Feedback Control of a Water Heater

Introduction

- A simple feeback control system involving a stirred tank, temperature measurement, controller and manipulated heater is shown in Figure

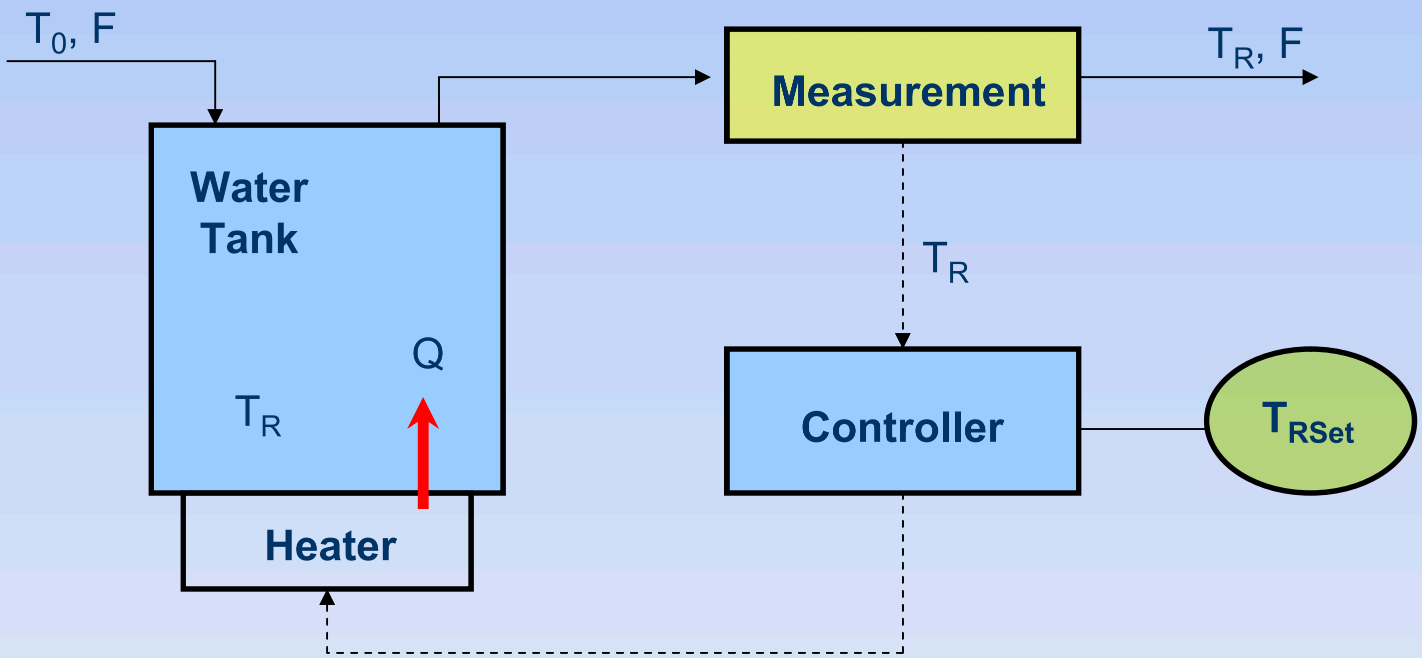

Figure. Feedback control of a simple continuous water heater.

Model

- The energy balance for the tank is

$$\frac{dT_R}{dt} = \frac{F}{V}(T_0 - T_R) + \frac{Q}{V\rho Cp}$$

- Where Q is the heat input from the controlled heater.
  A proportional-integral feedback controller can be modelled as

$$Q = Q_0 + Kp\varepsilon + \frac{Kp}{\tau_I}\int \varepsilon dt$$

- Where the control error is given by

$$\varepsilon = T_{RSet} - T_R$$

Model

- Nomenclature

Symbols

C_p	Specific heat	Kcal/(Kg C)
f	Frequency of oscillations	l/h
F	Flow rate	m^3 /h
K_P	Controller constant	Kcal/C
K_D	Controller constant for differential term	(Kcal h)/C
Q	Heat input	Kcal/h
T	Temperature	C
τ_I	Controller cinstant	h
V	Reactor volume	m^3
ϵ	Error	C
ρ	Density	Kg /m^3

Indices

M	Refers to measured value
R	Refers to reactor
Set	Refers to setpoint
0	Refers to inlet

Program

- Initial condition

Variables	Unit	Value
Inlet temperature	C	15-25
Set point	C	80
Reactor Volume	m^3	100
Inlet flow rate	m^3/hr	10
Density	Kg/m^3	1
Specific heat capacity	Kcal/Kg C	1
Initial Heat duty	Kcal/hr	6

- Initial condition parameters

Variables	Symbols	Unit	Value
Controller constant	K_P	Kcal/C	10
First order lag	τ_Q	hr	1
	τ_S	hr	2
Time constant	τ_I	hr	5
Error	ε	C	0

Program

- M-file

Editor - C:\Program Files\MATLAB71\work\controlled\Controlled_Reactors.m

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Stack: Base

```
% Program: Controlled Reactors
%        - Feedback Control of a Water Heater (TEMPCONT)
% Department of Chemical Engineering, Chungnam National University, Korea
%                                    written by Soyoyu (2008. 11. 1)
% Units: Degree (C), Heat duty (Kcal), Mass (Kg), Volume (m3), Time (Hr)

global T0 SC
clc

% Seletion of Controller
%  1: Propotional Control
%  2: Propotional-Integral Control
%  3: Propotional-IntegralContrl-Derivative Control
disp ('Seletion of Controller')
disp (' 1: Propotional Control')
disp (' 2: Propotional-Integral Control')
disp (' 3: Propotional-IntegralContrl-Derivative Control')
SC = input('Number ?');

if ((SC==1)|(SC==2)|(SC==3))
% Time Range
tf = 300;

% Temperature of Feed flow rate
T0 = 20;

% Output variables
x0=[20 6 20 0];    % Temp (Measurement), Heat duty, Temp (Tank), Control Error

% Solve initial value problems for ordinary differential equations
[t,x] = ode45('control',[0 tf],x0);

% distribution of output variables
x1=x(:,1);
x2=x(:,2);
x3=x(:,3);

% Plot results of measurment and tank temperatures
figure(1);
hold on
plot(t,x1,'r',t,x3,'b');
xlabel('Time (hr)')
ylabel('Temperature (C)')
legend('Measurement','Tank')
hold off

% Plot result of heat input to the water tank
figure(2);
hold on;
plot(t,x2,'c');
xlabel('Time (hr)')
ylabel('Heat duty (Kcal)')
hold off;
else
disp ('The controller selection became wrong.')
end
```

Program

- M-file

Editor - C:₩Program Files₩MATLAB71₩work₩controlled₩control.m

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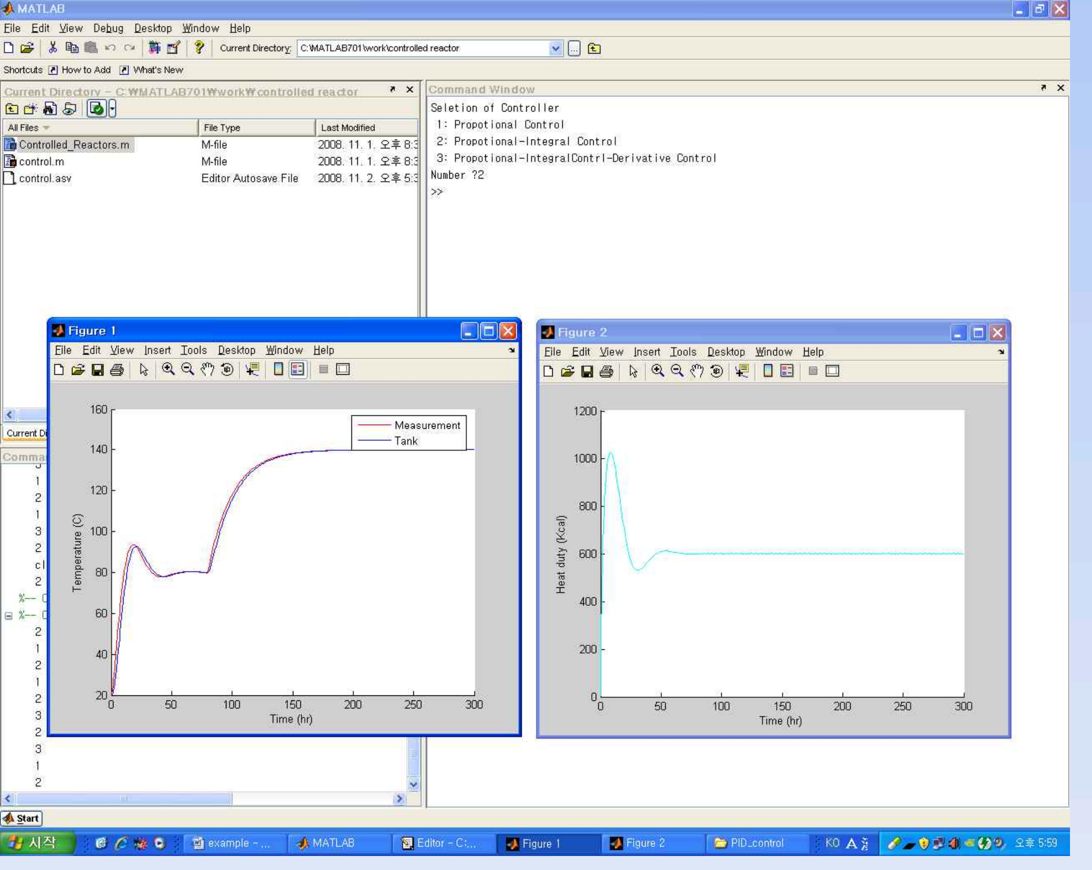
Stack: Base

```
% Program: Feedback Temperature Controller
%
% Department of Chemical Engineering, Chungnam National University, Korea
%                                written by Soyoyu (2008. 11. 1)
% units: Controller constant (Kcal/C), Time constant (Hr)

function xdot=control(t,x)
global TO SC

% Initial Values
V = 100;
F = 10;
QO = 6;
rho = 1;
cp = 1;
SEED = 1;

% Set point
TRset = 80;

%if (t > 80)
%     F = 25;
%end

% controller parameters ( Kp=50, TauI = 8, tauD = 2)
Kp = 50;
TauI = 8;
TauQ = 1;
TauS = 2;
TauD = 2;

switch SC
    case 1
        QC= QO + Kp*(TRset - x(3));
    case 2
        %if (t > 100)
        %QC = QO;
       % F = 9.5;
        %else
        %QC = QO;
        %end
        QC= QO + Kp*(TRset - x(3))+Kp/TauI*x(4)-0.2*x(2);
    case 3
        QC= QO + Kp*(TRset - x(3))+Kp/TauI*x(4)-Kp*TauD*1/TauS*(x(1)-x(3));
end
if (QC < 0)
    QC = 0;
end
xdot(1) = (F/V)*(TO-x(1))+x(2)/(V*rho*cp);  % Energy Balance
xdot(2) = (QC-x(2))/TauQ;                   % First order lag heater
xdot(3) = 1/TauS*(x(1)-x(3));               % First order lag measurement
xdot(4) = TRset - x(3);                     % Estimate control error
xdot=xdot';

% Random Inlet Temperature generator (15-25 C)
%TO = TO + ((rand(SEED)-0.5));
%if (TO >= 25)
% TO = 25.0;
%end
%if (TO <= 15)
% TO = 15.0;
%end
```

Exercises 1

- **Disconnect the controller (set K_P=0), allow the temperature to reach steady state, and measure the temperature response to changes in water flow.**

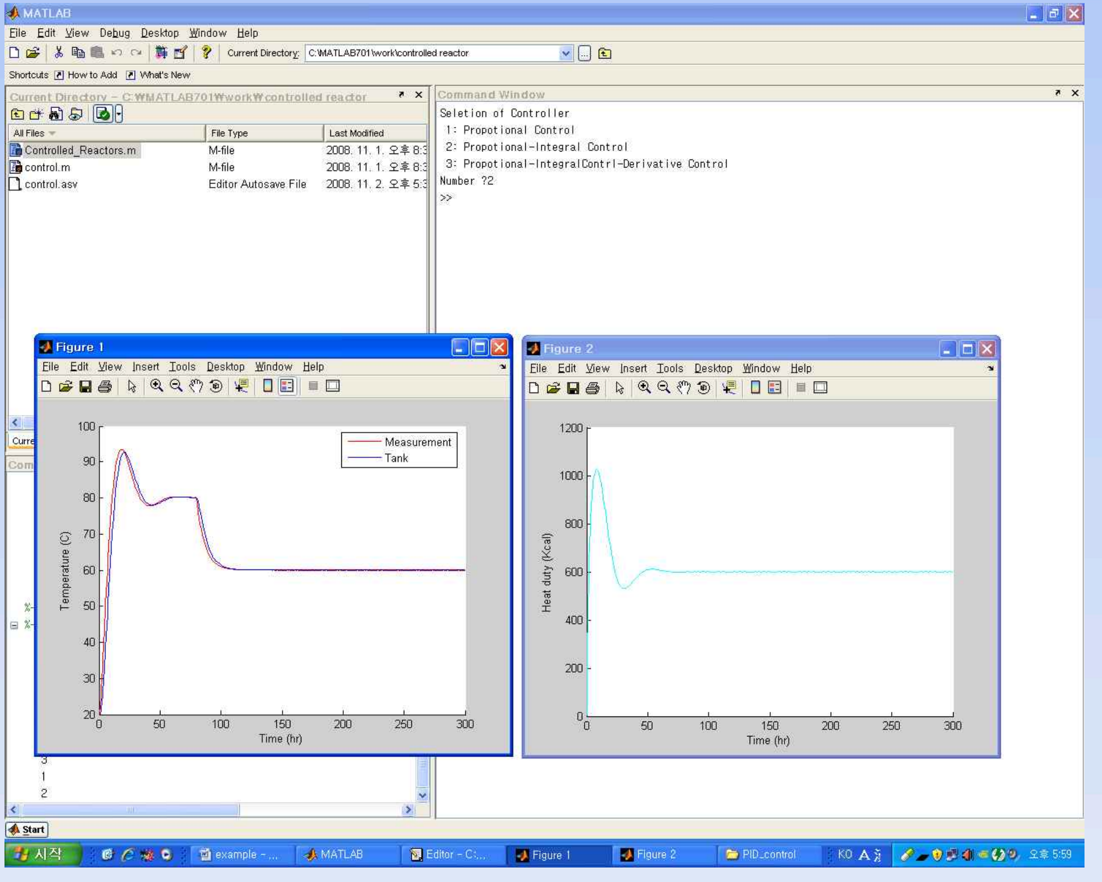
Water flow rate (5 m^3/hr)



Exercises 1

❖ Disconnect the controller (set K_P=0), allow the temperature to reach steady state, and measure the temperature response to changes in water flow.

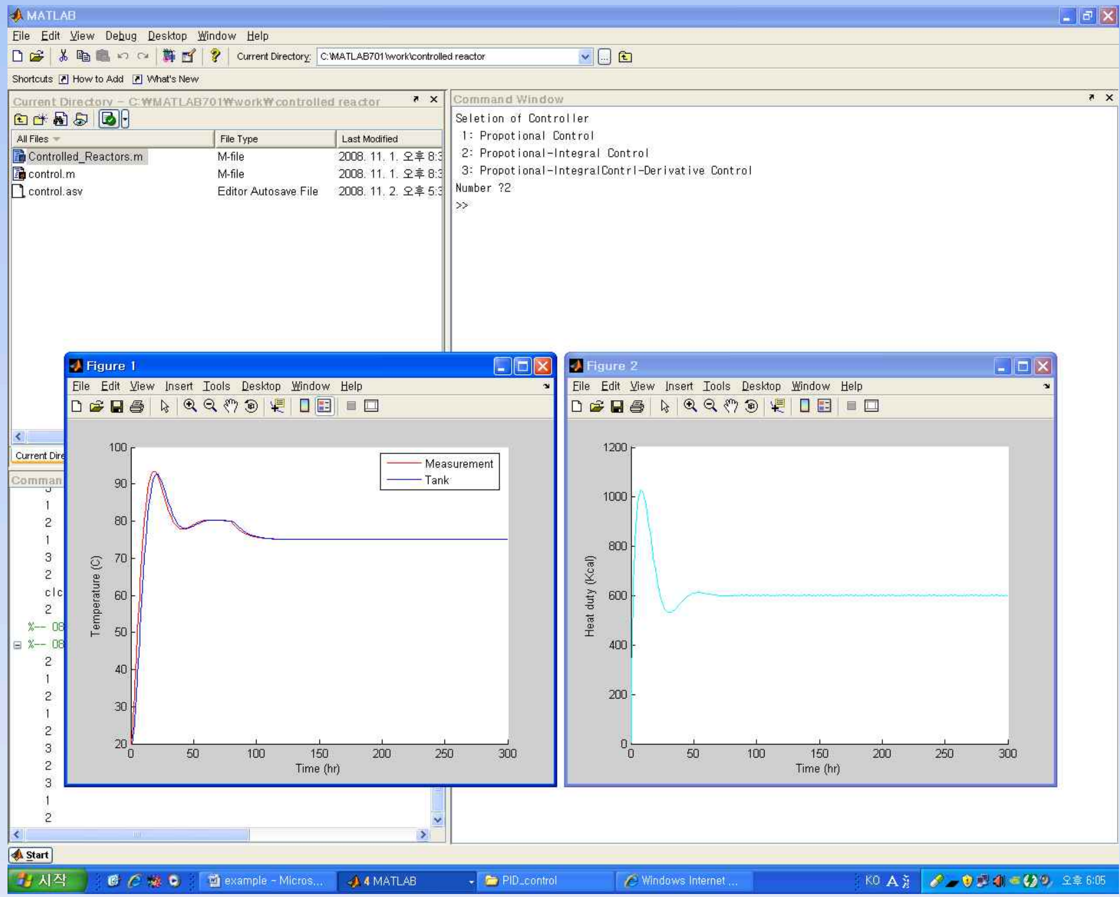
Water flow rate (15 m^3/hr)



Exercises 2

❖ **Repeat exercise 1 for a change in inlet temperature**

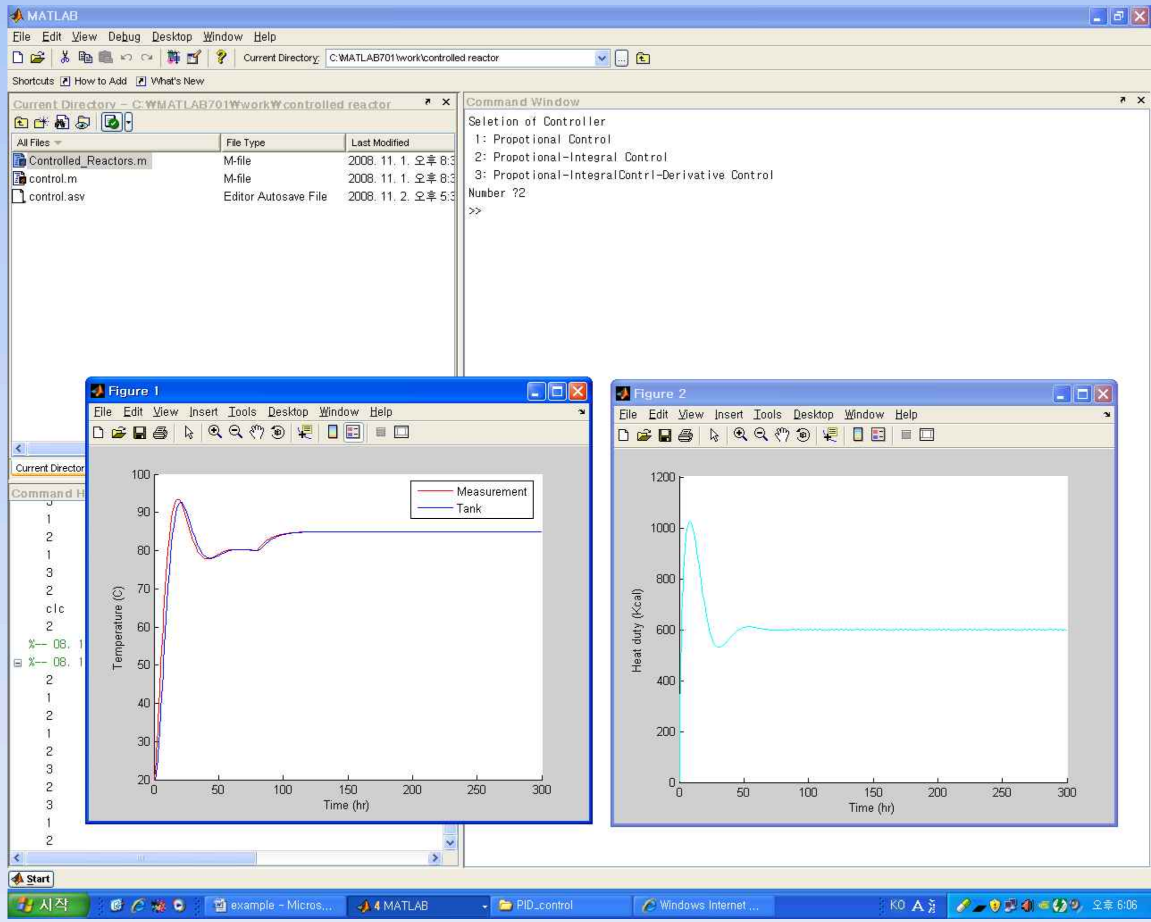
Inlet temperature (15°C)



Exercises 2

- Repeat exercise 1 for a change in inlet temperature

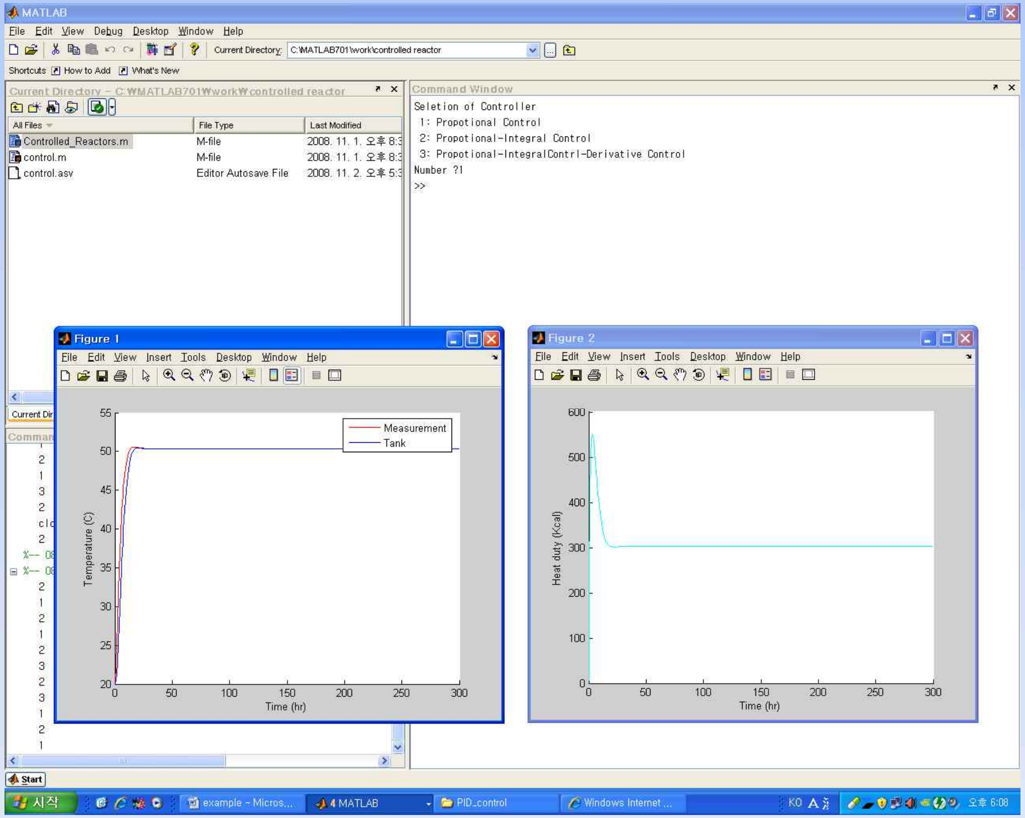
Inlet temperature (25°C)



Exercises 3

❖ **Measure the controlled response to a step change in F with proportional control only (set τ_I very high). Notice the offset error ϵ, and change τ_I to a low value. Dose the offset disappear?**

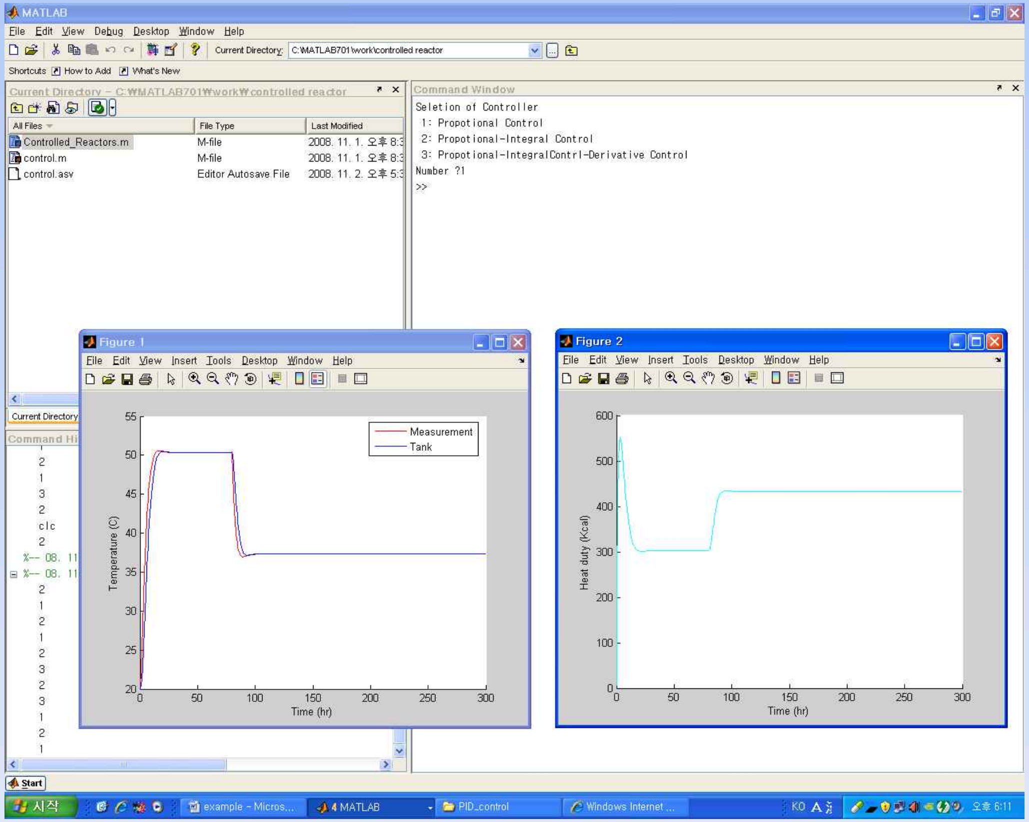
Proportional control only



Exercises 3

- Measure the controlled response to a step change in F with proportional control only (set τ_I very high). Notice the offset error ϵ, and change τ_I to a low value. Dose the offset disappear?

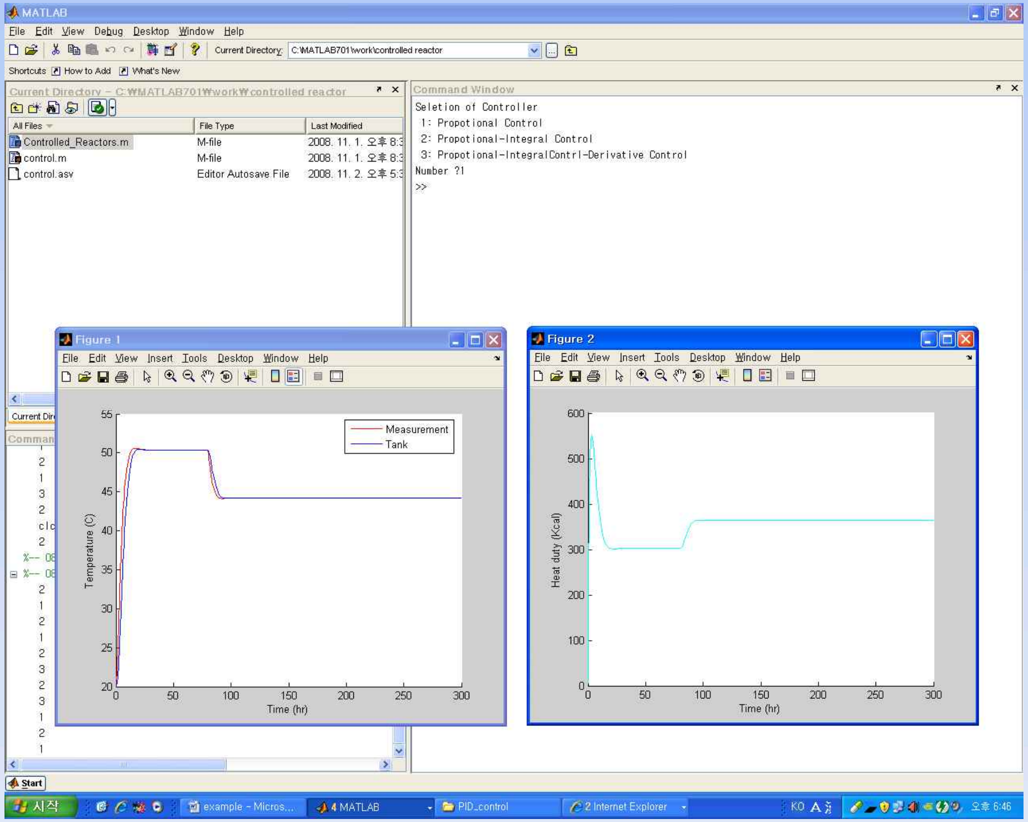
Step change in F with proportional control (F: 20 → 25 m^3/hr, after 80 hr)



Exercises 3

- **Measure the controlled response to a step change in F with proportional control only (set τ_I very high). Notice the offset error ε, and change τ_I to a low value. Dose the offset disappear?**

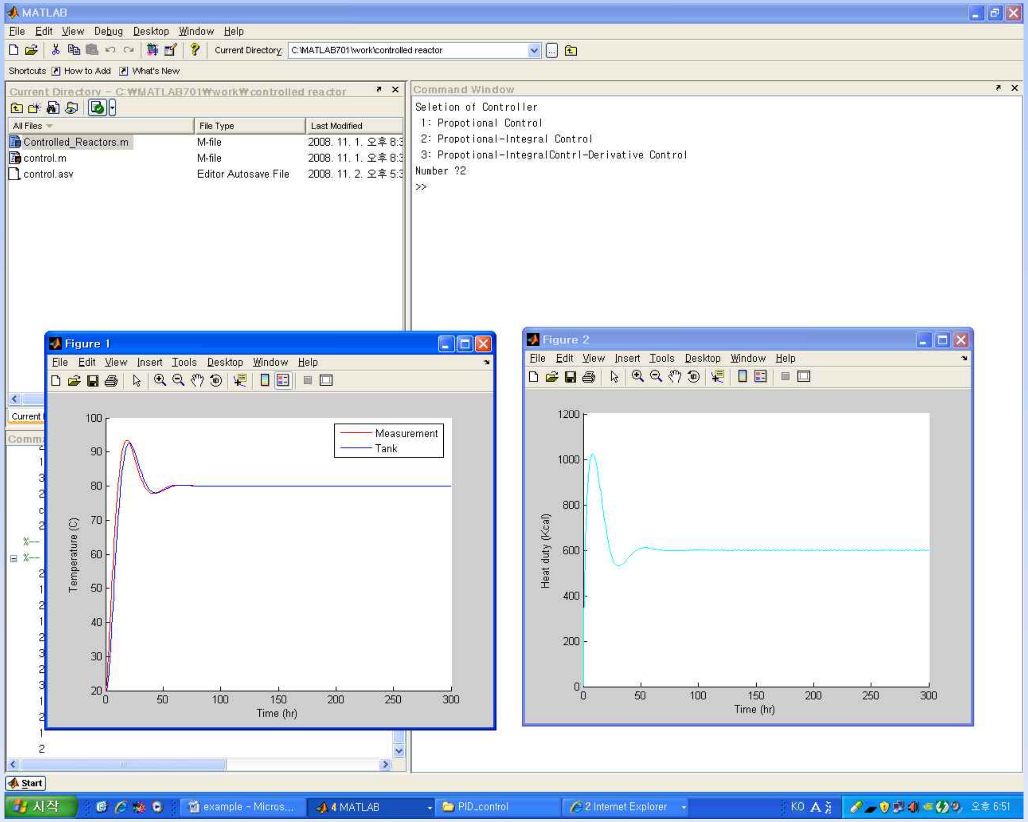
Step change in F with proportional control (F: 20 → 15 m^3/hr, after 80 hr)



Exercises 3

- Measure the controlled response to a step change in F with proportional control only (set τ_I very high). Notice the offset error ϵ, and change τ_I to a low value. Dose the offset disappear?

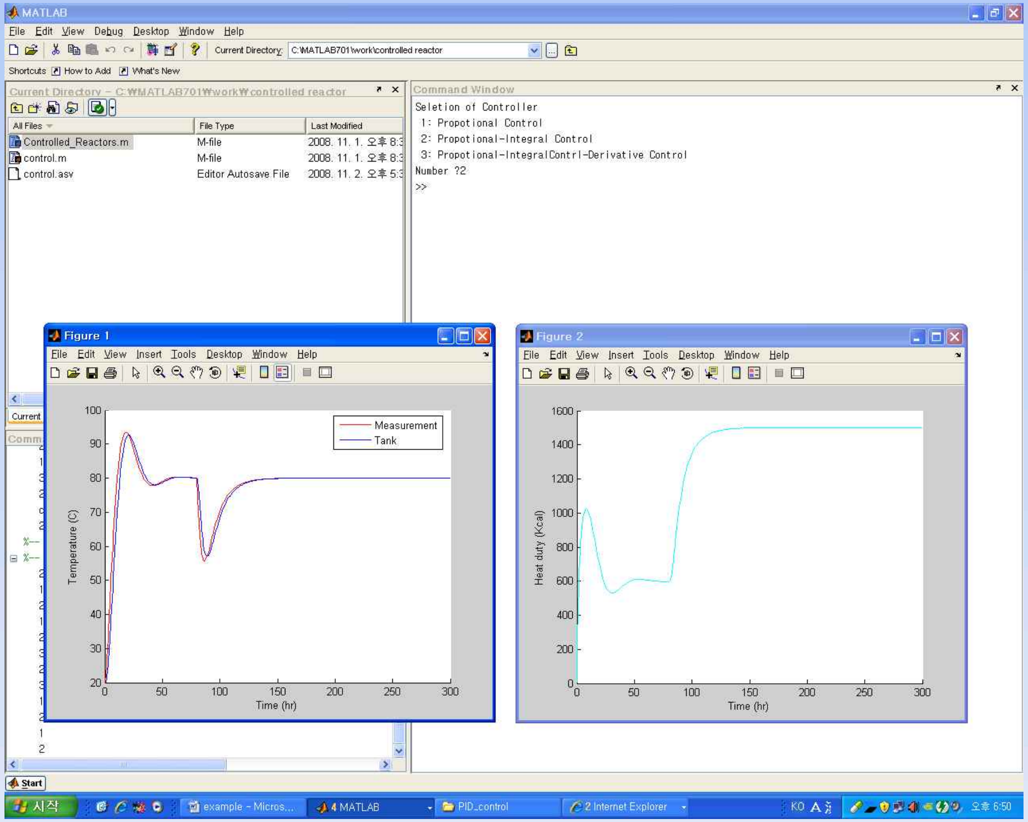
Proportional-integral control (F: 20 m^3/hr)



Exercises 3

- **Measure the controlled response to a step change in F with proportional control only (set τ_I very high). Notice the offset error ϵ, and change τ_I to a low value. Dose the offset disappear?**

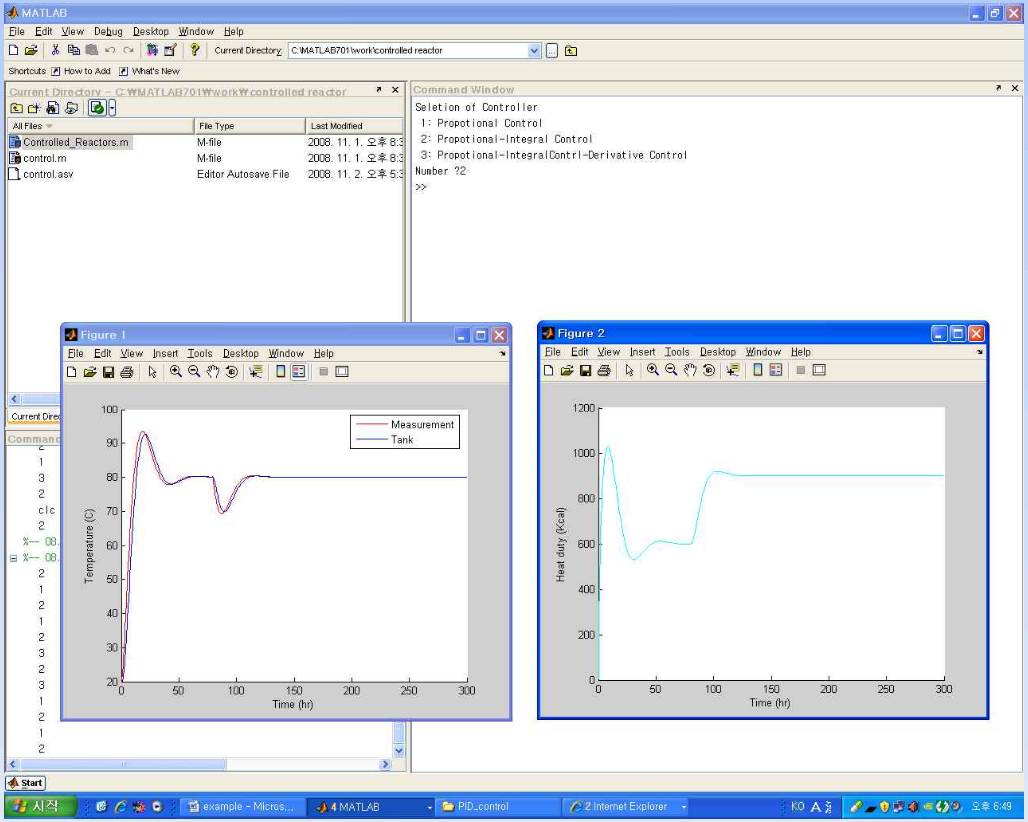
Step change in F with proportional-integral control (F: 20 → 25 m^3/hr, after 80 hr)



Exercises 3

- **Measure the controlled response to a step change in F with proportional control only (set τ_I very high). Notice the offset error ϵ, and change τ_I to a low value. Dose the offset disappear?**

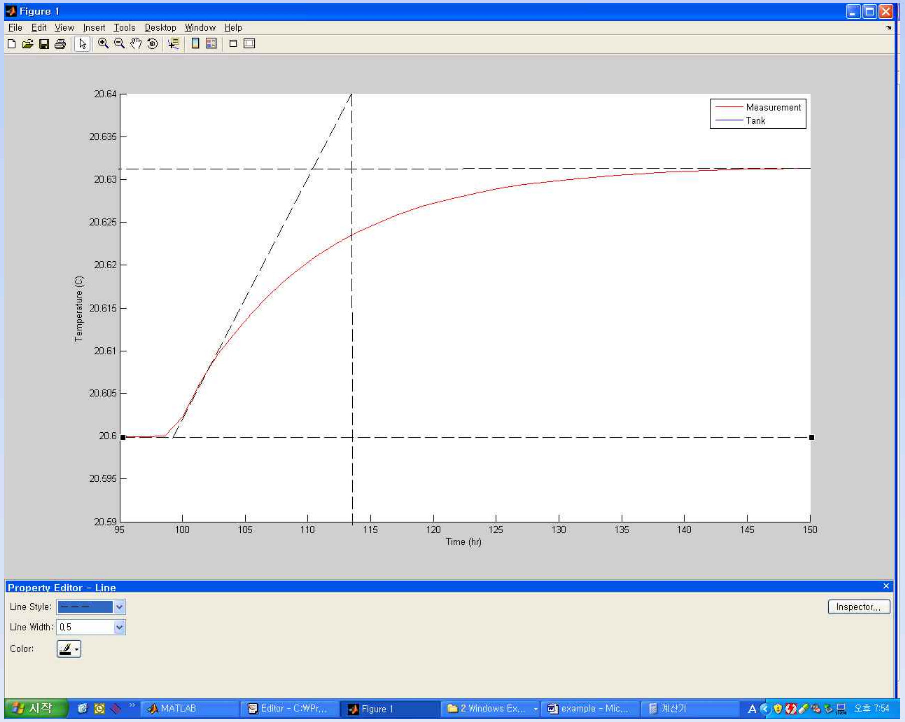
Step change in F with proportional-integral control (F: 20 → 15 m^3/hr, after 80 hr)



Exercises 4

- Disconnect the controller and by forcing the system with a stepwise change in water flow rate, τ_p obtain the process reaction curve as explained in τ_i Ch. 7. Analyze this curve to obtain the parameters for the Ziegler-Nichols Method. Use Table 7.1 to obtain the best controller setting for P and PI control. Try these out in a simulation.

Water flow change (process reaction curve)



$$S = \frac{Slope}{A} = \frac{0.0027}{0.5}$$

$$= 0.0053$$

Exercises 4

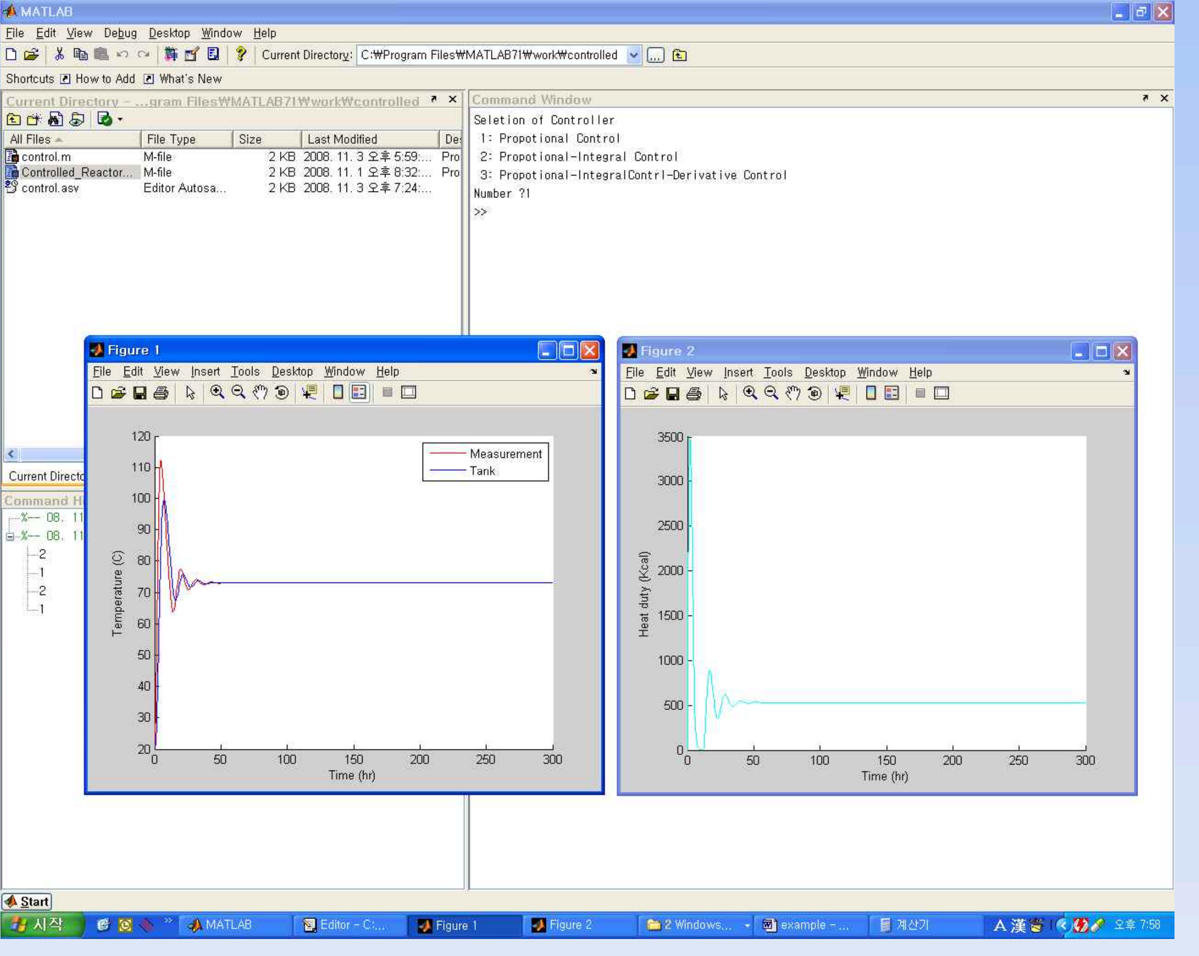
- Disconnect the controller and by forcing the system with a stepwise change in water flow rate, τ_I obtain the process reaction curve as explained in τ_I Ch. 7. Analyze this curve to obtain the parameters for the Ziegler-Nichols Method. Use Table 7.1 to obtain the best controller setting for P and PI control. Try these out in a simulation.

Table. Controller settings based on process responses (Zigler-Nichols)

Controller	Kp	τ_I	τ_D
P	$1/T_L S$		
PI	$0.9/T_L S$	$3.33T_L$	
PID	$1.2/T_L S$	$2T_L$	$T_L / 2$

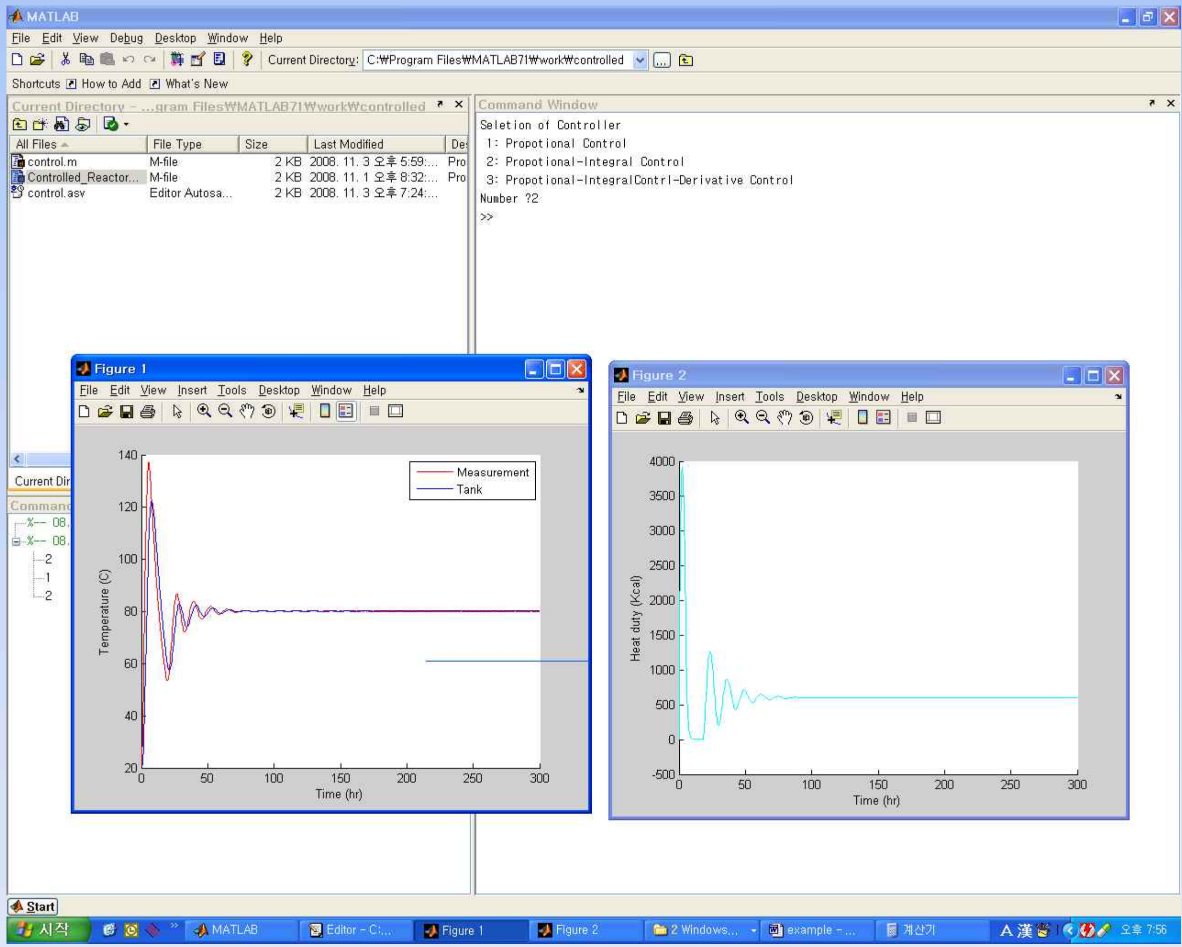
Exercises 4

P controller



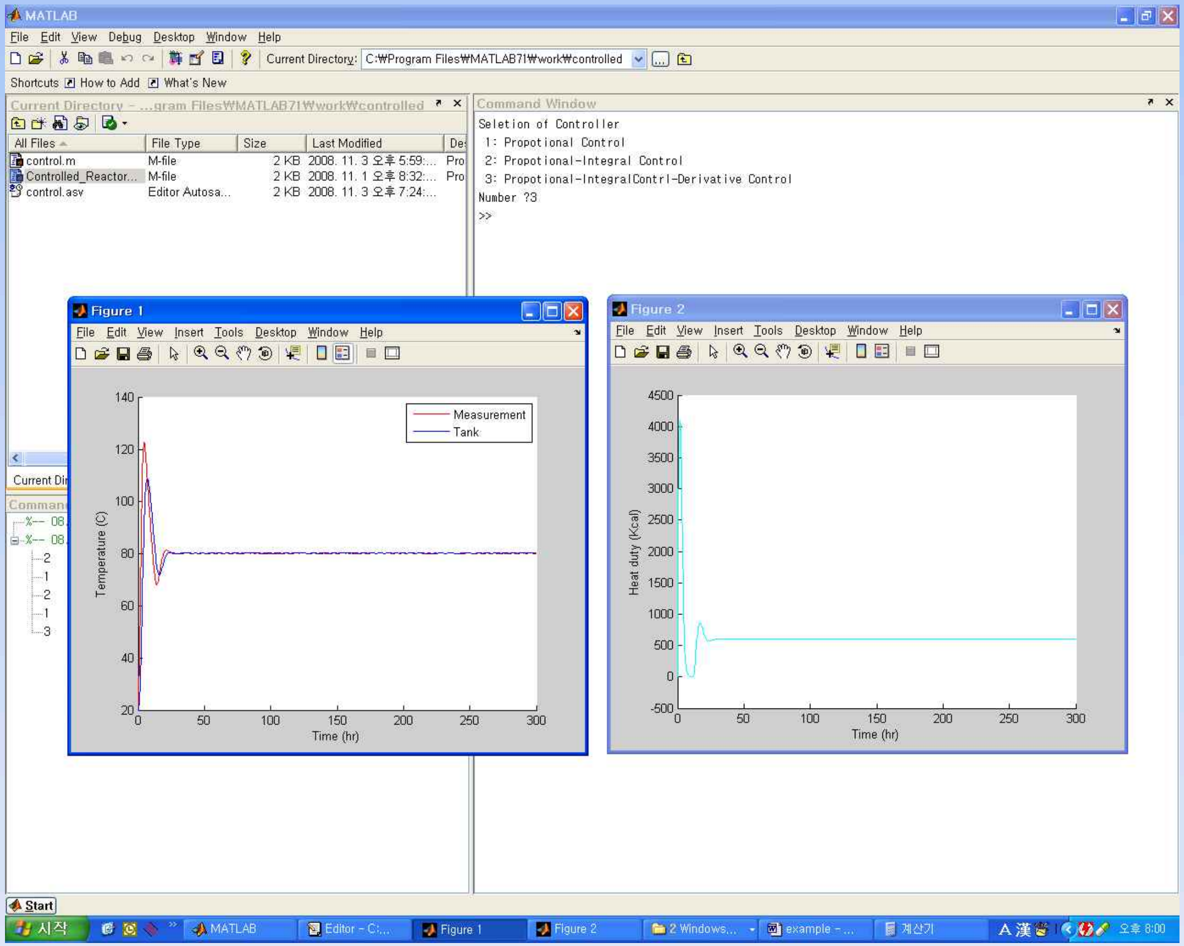
Exercises 4

PI controller



Exercises 4

PID controller



Exercises 5

- Proceed as in Example 4, but use the Cohen-Coon settings from Table 7.1

A=0.5 B=0.03 T_L=2.5 S=0.0053

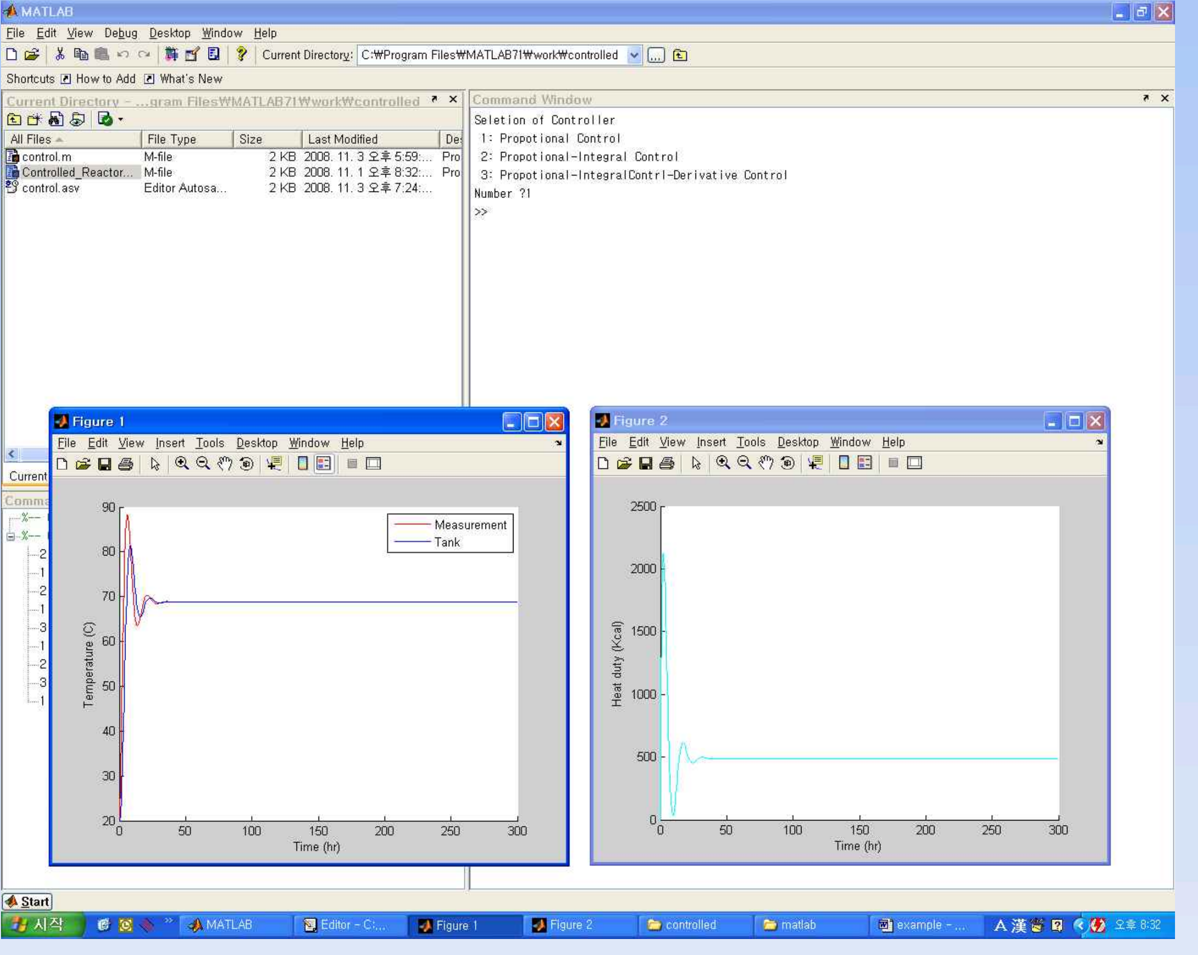
$$K = \frac{B}{A} = 0.06 \qquad \tau = \frac{B}{S} = 5.63$$

Table. Controller settings based on process responses (Cohen-Coon)

Controller	Kp	τ_I	τ_D
P	$\frac{\tau}{KT_L}(1+\frac{T_L}{3\tau})$		
PI	$\frac{\tau}{KT_L}(0.9+\frac{T_L}{12\tau})$	$T_L\frac{30+3T_L/\tau}{9+20T_L/\tau}$	
PID	$\frac{\tau}{KT_L}(\frac{4}{3}+\frac{T_L}{12\tau})$	$T_L\frac{32+6T_L/\tau}{13+8T_L/\tau}$	$T_L\frac{4}{12+2T_L/\tau}$

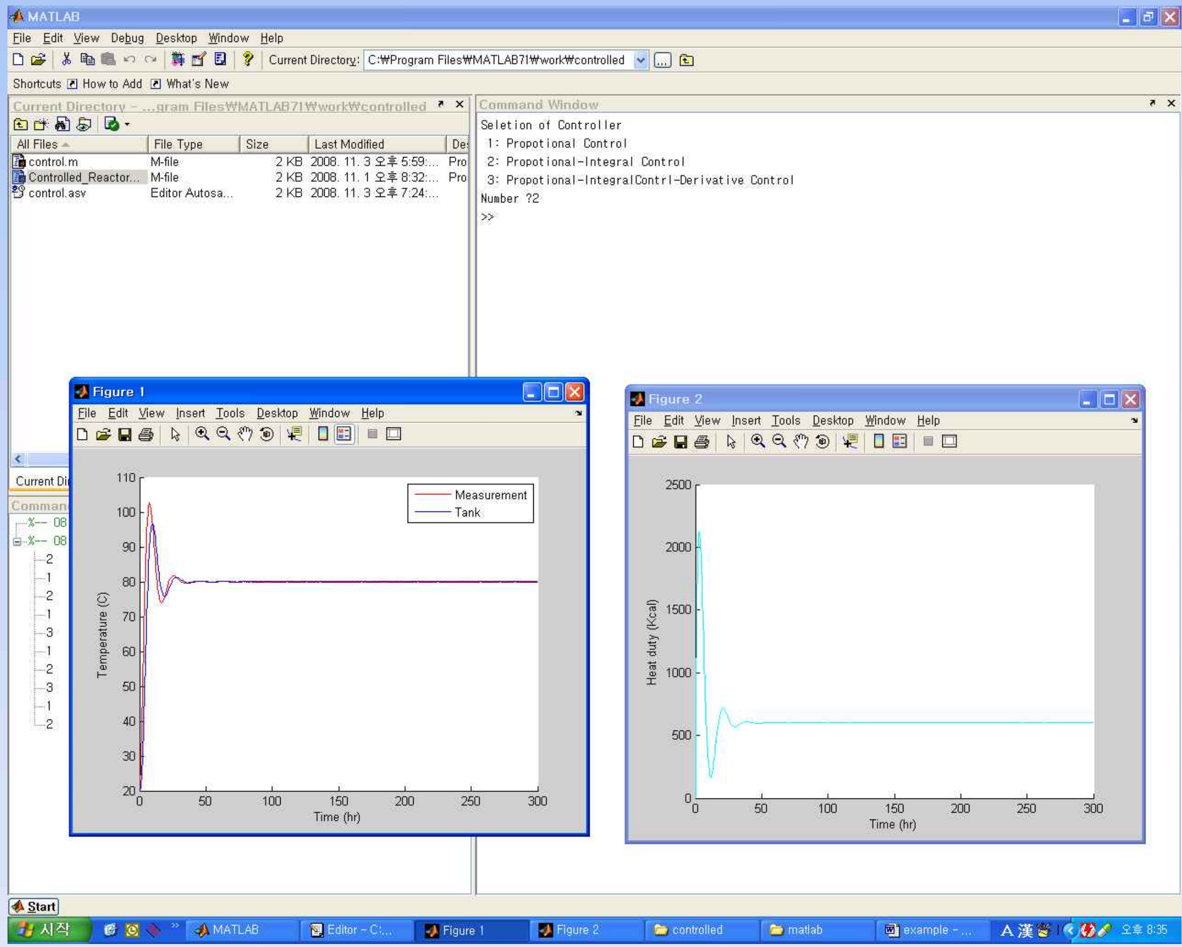
Exercises 5

P controller



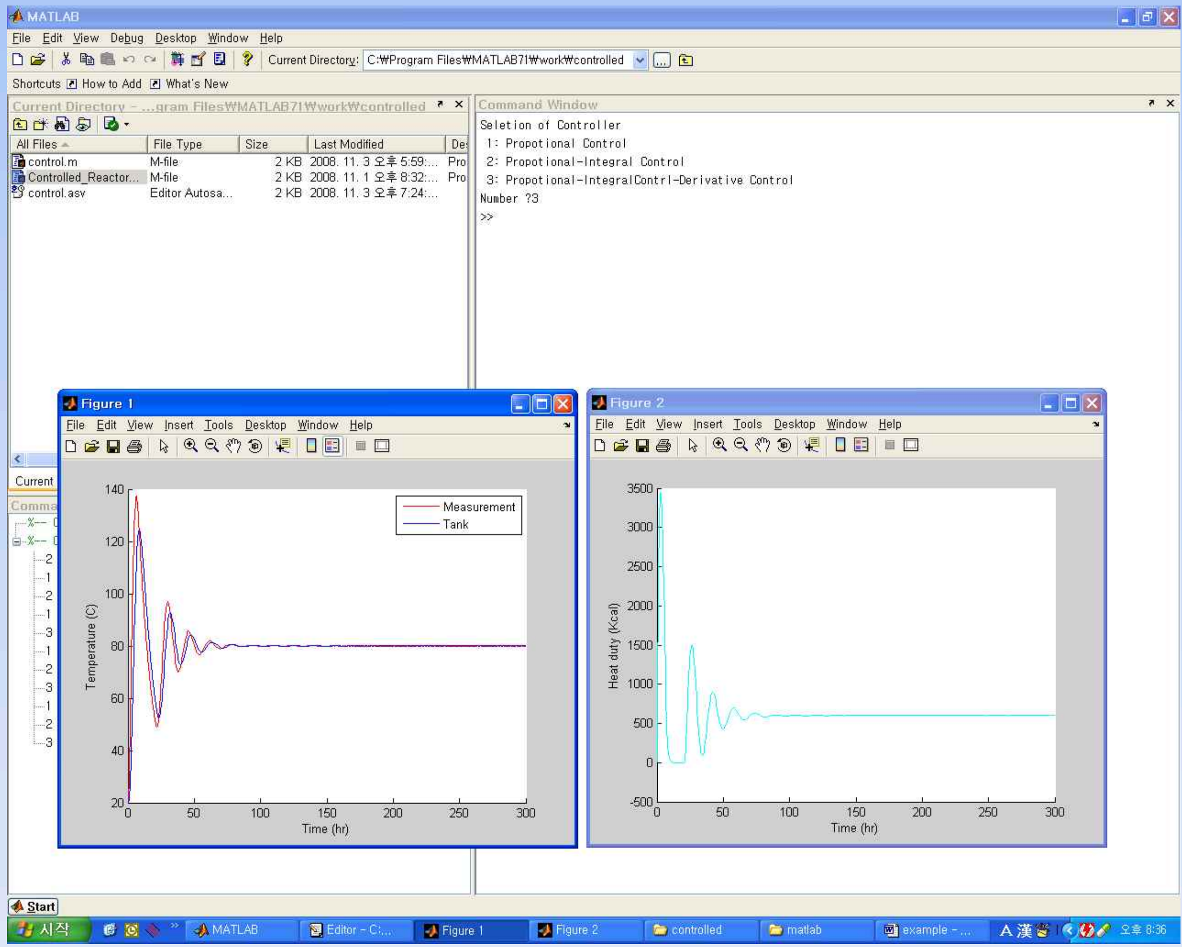
Exercises 5

PI controller



Exercises 5

PID controller



Exercises 6

- As explained in Ch. 7, try to establish the controller settings by trial and error.

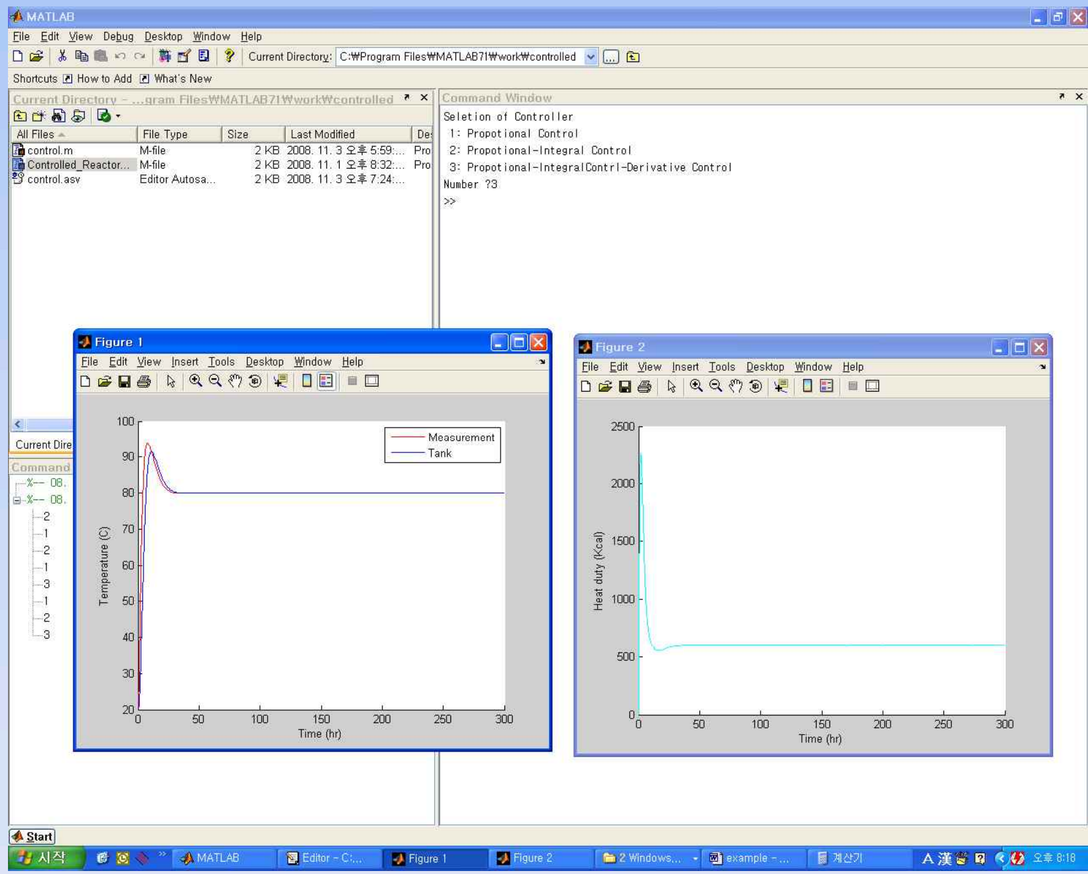
Trial and error method

Optimal parameter

$$Kp = 50$$

$$\tau_I = 8$$

$$\tau_D = 2$$



Exercises 7

- With proportional control only, increase K_P to K_{P0}, until oscillations in the response occur. Use this frequency, f_0, to set the controller according to the Ultimate Gain Method (K_P=0.45 K_{P0}, τ_I=f_0/1.2), where f_0 is the frequency of the oscillations at K_P=K_{P0} (see Ch. 7). Measure the response to a change in T_0

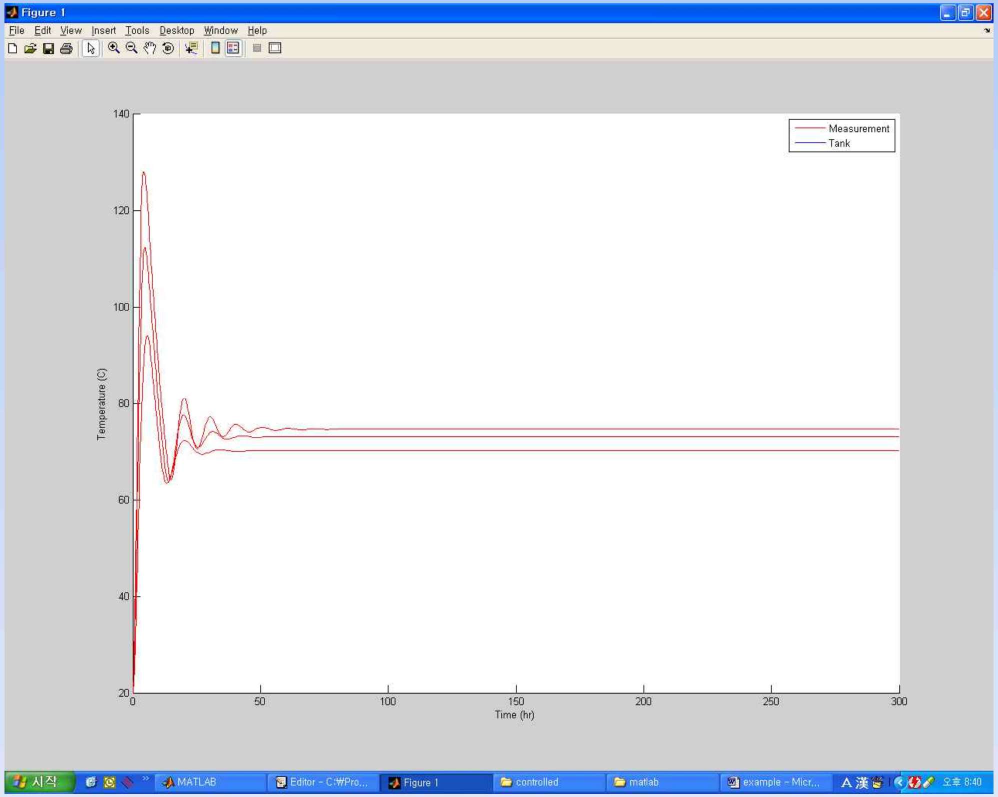
Table. Controller settings based on process responses (Ultimate Gain)

Controller	Kp	τ_I	τ_D
P	$0.5K_{P0}$		
PI	$0.45K_{P0}$	$f_0 / 1.2$	
PID	$0.6K_{P0}$	$f_0 / 2$	$f_0 / 8$

Exercises 7

- **With proportional control only, increase K_P to K_{P0}, until oscillations in the response occur. Use this frequency, f_0, to set the controller according to the Ultimate Gain Method (K_P=0.45 K_{P0}, τ_I=f_0/1.2), where f_0 is the frequency of the oscillations at K_P=K_{P0} (see Ch. 7). Measure the response to a change in T_0**

increase K_P to K_{P0}



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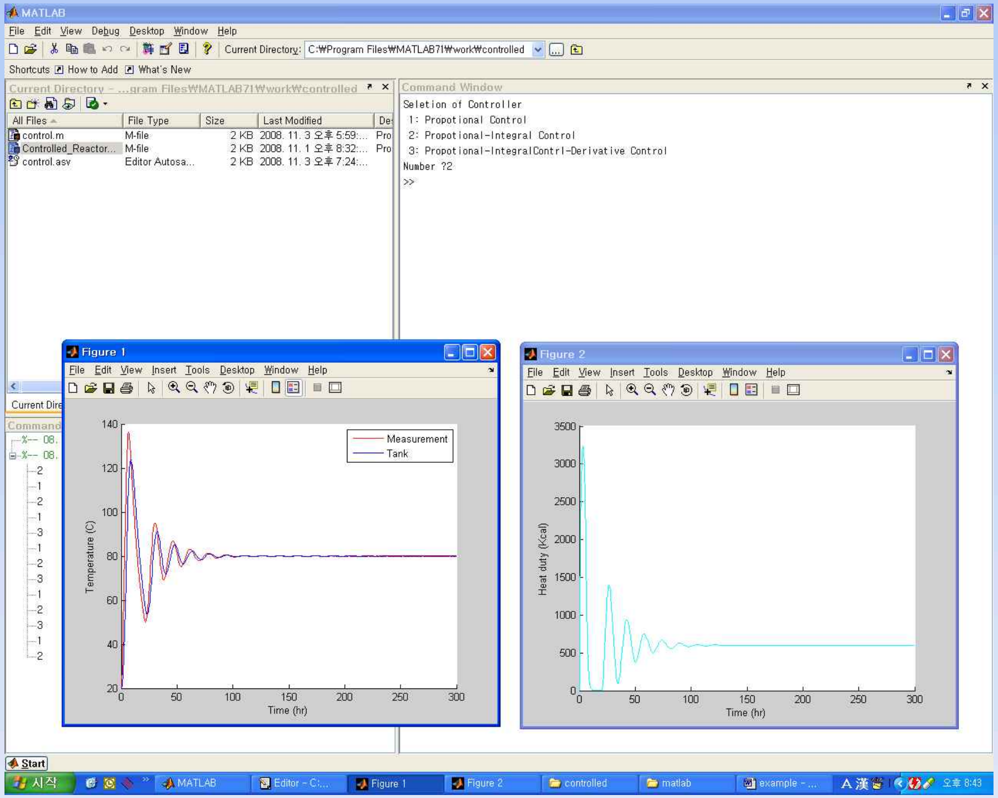
- With proportional control only, increase K_P to K_{P0}, until oscillations in the response occur. Use this frequency, f_0, to set the controller according to the Ultimate Gain Method (K_P=0.45 K_{P0}, τ_I=f_0/1.2), where f_0 is the frequency of the oscillations at K_P=K_{P0} (see Ch. 7). Measure the response to a change in T_0

Ultimate Gain method

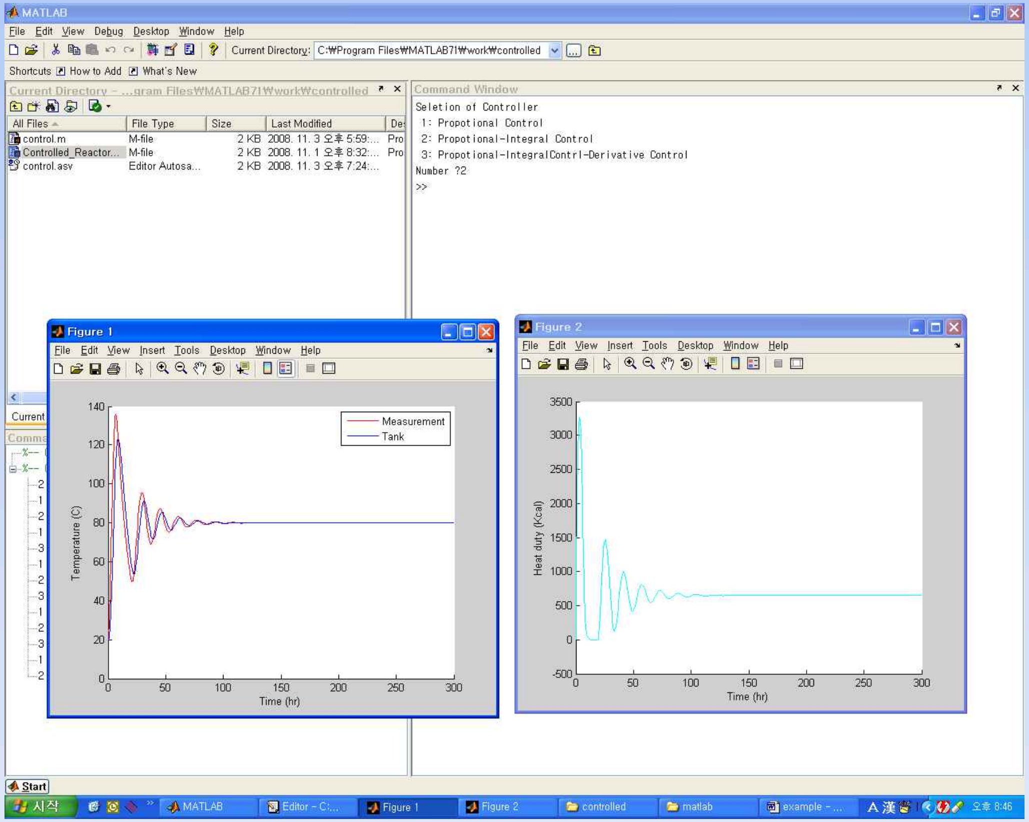
$$Kp = 45$$
$$\tau_I = 4.17$$
$$f_0 = 4$$



Exercises 7

- With proportional control only, increase K_P to K_{P0}, until oscillations in the response occur. Use this frequency, f_0, to set the controller according to the Ultimate Gain Method (K_P=0.45 K_{P0}, τ_I=f_0/1.2), where f_0 is the frequency of the oscillations at K_P=K_{P0} (see Ch. 7). Measure the response to a change in T_0

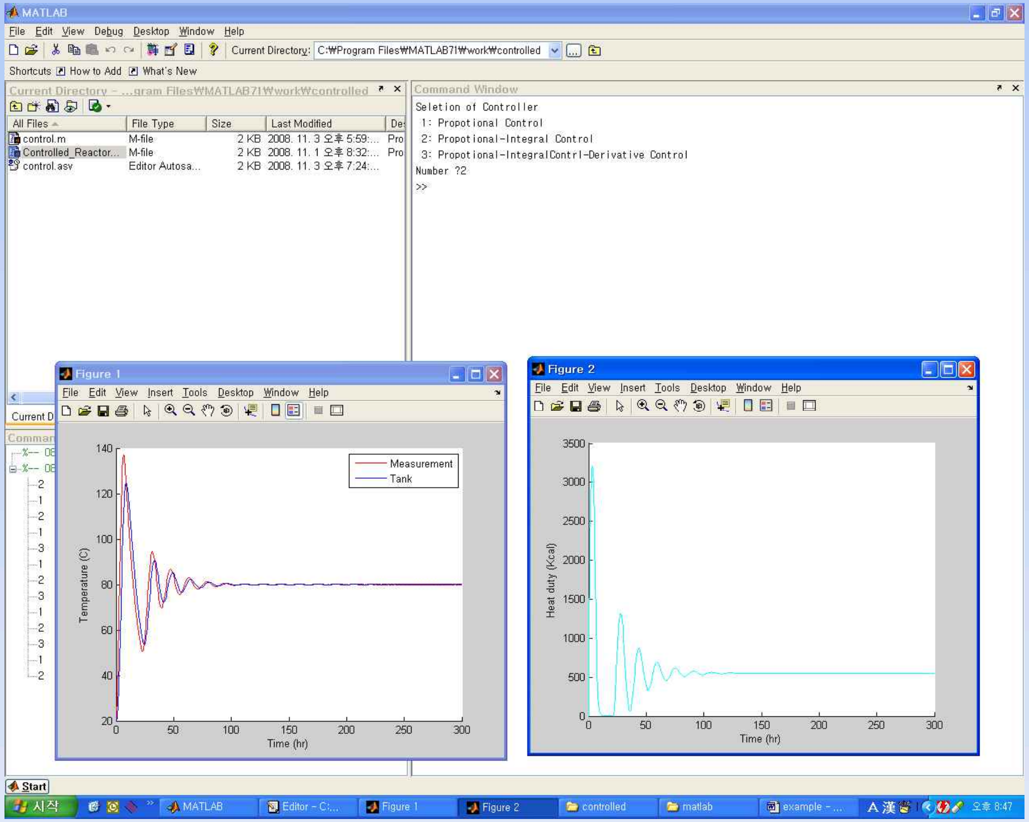
T_0 (20 → 15) with PI controller (Ultimate Gain method)



Exercises 7

- With proportional control only, increase K_P to K_{P0}, until oscillations in the response occur. Use this frequency, f_0, to set the controller according to the Ultimate Gain Method (K_P=0.45 K_{P0}, τ_I=f_0/1.2), where f_0 is the frequency of the oscillations at K_P=K_{P0} (see Ch. 7). Measure the response to a change in T_0

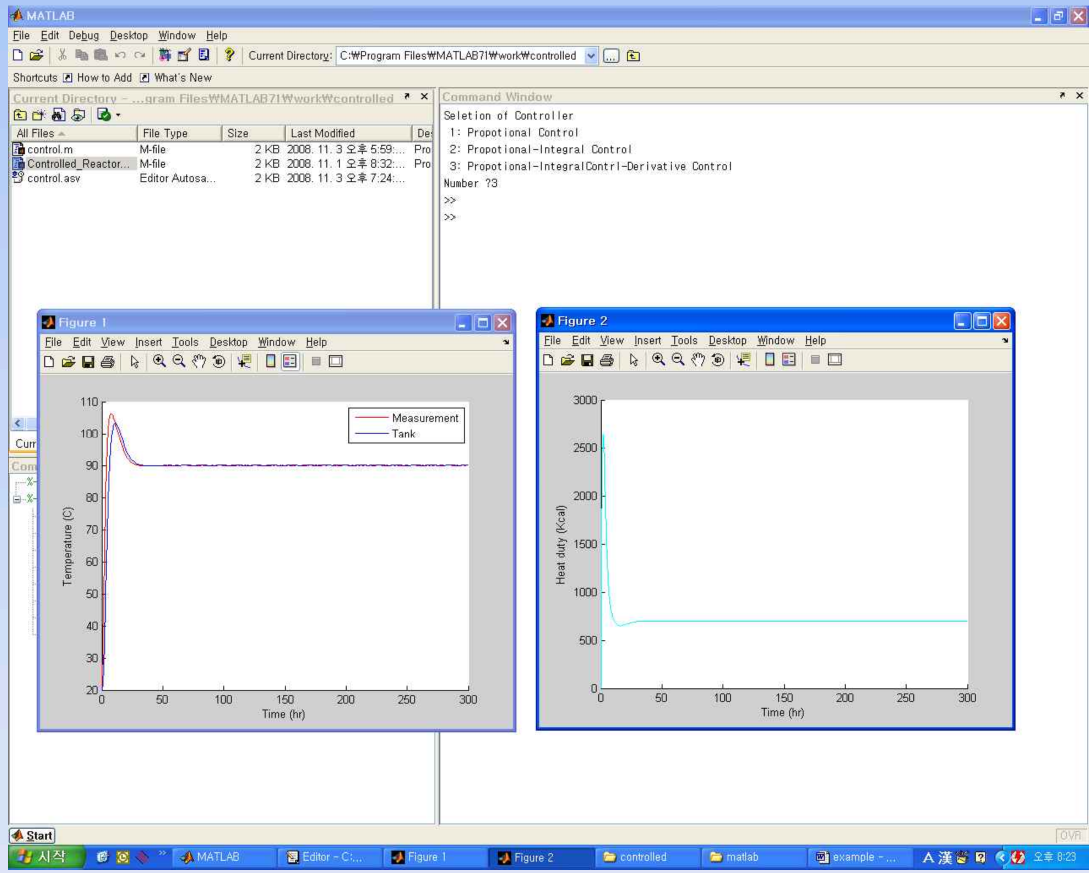
T_0 (20 → 25) with PI controller (Ultimate Gain method)



Exercises 8

- Adjust the controller by trial and error to obtain the best results to a change in set point.

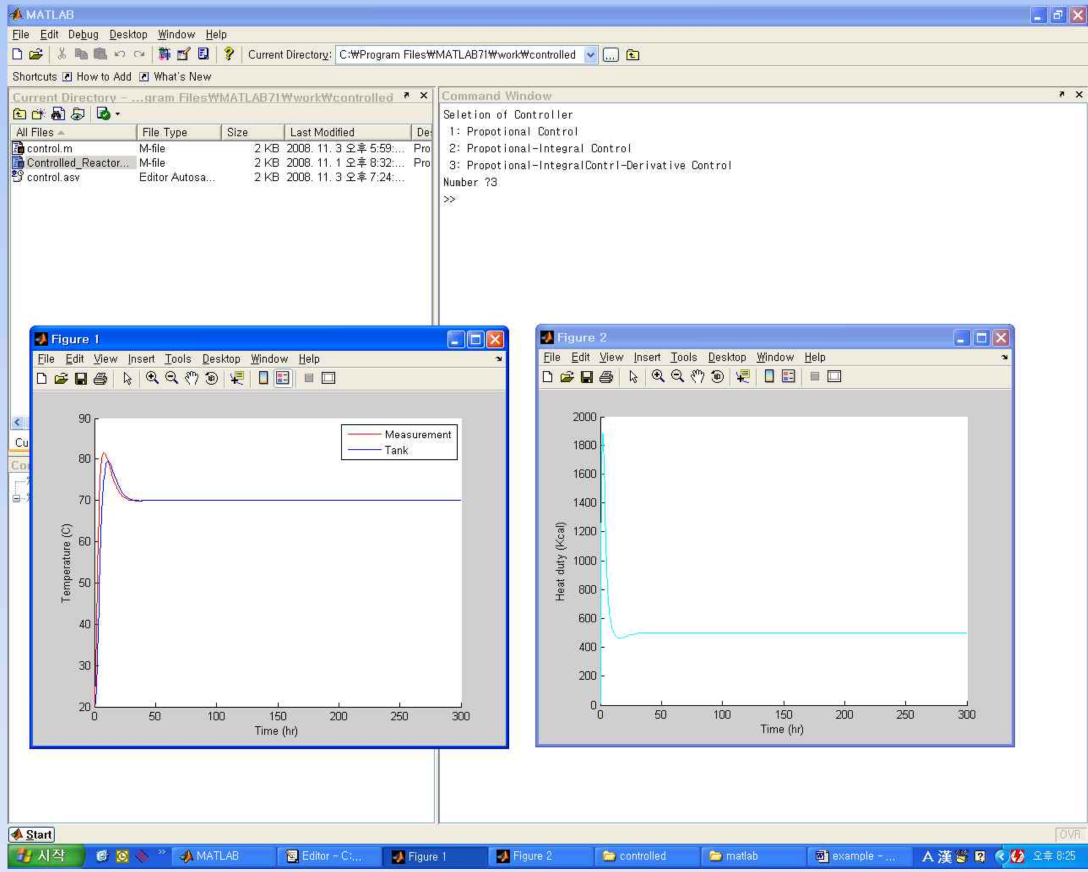
Set point change
(80 ºC → 90 ºC)



Exercises 8

- Adjust the controller by trial and error to obtain the best results to a change in set point.

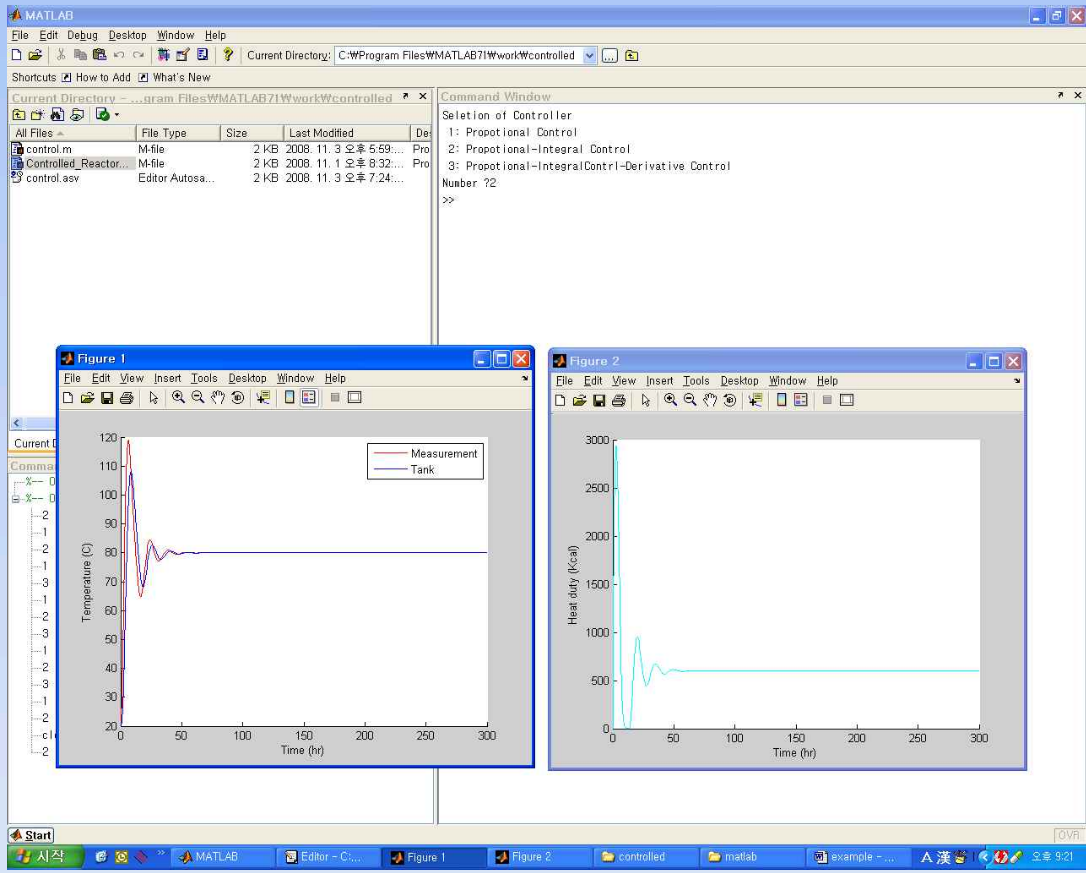
Set point change
(80 ºC → 70 ºC)



Exercises 9

- Include a loss term in the energy balance.

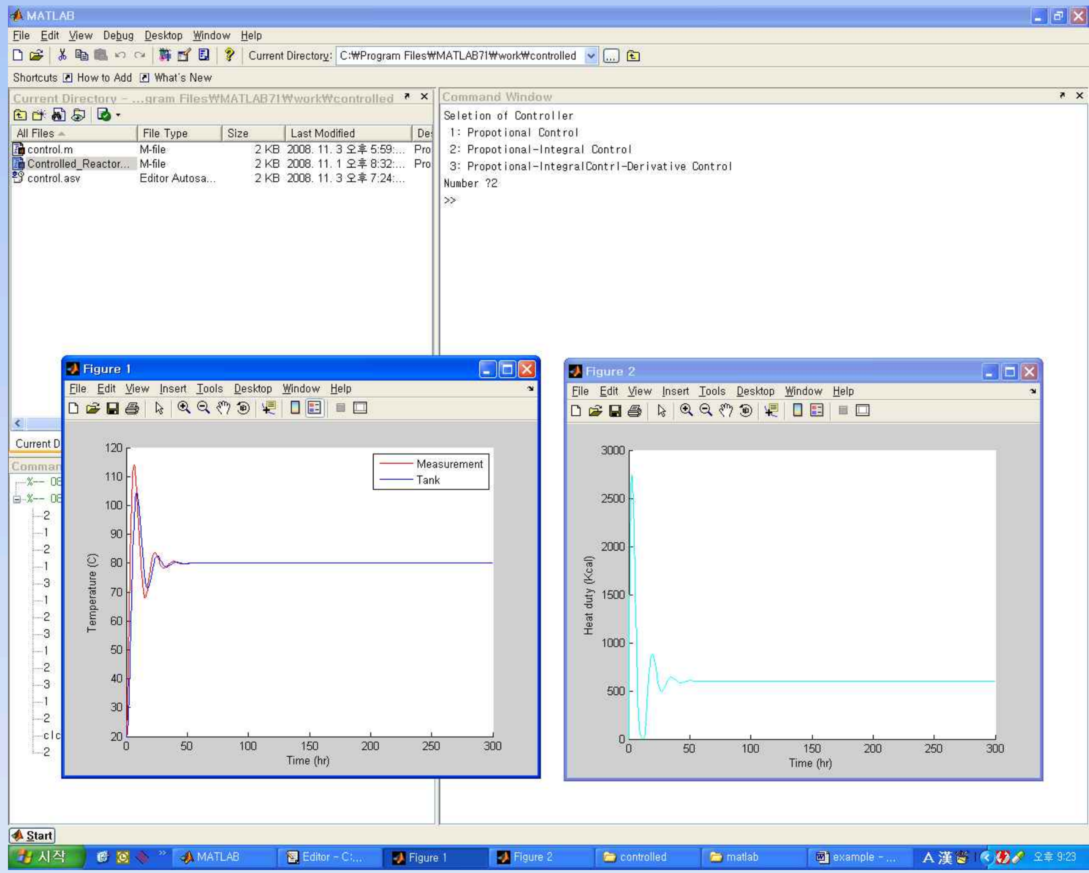
Trial and error method
(Q loss = 0)



Exercises 9

❖ **Include a loss term in the energy balance.**

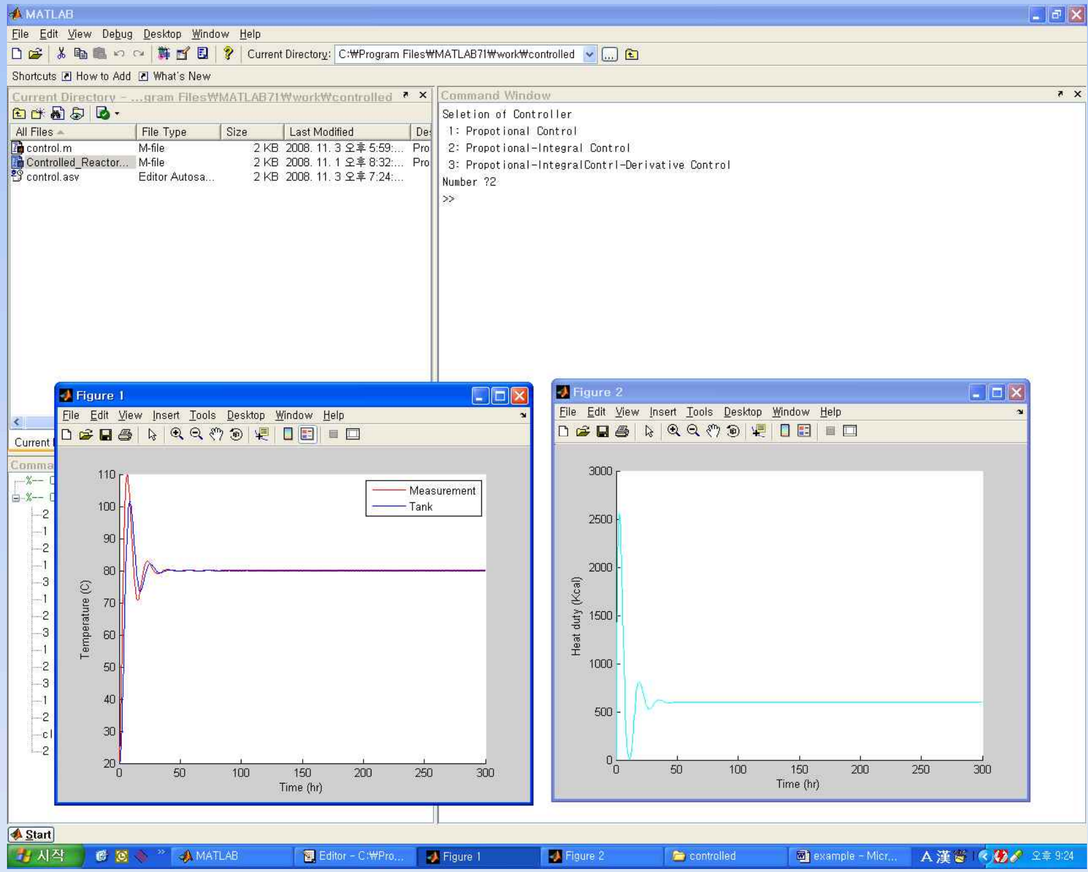
Trial and error method
(Q loss = 10%)



Exercises 9

❖**Include a loss term in the energy balance.**

Trial and error method
(Q loss = 20%)



Exercises 10

- Change the controller to give PID-control by adding a differential term,

Exercises 11

- Include a first order time lag in the temperature measurement using the equations :

$$\frac{dT_m}{dt} = \frac{T_R - T_m}{\tau_m}$$

$$\varepsilon = T_{RSet} - T_m$$

Conclusions

- A simple feedback control system involving a stirred tank, temperature measurement, controller and manipulated heater is shown .
- The Cohen-coon method were also compared with the predicted values by using the controllers of P, PI and PID. Among them, the P controller gave the best results.
- The best results of Ziegler-Nichols method follows in the order of PID > PI > P controller.
- Trial and error method of controllers can be adjusted by changing the values of gain K_P, reset time and derivative time .