Chapter 3 Multiple Particle Systems

3.1 Settling of a Suspension of Particles (slurry)

Hindered Settling

입자가 모여 있으면 서로의 영향이 침강에 미친다. 이를 간섭침강이라 한다.

Effective viscosity of suspension

$$\mu_e = \frac{\mu}{f(\varepsilon)}$$

Effective density of suspension

$$\rho_{ave} = \varepsilon \rho_f + (1-\varepsilon)\rho_p$$

Force balance

$$3\pi\mu_e (U_p - U_f)x = \frac{(\rho_p - \rho_{ave})\pi}{6} x^3 g$$

$$U_p - U_f = \frac{(\rho_p - \rho_{ave})\pi x^2 g}{18\pi\mu_e}$$

Substituting μ_e *and* ρ_{ave}

$$U_p - U_f = \frac{(\rho_p - \rho_f)\pi x^2 g}{18\pi\mu} \epsilon f(\epsilon) = U_T \epsilon f(\epsilon) \equiv U_{rel_T}$$

Corrected terminal settling velocity

U_p, U_f: *actual(interstitial) velocity of particles and fluid, respectively*

U_{ps}, U_{fs}: *superficial velocity*

$$U_{ps} = U_p(1-\varepsilon)$$

$$U_{fs} = U_f \varepsilon$$

where ε *: voidage or void fraction*

$$\therefore 1-\varepsilon = \frac{particle\ volume}{suspension\ volume} = volume\ concentration\ of\ particles = c_v$$

$\therefore U_{ps}$ *and* U_{fs}: *volume flux of particles and fluid*

$$\left(\frac{m}{s} \cdot \frac{m^3\ fluid\ or\ particles}{m^3\ suspension} = \frac{m^3\ fluid\ or\ particles}{m^2\ suspension \cdot s}\right)$$

☞ U_{ps}와 U_{fs}는 *superficial velocity*임과 동시에 부피 *flux* 임!

3.2 Batch Settling

(1) Settling Flux as a Function of Suspension Concnetration

Because of no net flow

$$U_{ps} + U_{fs} = 0$$

$$U_p(1-\varepsilon) + U_f\,\varepsilon = 0$$

$$\therefore U_p(1-\varepsilon) + [\,U_p - U_{rel_T}]\,\varepsilon = U_p(1-\varepsilon) + [\,U_p - U_T\ f(\varepsilon)\,]\,\varepsilon = 0$$

$$\therefore U_p = U_T\,\varepsilon^2 f(\varepsilon)$$

Richardson and Zaki(1954)

$$U_p = U_T\,\varepsilon^n$$

$$\text{where}\quad \frac{4.8-n}{n-2.4} = 0.043Ar^{0.57}\left[1-2.4\left(\frac{x}{D}\right)^{0.27}\right]$$

$$Ar = \frac{x^3\rho_f(\rho_p-\rho_f)g}{\mu^2}\quad \text{Archimedes number}$$

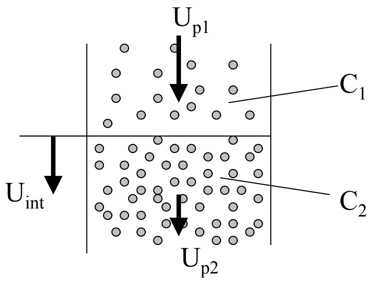
Superficial solid velocity or volumetric solid flux(m/s)

$$U_{ps} = U_p(1-\varepsilon) = U_T\,\varepsilon^2 f(\varepsilon)(1-\varepsilon)$$

$$= U_T(1-\varepsilon)\,\varepsilon^n$$

Settling flux curve (U_{ps} vs. $C_v(=1-\varepsilon)$): Figure 2.1

(2) Sharp Interfaces in Sedimentation



Material Balance over the interface

Since no mass accumulation at the interface

$$(\,U_{p1} - U_{int})C_1 = (\,U_{p2} - U_{int})C_2$$

$$\text{where}\quad C = 1-\varepsilon\ ,\ \text{solids fraction}$$

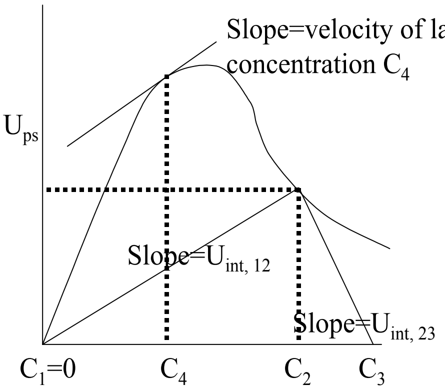
$$\therefore U_{int} = \frac{U_{ps_1} - U_{ps_2}}{C_1 - C_2}$$

Δ

$$As \quad C \to 0,$$

$$U = \frac{dU_{ps}}{dC}$$

where U: the velocity of layer of concentration C

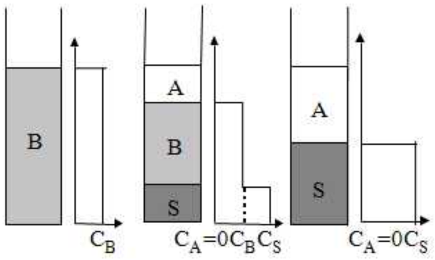


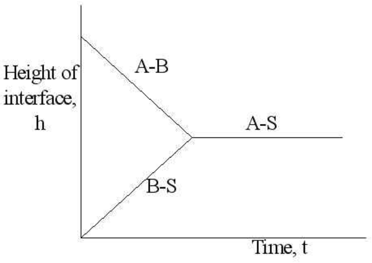
Worked Example 2.2

(3) Batch Settling

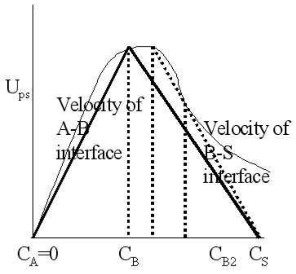
Supplying all the informations for the design of a thickener

<u>Type I Settling</u>



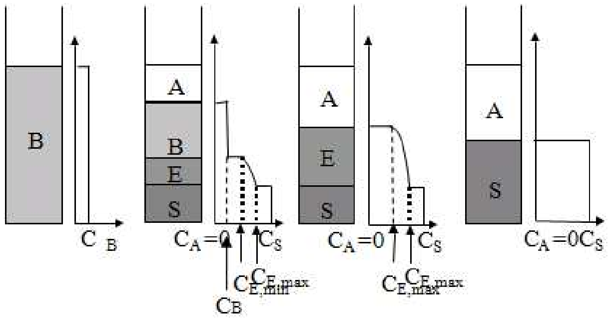


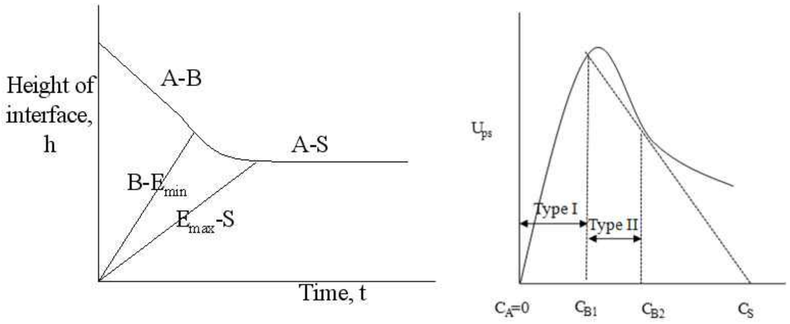
Variation of heights of interfaces with respect to time



Flux vs. concentration

Type II Settling





Variation in heights of interfaces with respect to time

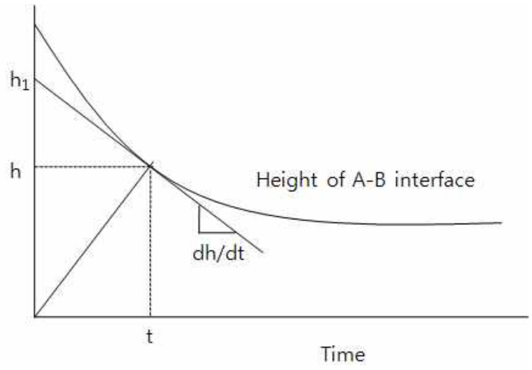
A descending layer or interface will only appear if it falls faster than and gets away from the more dilute layers descending from above them...

Worked Example 2.3

Recommended web site :
http://www.aem.umn.edu/Solid-Liquid_Flows/video.html

(4) Relationship between the height-time curve and the flux plot

Height of AB interface vs. time



Interface between clear liquid and c at h and t

Velocity of interface or U_p, the velocity of the particles at the interface

$$U_p = \frac{dh}{dt} = \frac{h_1 - h}{t}$$

The velocity at which a plane of concentration, c has risen a distance h from the base is $\frac{h}{t}$

The velocity of the particles relative to the plane

$$U_p + \frac{h}{t}$$

The volume of the particles which have passed through the plane

The total volume of the particles in the test

$$C_B h_0 t = A\left(U_p + \frac{h}{t}\right)Ct$$

$$\therefore C = \frac{C_B h_0}{h_1}$$

** This gives U_p vs. C (batch flux plot)*

Worked Example 2.1

3.3 Continuous Settling

(1) Settling of a Suspension in a Flowing Fluid

Thickener vs. Clarifier Figure 2.11

- Downward flow

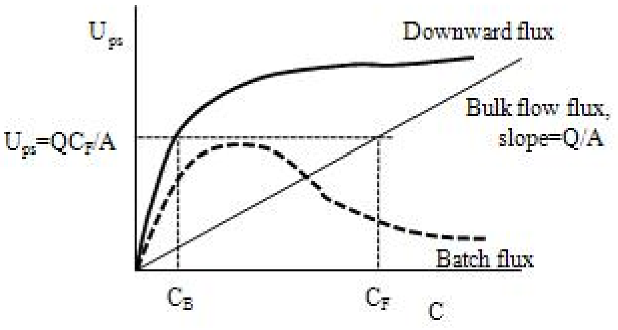
ε
ε

$$\underbrace{U_{ps}}_{\text{Total solid flux}} = \underbrace{\frac{Q(1- \quad)}{A}}_{\text{Flux due to bulk flow}} + \underbrace{U_T \quad {}^2 f(\quad)}_{\text{Flux due to settling}}$$

ε

Define $C_v \equiv 1 -$, particle volume concentration

Figure 2.12:



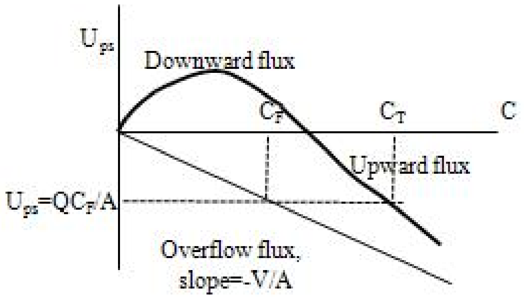
Feed concentration, C_F → Mean bottom section concentration, C_B

- *Upward flow*

εε

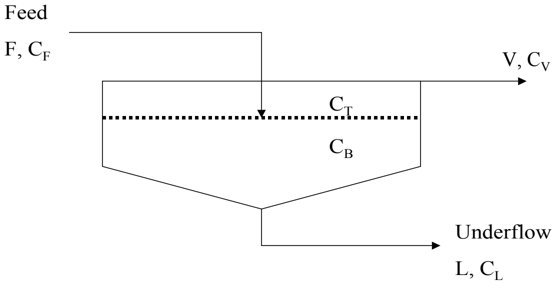
$$U_{ps} = \frac{-\ Q(1-\ \ )}{A} + U_T\ {}^2 f(\ \ )$$

Figure 2.13:



Feed concentration, C_F → Mean top section concentration, C_T

(2) Real Thickener



Feed/ Under(down)flow/ up(over)flow:

F(C_F) *L*(C_L) *V*(C_V)

Below feed

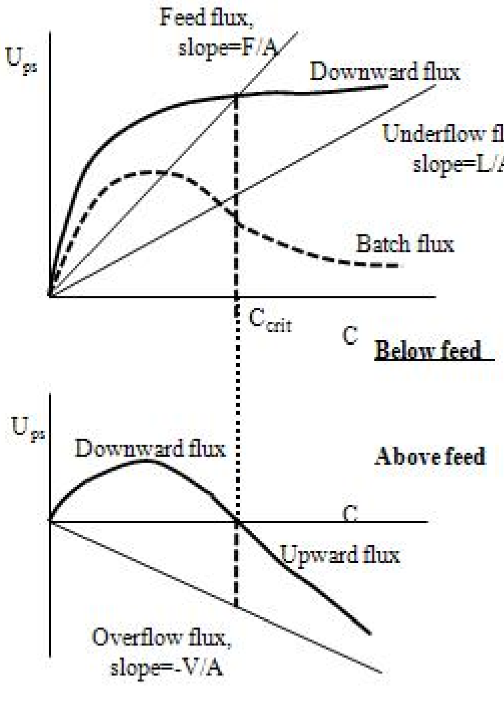
ε ε

$$U_{ps} = \frac{L(1-\ )}{A} + U_T\ {}^2 f(\ )$$

Above feed

ε ε

$$U_{ps} = \frac{-\ V(1-\ )}{A} + U_T\ {}^2 f(\ )$$



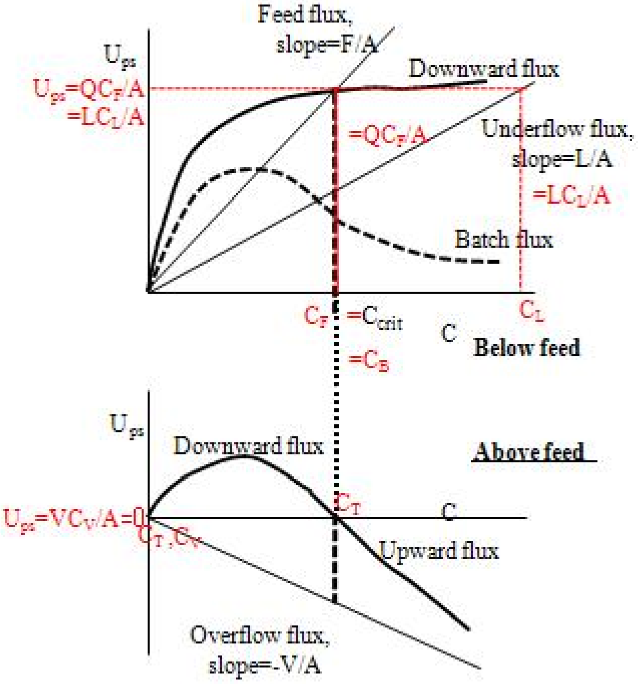
* *Critical concentration:* $C_{crit} \equiv C$ at $U_{p,\,upward} = 0$

(3) Critically Loaded Thickener

$$C_F = C_{crit}$$

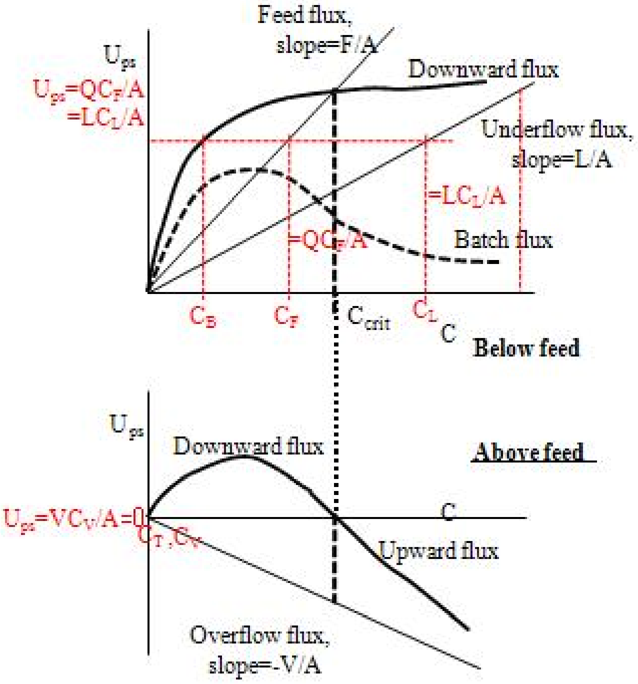
$$\therefore\quad U_{p,\,upward} = 0,\quad C_B = C_F\ ,\ C_T = C_V = 0\ \text{ and}$$

$$U_{ps,\,downflow} = \frac{FC_F}{A} = \frac{LC_L}{A}$$



(4) Underloaded Thickener

$$C_F < C_{crit}$$

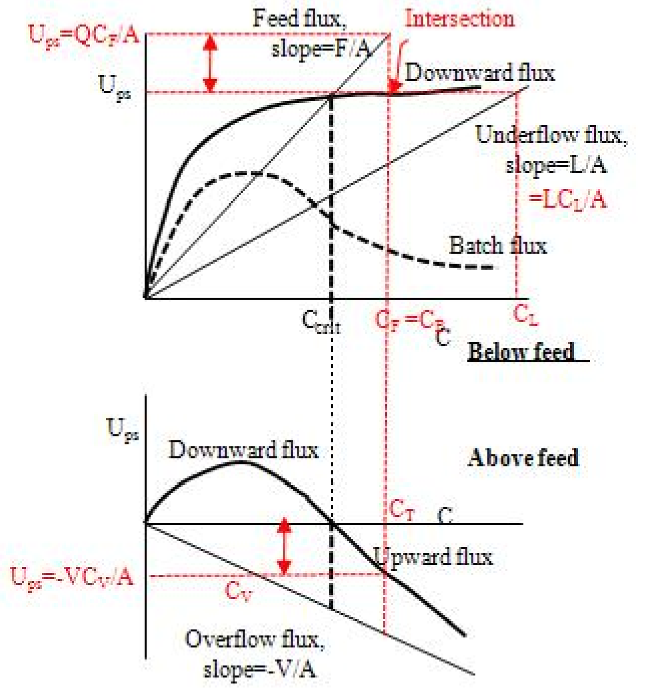


$$\therefore \quad U_{p,\,upward} = 0, \quad C_B < C_F \ \text{ and } \ C_T = C_V = 0$$

$$U_{ps,\,downflow} = \frac{FC_F}{A} = \frac{LC_L}{A}$$

(5) Overloaded Thickener

$$C_F > C_{crit}$$



$$\therefore \quad U_{p,\,upward} = \frac{FC_F}{A} - U_{ps,\,downflow}, \qquad C_B = C_F \ \text{and} \ \ C_T > C_V \neq 0$$

$$U_{ps,\,downflow} = \frac{LC_L}{A}$$

ω * *Centrifugal Sedimentation* r^{2} *instead of* g

* *When minimum total flux appears... ☞ Figure 2.18 and 2.19*

Worked Example 2.4