Chapter 8 Particle Size Reduction(Chapter 10)

8.1 Introduction

Preparation of particles

- *Size reduction (comminution, disintegration, breakdown): "3-μm wall"*
- *Growth (buildup): from atoms and molecules*

Why size reduction?

- *To create particles in a certain size and shape*
- *To increase the surface area available for next process*
- *To liberate valuable minerals held within particles*

** Size reduction process : extremely energy-intensive*

- *5 % of all electricity generated is used in size reduction*
- *Efficiency of size reduction < 1 %*

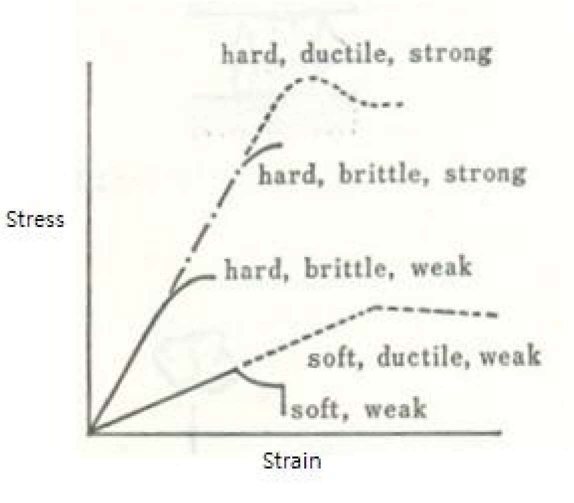
8.2 Particle Failure Mechanisms

(1) Stress-strain behavior

- *Interatomic force vs. interatomic distance*

Figure 10.1

Elastic vs. plastic



Classification of materials by stress-strain diagram

- *Yield strength*

  brittle vs. dutile(tough)

Strain energy : energy stored in a body under tension

: not uniform but concentrated in splits, cracks, hollow parts, foreign inclusions, and displacement

=the area under the stress-strain curve (Figure 10.1)

(2) Criteria for crack propagation *Griffith (1921)*

- *Strain energy > surface energy created*
- *For brittle fracture*

Fracture strength

$$\sigma_{ex} = \sqrt{\frac{2\gamma Y}{\pi L}}$$

where γ*: surface energy*

Y*: Young's modulus*

L*: half crack length*

cf. Theoretical strength of a crystalline material

$$\sigma_{th} = \sqrt{\frac{\gamma Y}{a_0}}$$

where a_0*: distance between two adjacent atomic planes in specific direction*

Multiple fracture

$\because$ *Dissipation velocity of excess strain energy = sonic*

> crack propagation velocity

(3) Size effect on fracture strength, S

- *Crack length distribution: Weibull distribution*

$$f(L) = \alpha m L^{m-1} \exp(-\alpha L^m)$$

where $\alpha > 0$ *and* $m > 1$*(shape parameter)*

- *By statistical analysis*

Compressive fracture strength for sphere, S

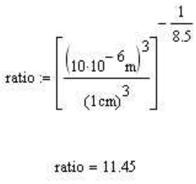
$$S \propto V^{-\frac{1}{m}} \sim x^{-\frac{3}{m}}$$

where V*: volume of specimen*

m*: shape factor*

Example

Weibull 분포함수의 shape factor m*이 8.5인 재료가 있다. 같은 재료로 이루어진 10*μm*인 입자와 1cm인 입자의 강도의 비를 구하여라.*



$$\text{ratio} := \left[\frac{\left(10\cdot 10^{-6}\,\text{m}\right)^3}{(1\,\text{cm})^3}\right]^{-\frac{1}{8.5}}$$

$$\text{ratio} = 11.45$$

- *Specific fracture energy for fracture between two parallel plates*

$$\frac{E}{M} = \frac{0.897\pi^{2/3}}{\rho_p}\left(\frac{1-\nu}{Y}\right)^{2/3} S^{5/3}$$

where M*: mass of the sphere*

ν*: Poisson ratio*

Substituting the expression for S

$$\frac{E}{M} \propto x^{-\frac{5}{m}}$$

* *For tough materials, plastic deformation rather than crack propagation*

8.3 Models Predicting Size Reduction and Product Size Distribution

(1) Energy Requirement

- *Rittinger(1867)*

$$E = C_R\left[\frac{1}{x_2} - \frac{1}{x_1}\right] \quad or \quad \frac{dE}{dx} = -C_R\frac{1}{x^2}$$

where x_1, x_2*: diameters of initial and final particles*

C_R: a constant

- Kick(1885)

$$E = C_K \ln\left(\frac{x_1}{x_2}\right) \quad or \quad \frac{dE}{dx} = C_K \frac{1}{x}$$

where C_K: a constant

- Bond(1952)

$$E = C_B\left(\frac{1}{\sqrt{x_2}} - \frac{1}{\sqrt{x_1}}\right) \quad or \quad \frac{dE}{dx} = C_B \frac{1}{x^{3/2}}$$

or

$$E_B = W_1\left(\frac{10}{\sqrt{x_2}} - \frac{10}{\sqrt{x_1}}\right)$$

where x_1, x_2 : top particle sizes before and after, or
µ the sieve sizes in m through which 80%
powders in the feed and product, respectively.

W_1 : Bond work index

e.g. $W_I = 9.45\,kWh/ton$ for bauxite
$= 20.7$ for coke from coal
$= 8.16$ for gypsum rock

- In general, Lewis

$$\frac{dE}{dx} = -\frac{C}{x^N}$$

where $N = 2$ for Rittinger
$= 1$ for Kick
$= 1.5$ for Bond

Figure 10.2

Kick → Bond → Rittinger as $x \downarrow$

Worked Example 10.1

(2) Fracture Kinetics

Based on feed particles

R*: mass fraction of feed particles*

$$\frac{dR}{dt} = -k_1 R$$

where k_1*: first-order rate constant,* $[s^{-1}]$

Based on fine particles under certain size, x_{fine}

$F_m(x_{fine})$*: cumulative mass fraction of fine particles under* x_{fine}

$$\frac{dF_m(x_{fine})}{dt} = k_x$$

where k_x*: zeroth-order rate constant,* $[s^{-1}]$

Based on specific surface area, $S_w\left(\frac{m^2}{g}\right)$

$$\frac{dS_w}{dt} = k_s(S_\infty - S_w)$$

where S_∞*: limit on specific surface area*

k_s*: rate constant,* $[s^{-1}]$

Initial rate

$$\frac{dS_w}{dt} = k_{s1}$$

Ultimate rate

$$\frac{dS_w}{dt} = k_{s2} S_w$$

Integrating

$$\ln\frac{S_\infty - S_w}{S_\infty - S_0} = -k_s t$$

Since $S_\infty \gg S_0$

$$S_w = S_\infty(1 - e^{-k_s t})$$

Example.

Derive the ultimate rate expression from Lewis equation.

From lewis equation

$$\frac{dE}{dx} = -\frac{C}{x^N} \quad \rightarrow \frac{dx}{dE} = -C' x^N$$

$x \sim x_{sv} \sim S_w^{-1}$ *and* $E \propto t$

$$\frac{d\left(\frac{1}{S_w}\right)}{dt} = -C\left(\frac{1}{S_w}\right)^N$$

or

$$\frac{dS_w}{dt} = k_s S_w^{2-N}$$

If N=1, the equation turns to $\frac{dS_w}{dt} = k_{s2} S_w \ldots$

(3) Prediction of the Product Size Distribution

Grouping of particles with respect to size interval (i=1,2,3....)

Definitions

m_i*: mass of particles of size interval i*

S_j *: the specific rate of breakage*

- probability of a particle of size j being broken in unit time

$b(i, j)$ *: breakage distribution function*

- fraction of size i from the breakage of mother particle j

$$i < j$$

Then population balance:

$$\frac{dm_i}{dt} = \sum_{j=1}^{j=i-1} b(i, j) S_j m_j - S_i m_i$$

where i < j

Figure 10.3

* $B(i,j)$*: fraction of i from* $j \rightarrow j$ *to n*

In terms of mass fraction

$$\frac{dx_i}{dt} = \sum_{j=0}^{j=i-1} b(i, j) S_j x_j - S_i x_i$$

* $b(i,j)$, S_j*: determined from lab-scale experiment*

** Size distribution from comminution*

Rosin-Rammler equation

$$1 - F_m(x) = 100\exp(-ax^n)$$

8.4 Types of Comminution Equipment

(1) Factors Affecting Choice of Machine

- *Stressing mechanism*
- *Mode of operation : batch/continuous or open/closed circuit*
- *Capacity*
- *Size of feed and product*
- *Material properties*
- *Carrier medium : air/inert gas/water/oil*
- *Integration with other unit operation : drying, classification, mixing, transportation, storage*

(2) Stressing Mechanisms

Between two solid surfaces *Figure 10.4*

- *Particle-surface and particle-particle*
- *By crushing(pressure) and attrition(friction)*
- *0.01 - 10m/s*
- *For coarse(< 100mm) and medium-coarse size reduction (< 10mm)*
- *For medium-hard(Moh's:4-6) to medium materials(Moh's:7-10)*

Jaw crusher(Figure 10.6)
Gyratory crusher(Figure 10.7)
Crushing roll(Figure 10.8)
Horizontal table mill(Figure 10.9)

Against a solid surface *Figure 10.5*

- Particle against surface and another particle

- 10-200m/s, high velocity impact

- By impact + attrition

- Medium-fine to ultrafine comminution

Hammer mill(Figure 10.10)
Pin mill(Figure 10.11)
Fluid energy mill(Jet mill)(Figure 10.12)

Applied by carrier medium (water or oil)

- Wet grinding

- Crushing + impact + shearing(attrition)

- Finer products/lowering dust emission/30% energy saving*

- Higher wear/needs wastewater treatment

Sand mill(Figure 10.13)
Colloid mill(Figure 10.14)
Ball mill(Figure 10.15) - both wet and dry

** Ultrafine grinding(<1μm)*

Applied by nonmechanical introduction of energy

- Thermal shock, explosive shattering or electrohydraulic

(3) Particle Size

Terminologies of comminution according to particle size Table 10.1

crushing>grinding
coarse>intermediate>fine

Comminution equipment according to particle size Table 10.2

(4) Material Properties

- *Abrasive: low-speed operation*

  Hardness: resistance to abrasion
- *Tough: reducing temperature*
- *Co-adhesive(adhesive): wet grinding*
- *Fibrous: shredders, cutters*
- *Low melting: using cold air*
- *Thermally sensitive, flammabe: inert carrier medium(N_2)*
- *Toxic/radioactive: closed system*

(5) Carrier Medium

- *Air, inert gas*
- *Water, oil*

(6) Mode of Operation

- *Batch vs. continuous*

(7) Types of Milling Circuits

- *Open circuit vs. closed circuit*

  Figure 10.16 Figure 10.17, 10.18