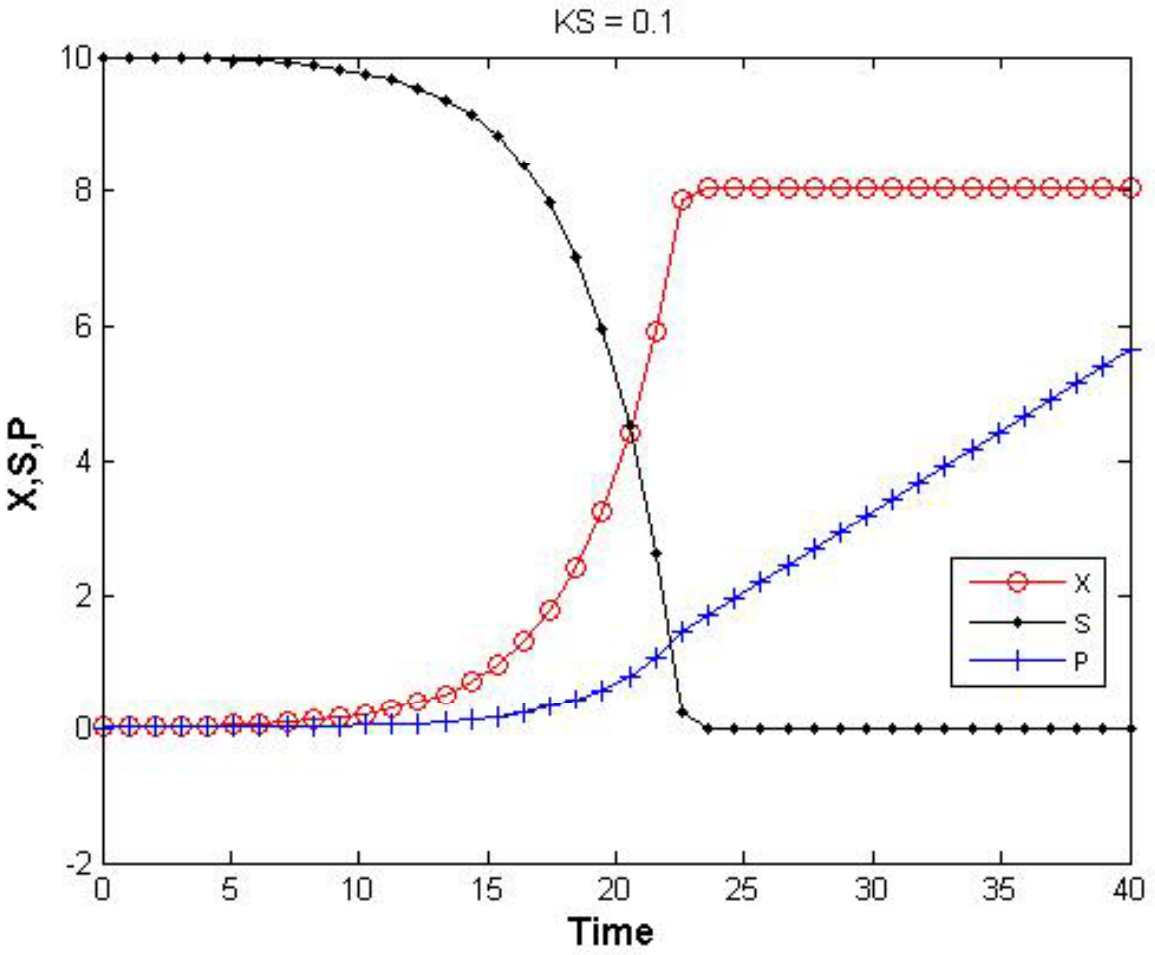
Reaction Duration

```
Tint = 1;
Tfin = 40;
```

Initial values of variables

```
X0 = 0.01;
S0 = 10;
P0 = 0;
```

Plot of X, S and P during the batch growth



Batch Fermentation: Results

Plot of μ vs. S and r_X vs. r_S