Chapter 6 Storage and Flow of Powders - Hopper Design (Chapter 8)

☞*http://www.dietmar-schulze.de/storage.html*

6.1 Introduction

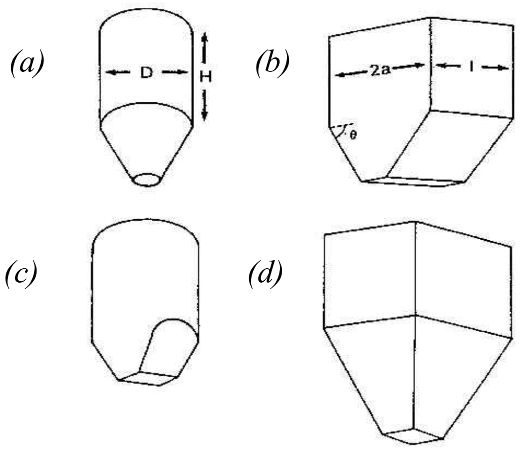
Storage tanks

Silos : section of constant cross sectional area

- *Bins : H > 1.5 D*
- *Bunker : H < 1.5 D*

Hopper : section of reducing cross sectional area downwards

Typical bulk solids storage vessels



(a) conical and axisymmetric hopper; (b) plane-flow wedge hopper; (c) plane flow chisel hopper; (d)pyramid hopper

6.2 Mass Flow vs. Core(Funnel) Flow

Mass flow vs. core flow : Figure 8.1

Figure 8.2 Figure 8.3

To see the mass flow in hopper ☞

http://www.cco.caltech.edu/~granflow/movies.html

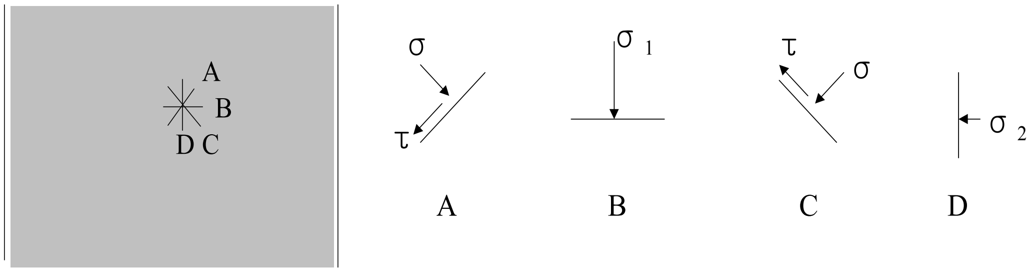
Table *Comparison of mass flow and core flow of particulate materials.*

Mass flow	*Core flow*
Characteristics	
No stagnant	*Stagnant zone formation*
Uses full cross-section of vessel	*Flow occurs within a portion of vessel cross-section*
First-in, first-out flow	*First-in, last-out flow*
Advantages	
Minimises segregation, agglomeration of materials during discharge	*Small stress on vessel walls during flow due to the 'buffer effect' of stagnant zones*
	Very low particle velocities close to vessel walls; reduced particle attrition and wall wear
Disadvantages	
Large stresses on vessel walls during flow	*Promotes segregation and agglomeration during flow*
Attrition of particles and erosion and wear of vessel wall surface due to high particle velocities	*Discharge rate less predictable as flow region boundary can alter with time.*
Small storage volume to vessel height ratio	

6.2S Stresses in Bulk Solids

(1) Mohr Stress Circle

Two dimensional stresses in the powder bed



σ - *Normal stress,*

τ - *Shear(tangential) Stress,*

분체층은 정지되어 있어도 유체와는 달리

- *수직응력(유체에서 보통 압력으로 부름)이외에 전단력이 존재하고,*
- *수직응력과 전단응력은 면의 배향에 따라서도 달라진다.*

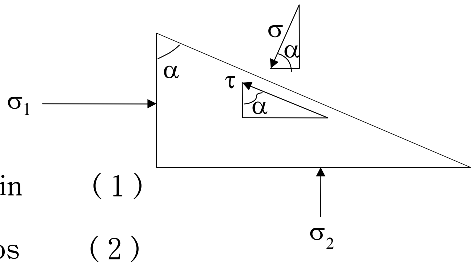
- Principal stresses (major, minor), σ_1, σ_2

: normal stresses to the plane in which shear stresses vanish

where $\sigma_1 \perp \sigma_2$

면의 배향에 따라서는 전단응력이 없어지는 경우가 두 개 생기며 이 두면은 서로 수직하고, 하나는 최대, 다른 하나는 최소 응력을 가진다.

Correlating σ *and* τ*, in terms of principal stresses*



From force balances,

$$\sigma_1 \cos\alpha = \sigma\cos\alpha + \tau\sin\alpha \qquad (1)$$

$$\sigma_2 \sin\alpha = \sigma\sin\alpha - \tau\cos\alpha \qquad (2)$$

$(1)\times\cos\alpha + (2)\times\sin\alpha$

$$\sigma_1\cos^2\alpha + \sigma_2\sin^2\alpha = \sigma$$

$(1)^2 + (2)^2$

$$\sigma_1^2\cos^2\alpha + \sigma_2^2\sin^2\alpha = \sigma^2 + \tau^2$$

Eliminating $\sin\alpha$ *and* $\cos\alpha$

$$\left(\sigma - \frac{\sigma_1 + \sigma_2}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2$$

Mohr Stress Circle

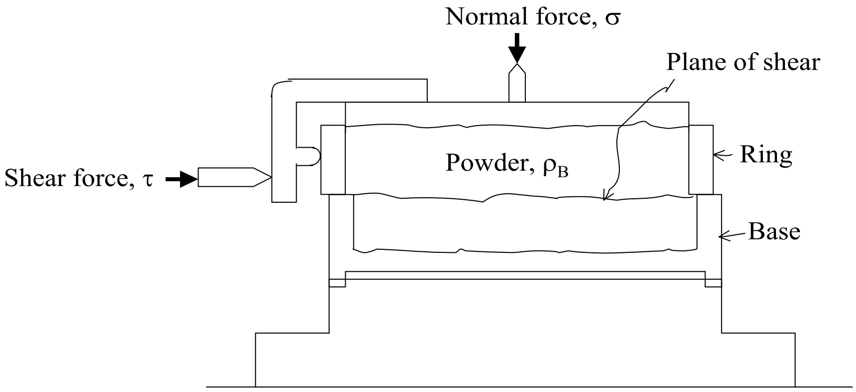
6.4 Shear Cell Tests - Yield Behavior of Bulk Powder

Powder Bed :

- Fixed : Adsorption beds, catalyst beds, packed beds for absorber

- Moving : Feeding in storage tank

Jenike shear cell



Jenike yield locus

**Construction method*

ρ - *Put the powder sample of* $_B$ *in the cell.*

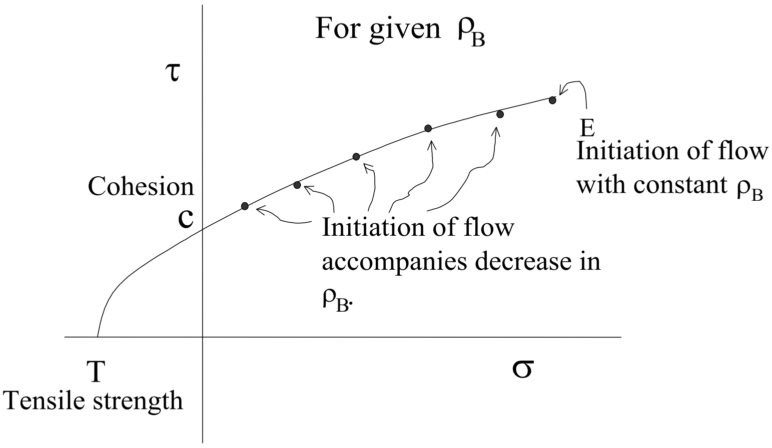
σ - *Note the horizontal stress( ) to initiate flow for the given normal stress( ).*

ρσ *(must be low enough for* $_B$ *to decrease during application of)*

ρ - *Repeat this procedure for each identical powder sample(* $_B$*) with greater*

ρσ *until* $_B$ *does not decrease. Five or six pairs of (,) should be generated.*

ρ - *Particle bed is about to move on the Janike yield locus for given* $_B$.



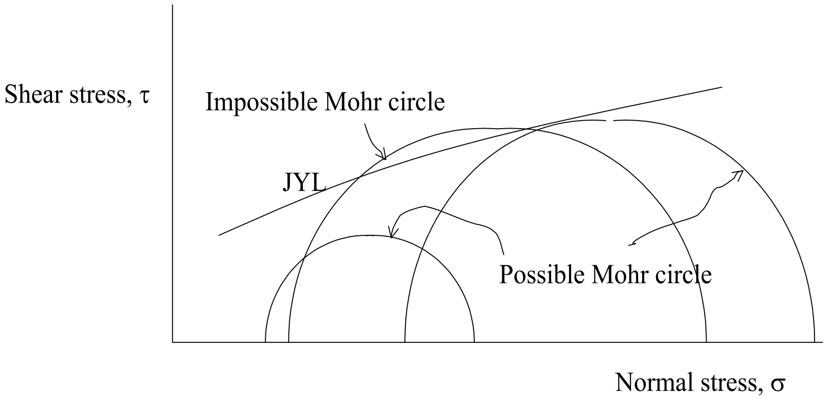
Janike yield locus

** Definition*

- *Expanded flow(at the points up to E on the curve)*
- *Free flow (at point E): critical flow*
- *Cohesion*
- *Tensile strength*

6.5 Analysis of Shear Cell Test Results

(1) JYL vs. Mohr stress circle



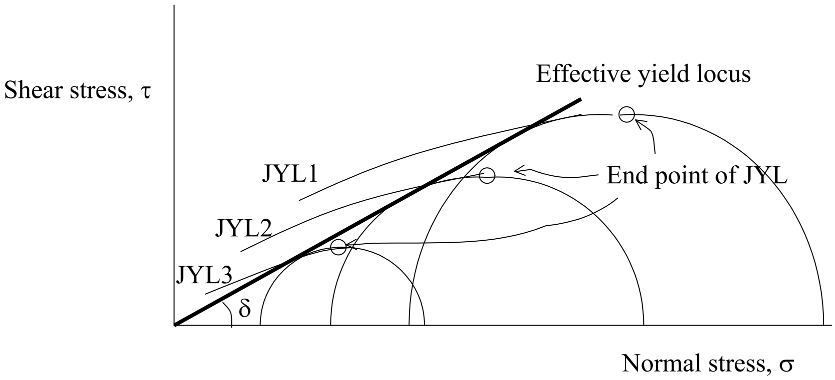
All the Mohr circles exist below JYL...

(2) Determination of δ from Shear Cell Tests

* *Effective Yield Locus*

- *Tangent line of the Mohr circles passing E's (end points) of JYL's for different*
ρ *$_B$'s is straight...*
σ - *From its slope, the effective internal angle of friction is obtained by*
δσ

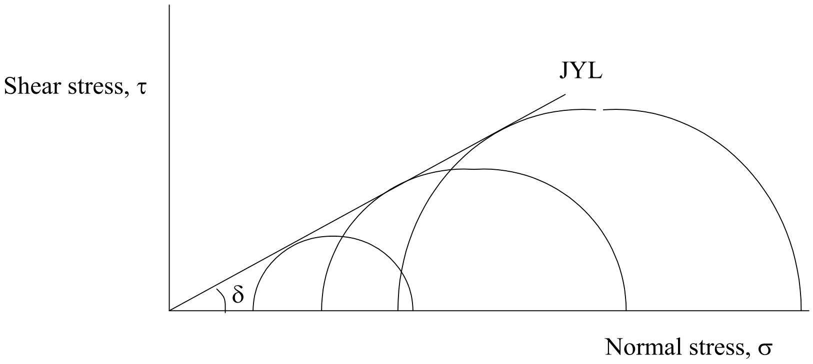
$$\therefore \; = \; \tan$$



Worked Example 8.1(a)

Ex.8.4, 8.5

* *For free flowing powder*

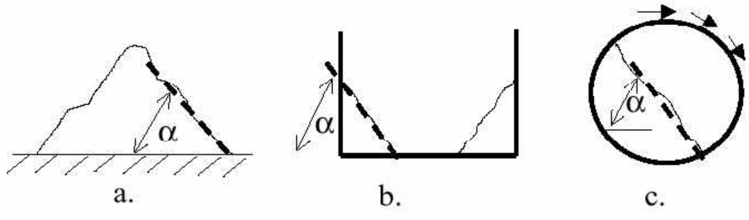


Yield locus for noncohesive (free-flow) powder

α * *Angle of Repose, (安息角)*

For noncohesive(free-flowing) particles

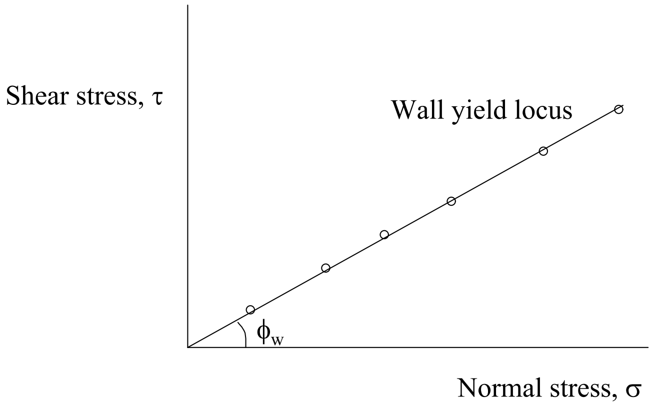
ɸ ~



Angle of repose, α, of (a) a pile of powder, (b) powder in a container, and (c) powder in a rolling drum.

Wall Yield Locus

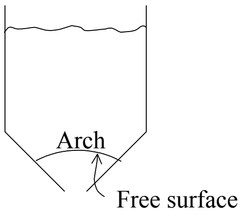
- *Similarly yield locus of powder bed against wall can be found from wall shear test...☞ Figure 8.16*
- *Straight line passing the origin....*



$$\Phi \qquad {}_w = {}_w \tan {}_w$$

6.3 Design Philosophy

Arching

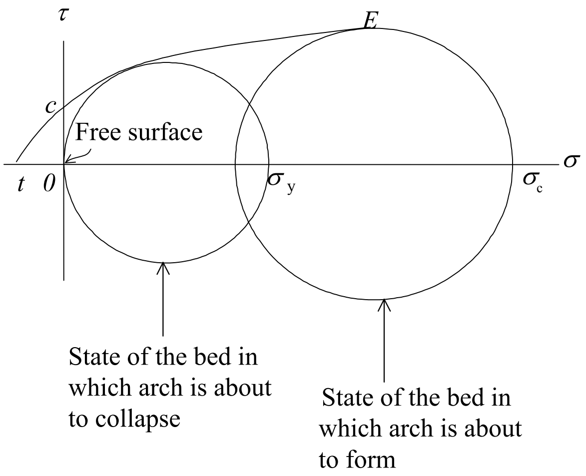


Arch - free surface, no flow

e.g. a salt shaker (a salt pourer?)

(1) Determination of σ_y and σ_c

- Two limiting Mohr circles for arch formation



σ_y: *Unconfined yield strength (UYS),*

σ_C: *Compacting(consolidating) stress*

(3) Flow-nonflow condition *(for breaking arch)*

For flow

$$\sigma \qquad {}_D \;\rangle\; \qquad {}_y$$

where σ_D*: actual stress of the powder developed under hopper condition*

(4) Critical Outlet Diameter

To avoid the formation of arch...

Langmaid

$$\frac{S}{x_{sv}} = 1.8 + 0.038\left(\frac{\phi_s}{\phi_v}\right)^{1.8}$$

$$\frac{D_c}{x_{sv}} = 2.3 + 0.071\left(\frac{\phi_s}{\phi_v}\right)^{1.8}$$

where S*: critical slit width and* D_c*: critical orifice diameter*

ϕ_s*: specific surface area shape coefficient*

$$\phi_s = \frac{S}{x_v^2}$$

ϕ_v*: specific volume shape coefficient*

$$\phi_v = \frac{V}{x_v^3}$$

For spheres, since $\frac{\phi_s}{\phi_v} = 6$

$$\frac{S}{x_{sv}} = 2.8 \quad \text{and} \quad \frac{D_c}{x_{sv}} = 4.1$$

For crushed particles, $\frac{\phi_s}{\phi_v} = 10$

$$\frac{S}{x_{sv}} = 4.2 \quad \text{and} \quad \frac{D_c}{x_{sv}} = 6.8$$

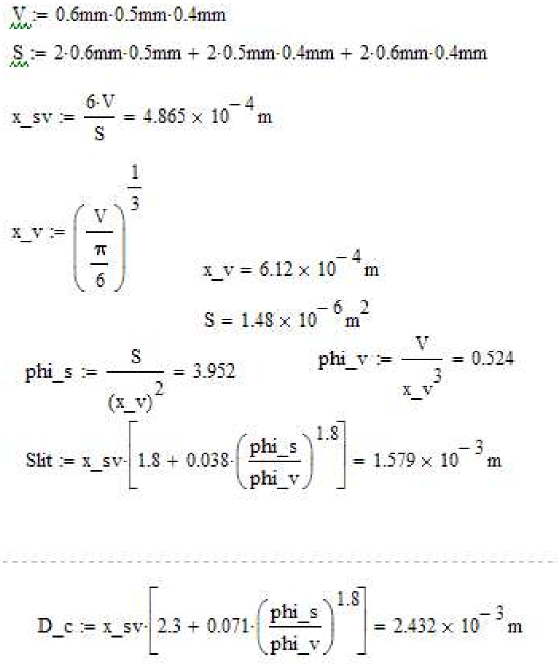
Pokrovski

$$\frac{D_c}{x_{sv}} = \frac{2\mu_i^2}{1-\mu_i^2}\left(0.5 + \frac{1-\mu_i}{\sqrt{(1+\mu_i^{2)}}}\right)$$

where μ_i*: internal friction coefficient*

Example

식탁소금병의 구멍크기를 설계하여라. 소금의 크기는 $0.6\times0.5\times0.4mm$ *직육면체라고 한다.*



V := 0.6mm·0.5mm·0.4mm

S := 2·0.6mm·0.5mm + 2·0.5mm·0.4mm + 2·0.6mm·0.4mm

$$x\_sv := \frac{6\cdot V}{S} = 4.865\times10^{-4}\,m$$

$$x\_v := \left(\frac{V}{\frac{\pi}{6}}\right)^{\frac{1}{3}} \qquad x\_v = 6.12\times10^{-4}\,m$$

$$S = 1.48\times10^{-6}\,m^2$$

$$phi\_s := \frac{S}{(x\_v)^2} = 3.952 \qquad phi\_v := \frac{V}{x\_v^3} = 0.524$$

$$Slit := x\_sv\cdot\left[1.8 + 0.038\cdot\left(\frac{phi\_s}{phi\_v}\right)^{1.8}\right] = 1.579\times10^{-3}\,m$$

$$D\_c := x\_sv\cdot\left[2.3 + 0.071\cdot\left(\frac{phi\_s}{phi\_v}\right)^{1.8}\right] = 2.432\times10^{-3}\,m$$

6.8 Pressure on the Base of a Tall Cylindrical Bin - Stresses in the Storage Tank

Vertical stress, σ_v

- *In the cylindrical bins*

Force balance on a slice of thickness ΔH *in the powder bed,*

$$\frac{\pi D^2}{4}\Delta\sigma_v + \pi D\tan\phi_w\sigma_h\Delta H = \frac{\pi D^2}{4}\rho_B g\Delta H$$

$$D\Delta\sigma_v + 4\tan\phi_w\sigma_h\Delta H = D\rho_B g\Delta H$$

Assuming $\sigma_H = k\sigma_v$ *and* $\Delta H \to 0$

$$\frac{d\sigma_v}{dH} + \left(\frac{4\tan\phi_w k}{D}\right)\sigma_v = \rho_B g$$

Integrating

ρ σ Φ

$$ {}_{v} = \frac{D \quad {}_{B} g}{4 \tan \quad {}_{w} k} [1 - e^{-4 \tan \quad {}_{w} kH/D}] + \quad {}_{v0} e^{-4 \tan \quad {}_{w} kH/D} $$

σ *When no force acting on the free surface of the powder* $\quad {}_{v0} = 0$,

ρ σ Φ

$$ {}_{v} = \frac{D \quad {}_{B} g}{4 \tan \quad {}_{w} k} [1 - e^{-4 \tan \quad {}_{w} kH/D}] $$

Janssen equation

For small H,

ρ

$$ {}_{v} \cong \quad {}_{B} Hg $$

(liquid-like)

For large H $(> 4D)$

ρ σ Φ

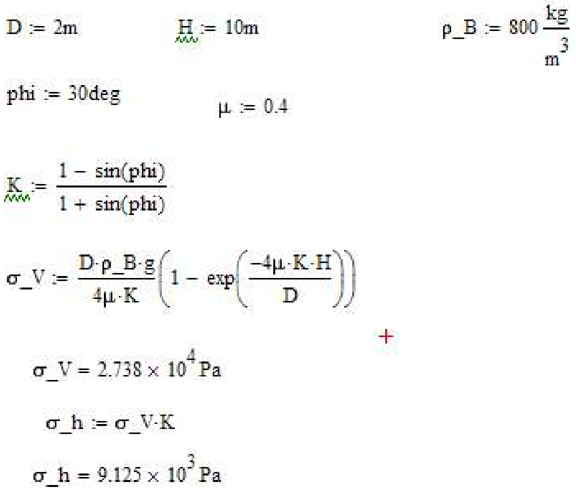
$$ {}_{v} \cong \frac{D \quad {}_{B} g}{4 \tan \quad {}_{w} k} $$

σ *independent of H and* $\quad {}_{v0}$

Figure 8.21

Example

직경 2m, 높이 10m의 원형 bin에 층비중 0.80ton/m³, 내부마찰각 30°의 분체를 채울 때 바닥에 미치는 수직압력과 수평압력을 구하여라. 벽마찰계수는 0.4라고 한다.



D := 2m

H := 10m

ρ_B := 800 $\frac{kg}{m^3}$

phi := 30deg

μ := 0.4

$$ K := \frac{1 - \sin(phi)}{1 + \sin(phi)} $$

$$ \sigma\_V := \frac{D \cdot \rho\_B \cdot g}{4\mu \cdot K}\left(1 - \exp\left(\frac{-4\mu \cdot K \cdot H}{D}\right)\right) $$

$\sigma\_V = 2.738 \times 10^4$ Pa

σ_h := σ_V·K

$\sigma\_h = 9.125 \times 10^3$ Pa

- *In hopper*

$For\quad C' \neq 1,$

ρ σ $$_{v,2} = \frac{_{B}gh}{C-1}\left\{1-\left(\frac{h}{H_2}\right)^{C-1}\right\} + \ _{0}\left(\frac{h}{H_2}\right)^{C}$$

$For\quad C' = 1$

ρ $$_{v,2} = \ _{B}h\ln\left(\frac{H_2}{h}\right) + \ _{0}\left(\frac{h}{H_2}\right)$$

⊕ $$where\quad C' \equiv 2\tan\ _{W}\cot\ _{1}(K\cos^{2}\ _{1} + \sin^{2}\ _{1})$$

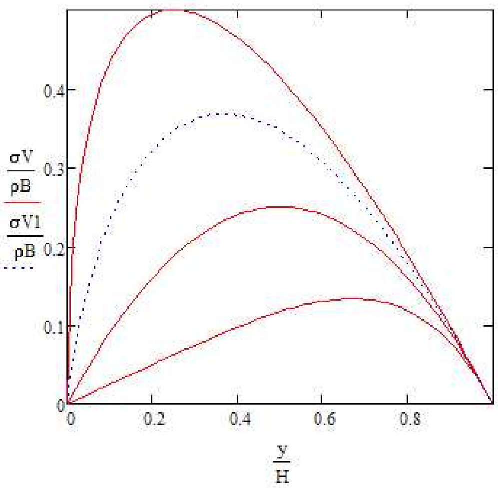
$\rho B := 800\frac{kg}{m^3}$ $\quad H := 1m$ $\quad N := 100$

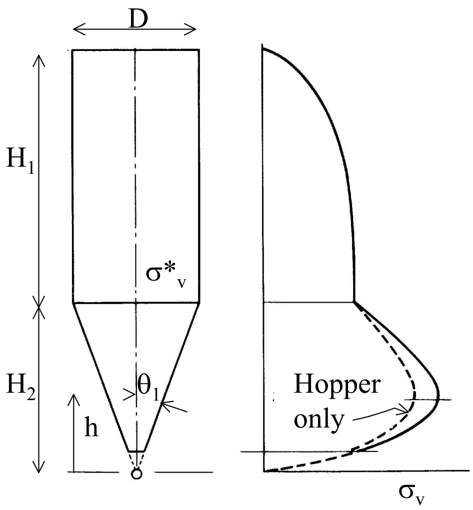
$j := 0..2$ $\quad C := \begin{pmatrix}0.5\\2\\5\end{pmatrix}$

$C = \begin{pmatrix}0.5\\2\\5\end{pmatrix}$ $\quad C_0 = 0.5$ $\quad y_1 := 0m$ $\quad i := 1..N$

$\sigma V_{i,j} := \frac{\rho B\cdot y_i}{C_j - 1}\left[1-\left(\frac{y_i}{H}\right)^{C_j-1}\right]$ $\quad y_i := y_{i-1} + \frac{1}{N}m$

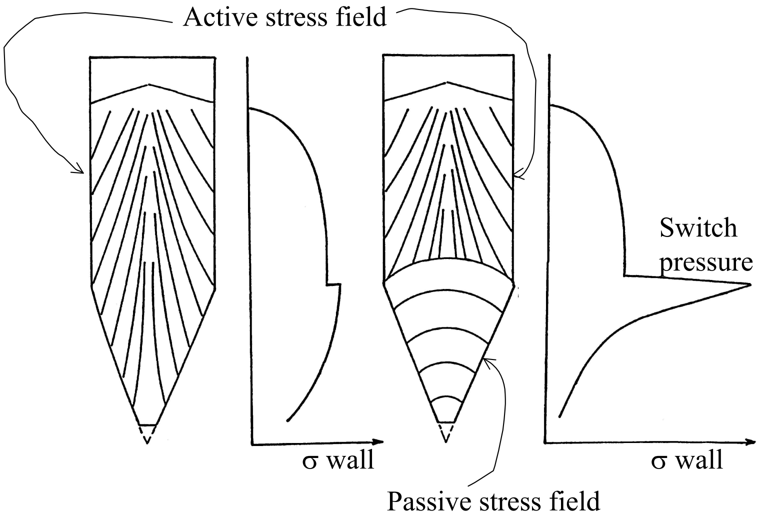
$\sigma V1_i := \rho B\cdot y_i\cdot\ln\left(\frac{H}{y_i}\right)$





Wall stress distribution in silo-hopper

Storage tank내의 압력은 저장 중에는 위의 정압과 일치하나 feeding과 discharge시에는 달라진다.



Initial filling

Flow

6.9 Mass Flow (Discharge) Rate

For cylindrical and conical hoppers

Beverloo(1961)

Dimensional analysis

ρ

$$M_p = C\ \ _Bg^{1/2}(B_0 - kx)^{5/2}$$

where C *: 0.55 ~ 0.65*

k *: 1.5 or somewhat larger depending on particle shape*

- Independent of H, D

For cohesionless coarse particles

π
ρ

$$M_p = \frac{}{4}\sqrt{2}\ \ _Bg^{0.5}h^{0.5}B^2$$

Other empirical equations

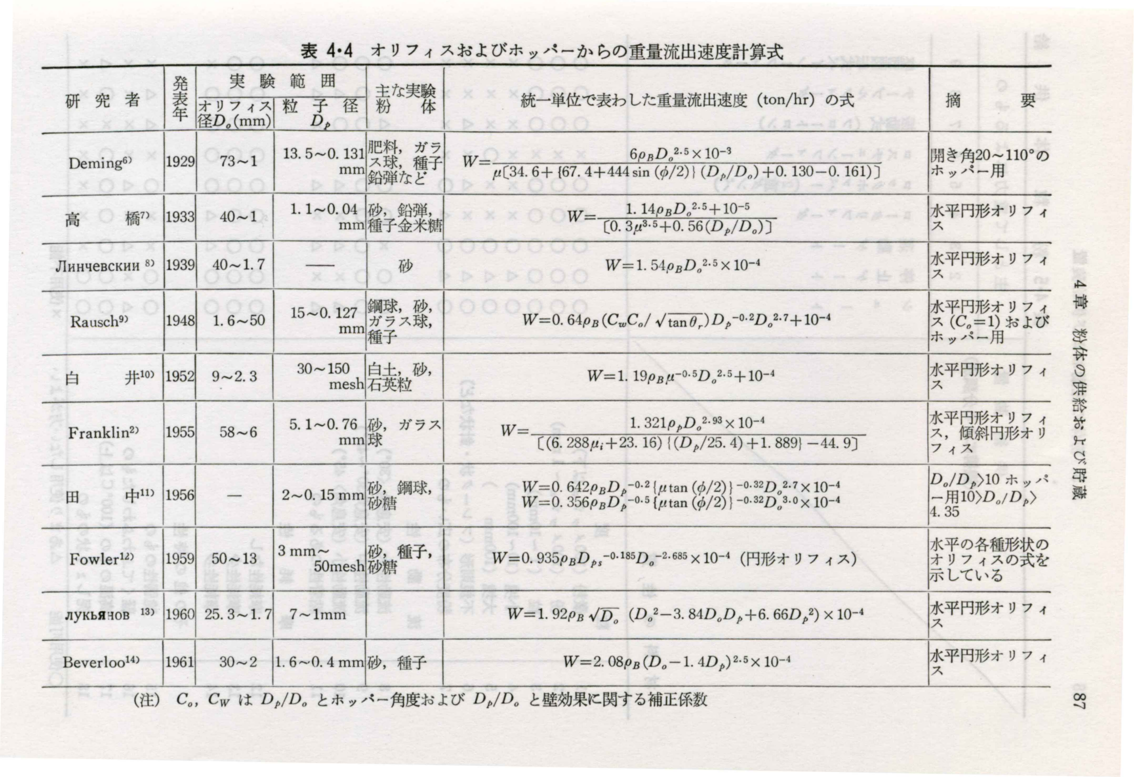


表 4·4 オリフィスおよびホッパーからの重量流出速度計算式

研究者	発表年	実験範囲		主な実験粉体	統一単位で表わした重量流出速度 (ton/hr) の式	摘要
		オリフィス径D_o(mm)	粒子径 D_p			
Deming[6)]	1929	73~1	13.5~0.131 mm	肥料, ガラス球, 種子鉛弾など	$W = \frac{6\rho_B D_o^{2.5}\times 10^{-3}}{\mu[34.6+\{67.4+444\sin(\phi/2)\}(D_p/D_o)+0.130-0.161)]}$	開き角20~110°のホッパー用
高　橋[7)]	1933	40~1	1.1~0.04 mm	砂, 鉛弾, 種子金米糖	$W = \frac{1.14\rho_B D_o^{2.5}+10^{-5}}{[0.3\mu^{3.5}+0.56(D_p/D_o)]}$	水平円形オリフィス
Линчевскии[8)]	1939	40~1.7	——	砂	$W = 1.54\rho_B D_o^{2.5}\times 10^{-4}$	水平円形オリフィス
Rausch[9)]	1948	1.6~50	15~0.127 mm	鋼球, 砂, ガラス球, 種子	$W = 0.64\rho_B(C_w C_o/\sqrt{\tan\theta_r})D_p^{-0.2}D_o^{2.7}+10^{-4}$	水平円形オリフィス ($C_o=1$) およびホッパー用
白　井[10)]	1952	9~2.3	30~150 mesh	白土, 砂, 石英粒	$W = 1.19\rho_B\mu^{-0.5}D_o^{2.5}+10^{-4}$	水平円形オリフィス
Franklin[2)]	1955	58~6	5.1~0.76 mm	砂, ガラス球	$W = \frac{1.321\rho_p D_o^{2.93}\times 10^{-4}}{[(6.288\mu_i+23.16)\{(D_p/25.4)+1.889\}-44.9]}$	水平円形オリフィス, 傾斜円形オリフィス
田　中[11)]	1956	—	2~0.15 mm	砂, 鋼球, 砂糖	$W = 0.642\rho_B D_p^{-0.2}\{\mu\tan(\phi/2)\}^{-0.32}D_o^{2.7}\times 10^{-4}$ $W = 0.356\rho_B D_p^{-0.5}\{\mu\tan(\phi/2)\}^{-0.32}D_o^{3.0}\times 10^{-4}$	$D_o/D_p>10$ ホッパー用$10>D_o/D_p>4.35$
Fowler[12)]	1959	50~13	3 mm~50mesh	砂, 種子, 砂糖	$W = 0.935\rho_B D_{ps}^{-0.185}D_o^{-2.685}\times 10^{-4}$ (円形オリフィス)	水平の各種形状のオリフィスの式を示している
лукьянов[13)]	1960	25.3~1.7	7~1mm		$W = 1.92\rho_B\sqrt{D_o}\ (D_o^2-3.84D_oD_p+6.66D_p^2)\times 10^{-4}$	水平円形オリフィス
Beverloo[14)]	1961	30~2	1.6~0.4 mm	砂, 種子	$W = 2.08\rho_B(D_o-1.4D_p)^{2.5}\times 10^{-4}$	水平円形オリフィス

(注) C_o, C_W は D_p/D_o とホッパー角度および D_p/D_o と壁効果に関する補正係数

4章 粉体の供給および貯蔵

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W: Mass flow rate(ton/hr); ϕ*:hopper angle(°);* D_o*:orifice diameter(mm);* $\mu = \tan\theta_r$*;* D_p*: equivalent volume diameter(mm);* $\mu_i = \tan\phi_i$*;* ρ_B*: bed density(*ton/m^3*);* ϕ_i*: internal friction angle(°);* θ_r*: repose angle(°);* C_o*: correction factor for* $\frac{D_p}{D_o}$ *and hopper angle;* C_W*: correction factor for* $\frac{D_p}{D_o}$ *and wall effect...*

- Be careful for application!