Chapter 2. Single Particles In a Fluid

2.1 Motion of Solid Particles in a Fluid

1) Drag Force, F_D:

Net force exerted by the fluid on the spherical particle(diameter x) in the direction of flow

$$F_D = C_D\left(\frac{\pi}{4}x^2\right)\frac{\rho_f U^2}{2}$$

Area exerted friction | *Kinetic by energy of unit mass fluid*

where C_D *: Drag coefficient*

cf. For pipe flow

$$\tau_w = f\frac{\rho_f U^2}{2} \quad \rightarrow \quad F_w = f(\pi DL)\frac{\rho_f U^2}{2}$$

where f*: Fanning friction factor*

C_D *vs.* Re_p *- Figure 1.3*

$$Re_p = \frac{xU\rho_f}{\mu}$$

where U : the relative velocity of particle with respect to fluid

- Stokes' law range: $Re_p < 1$ *(creeping flow region)*

$$F_D = 3\pi\mu xU$$

Stokes' law

$$= \left(\frac{24}{Re_p}\right)\left(\frac{\pi}{4}x^2\right)\frac{\rho_f U^2}{2}$$

$$\therefore C_D = \frac{24}{Re_p}$$

$$cf.\ f = \frac{16}{Re} \quad \text{for pipe flow}$$

- Intermediate range

$$C_D = \frac{24}{Re_p}(1 + 0.15\,Re_p^{0.687})$$

or

Table 1.1

- *Newton's law range:* $500 < Re_p < 2 \times 10^5$

$$C_D \approx 0.44$$

Newton's law

2) Non-continuum Effect for nanoparticles

- *Mean-free path of fluid*

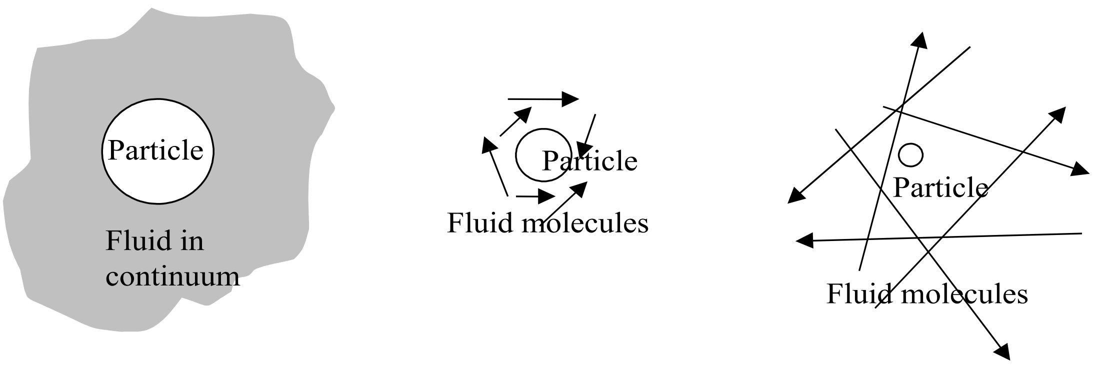
$$\lambda = \frac{1}{\sqrt{2}\,\pi\, n_m\, d_m^2}$$

where

n_m : *number concentration of molecules*

d_m : *diameter of molecules*

For air at 1 atm and 25°C λ *= 0.0651μm=65.1nm*



Continuum regime　　*transition regime*　　*free-molecule regime*

- *Knudsen number,* Kn

$$Kn = \frac{\lambda}{x}$$

· *Continuum regime* : $Kn \sim 0$ *(<0.1)*

· *Transition regime* : $Kn \sim 1$ *(0.1 ~10)*

· *Free molecule regime* : $Kn \sim \infty$ *(>10)*

- *Drag force corrected for noncontinuum effect*

$$F_D = \frac{3 \quad x \;\, U}{C_c}$$

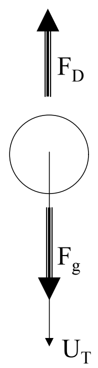
where C_c : *Cunningham correction factor*

$$C_c = 1 + Kn[2.34 + 1.05\exp(-0.39/Kn)]$$

x, m	C_c
0.01	22.7
0.05	5.06
0.1	2.91
1.0	1.168
10	1.017

In air at 1atm and 25°C

2.2 Particle Falling Under Gravity Through a Fluid



The velocity of free falling particle when

$$F_D = F_g - F_B$$

$$\therefore C_D\left(\frac{}{4} x^2\right)\frac{{}_f U^2}{2} = \frac{}{6}(\;{}_p - \;{}_f)x^3 g$$

$$\therefore U_T = \left[\frac{4}{3}\frac{gx}{C_D}\left(\frac{{}_p - \;{}_f}{{}_f}\right)\right]^{1/2}$$

For Stokes law regime

$$\frac{3 \quad x \;\, U}{C_c} = \frac{}{6}(\;{}_p - \;{}_f)x^3 g$$

$$\therefore U_T = \frac{(\;{}_p - \;{}_f)gx^2 C_c}{18}$$

d_p, m	U_T, cm/s[a]
0.1	8.8×10^{-5}
0.5	1.0×10^{-3}
1.0	3.5×10^{-3}
5.0	7.8×10^{-2}
10.0	0.31

[a]For unit density particle in air at 1 atm and 25°C

Example. In 1883 the volcano Krakatoa exploded, injecting dust 32km up into the atmosphere. Fallout from this explosion continued for 15 months. If one assumes settling velocity was constant and neglects slip correction, what was the minimum particle size present? Assume particles are rock spheres with a specific gravity of 2.7.

ρ μ

$$\therefore U_T = \frac{(\ _p - \ _f) g x^2 C_c}{18} = \frac{2.7 \cdot 980 \cdot x^2 \cdot 1}{18 \cdot 1.81 \cdot 10^{-4}}$$

$$= \frac{32 \cdot 10^3 \cdot 10^2}{15 \cdot 30 \cdot 24 \cdot 3600} = 0.0823\ cm/s$$

μ

$$\therefore d_p = 3.19\ \ m$$

Worked Example 1.1

Worked Example 1.4

Worked Example 1.5: Use MathCad

Worked Example 1.6: Use MathCad

* C_D *for nonspherical particles - Figure 1.3*

in terms of sphericity

Ψ

$$= \frac{surface\ area\ of\ a\ sphere\ having\ the\ volume\ of\ the\ particle}{surface\ area\ of\ the\ particle}$$

- *Dynamic shape factor,* χ

$$\chi \equiv \frac{F_D}{3\pi\mu U x_v}$$

- *Equivalent diameters related with terminal settling velocity*

·*Stokes diameter:*

the diameter of the hypothetical sphere having the same terminal settling *velocity*

- *Aerodynamic diameter:*

  the diameter of the hypothetical unit-density sphere having the same terminal settling velocity

By definition

$$U_T = \frac{\rho_p x_v^{2g}}{18\mu\chi} = \frac{pho_0 x_a^2 g}{18\mu} = \frac{\rho_p x_s^2 g}{18\mu}$$

or

$$\rho_p \frac{x_v^2}{\chi} = \rho_0 x_a^2 = \rho_p x_s^2$$

Examples

Calculate the aerodynamic diameter(um) for a quartz particle with $x_e = 20\mu m$ *and* $\rho_p = 2700 kg/m^3$. *For the quartz particle,* $\chi = 1.36$.

$$\left[\frac{2700\,\frac{\mathrm{kg}}{\mathrm{m}^3}\,(20\mu\mathrm{m})^2}{1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}}\right]^{\frac{1}{2}} = 3.286 \times 10^{-5}\,\mathrm{m}$$

$$20\mu\mathrm{m} = 2 \times 10^{-5}\,\mathrm{m}$$

2.S1 Motion of a Single Particle in an General External Force Field

μ

$$m_p \frac{d\overrightarrow{U}}{dt} = -\frac{3 \quad x(\overrightarrow{U} - \overrightarrow{V})}{C_c} + \sum_i \overrightarrow{F_{ei}}$$

where $\overrightarrow{F_{ei}}$: *external force field I*

e.g. gravity force, electrical force....

- *Its solution gives the motion(trajectory, displacement) of a single particles with respect to time...*
- *Steady-state solution gives the migration velocity exerted by the external force fields specified...*

External force field	Expression	Migration velocity	Remarks
Gravitational	$m_p g$	$U_T = \frac{(\rho_p - \rho_f) g x^2 C_c}{18\mu}$	Terminal settling velocity
Centrifugal	$F_c = m_p\left(1 - \frac{\rho_f}{\rho_p}\right) r\omega^2$	$U_{cf} = \frac{(\rho_p - \rho_f) x^2 r\omega^2 C_c}{18\mu}$	r: radius of centrifuge ω: angular velocity
Electrical	$\vec{F} = q\vec{E} = n_e e E$	$U_e = \frac{n_e e E C_c}{3\pi\mu x}$	q: charge of particles E: strength of electric field e : charge of electron n_e : number of the units

- Applied to particle classification and separation from fluid

2.S2 Inertial Motion of Particles

1) Stop Distance

For Stokesian particles

Momentum(force) balance for a single sphere

$$m_p \frac{dU}{dt} = -\frac{3\pi\mu x U}{C_c}$$

Integrating once

$$U = U_0 e^{-t/\tau}$$

$$\text{where} \quad \tau = \frac{m_p C_c}{3\pi\mu x} = \frac{\rho_p x^2 C_c}{18\mu}$$

relaxation time

Integrating twice

$$x = U_0\tau\,(1 - e^{t/\tau})$$

As $\frac{t}{\tau} \to \infty$,

$$x \sim U_0\tau = \frac{\rho_p x^2 U_0 C_c}{18\mu} \equiv s$$

stop distance

Example.

What is the stopping distance pf a 1-um aluminum-oxide sphere thrown from a grinding wheel at 9m/s into still air. The particle density is 4000kg/m^3. Neglect slip correction.

$$s := \frac{4000\,\frac{kg}{m^3}\,(1\mu m)^2\,9\,\frac{m}{s}}{18\cdot 18.1\cdot 10^{-6}\,Pa\cdot s} = 1.105\times 10^{-4}\,m$$

$$18.1\cdot 10^{-6}\,Pa\cdot s = 2.21\times 10^{-13}\,\frac{m^2}{s^2}\,\frac{kg}{m\cdot s}$$

$$Re0 := \frac{1\mu m\cdot 9\,\frac{m}{s}\cdot 1.2\,\frac{kg}{m^3}}{18.1\cdot 10^{-6}\,\frac{kg}{m\cdot s}} = 0.597$$

Stokes law can apply.

2) Simiulitude Law for Impaction *: Stokesian Particles*

If no external force

For Re < 1

Force balance around a particle (equation of particle motion)

$$m_p \frac{d\vec{U}}{dt} = -3 \quad x(\vec{U} - \vec{U_f})$$

Defining dimensionless variables

$$\vec{U_1} \equiv \frac{\vec{U}}{U}, \quad \vec{U_{f1}} \equiv \frac{\vec{U_f}}{U} \quad and \quad \equiv \frac{tU}{L}$$

where U, L: characteristic velocity and length of the system

Substituting

$$m_p \frac{d(\vec{U_1}U)}{d\left(\frac{L}{U}\right)} = -3 \quad x(\vec{U_1}U - \vec{U_{f1}}U)$$

$$\therefore St \frac{d\vec{U_1}}{d} = -(\vec{U_1} - \vec{U_{f1}})$$

$$where \quad St \equiv \frac{m_p U}{3 \ xL} = \frac{\frac{{}_p \ x^3}{6} U}{3 \ xL} = \frac{{}_p x^2 U}{18 \ L} = \frac{U}{L} \equiv \frac{particle\ persistence}{size\ of\ obstacle}$$

Stokes number

In terms of displacement $\vec{U} = \frac{d\vec{r}}{dt}$

where $\vec{r}$*: dispacement vector (* $\vec{r_1} \equiv \frac{\vec{r}}{L}$ *)*

$$\rightarrow \vec{U_1}U = \frac{d(\vec{r_1}L)}{d\left(\frac{\theta L}{U}\right)} \rightarrow \vec{U_1} = \frac{d\vec{r_1}}{d\theta}, \ and \ \frac{d\vec{U_1}}{d\theta} = \frac{d^2 r_1}{d\theta^2}$$

$$\therefore \quad St\frac{d^2\overrightarrow{r_1}}{d\ \ ^2}+\frac{d\overrightarrow{r_1}}{d}=\overrightarrow{U_{f1}}$$

If solved, the solution would be

$$\therefore \overrightarrow{r_1} \;=\; f\ (St,\ \ Re,\ \ R)$$

$$\begin{array}{cccc} \uparrow & & \uparrow & \uparrow \\ \text{particle trajectory} & & \overrightarrow{U_{f1}} & B.C. \end{array}$$

Trajectory of a particle under no external force

(Inertial motion)

* *For the two particle systems*

If Re, St and B.C. are the same, particle trajectories are the same.

* *Applications*

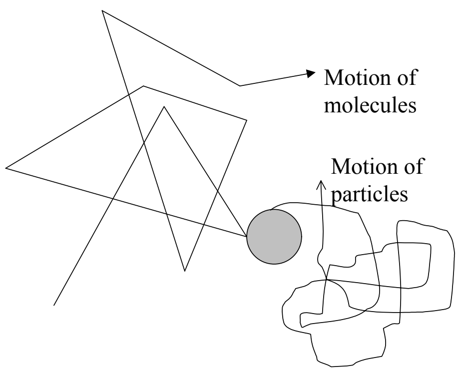
- *Cyclone*
- *Particle impactor*
- *Filter*

* *Isokinetic sampling*

2.S3 Diffusion and Phoresis

(1) Particle (Brownian) Diffusion

<u>*Brownian motion*</u>



- *Random wiggling motion of particles by collision of fluid molecules on them*
- *Important for motion of nanoparticles*
- *Stochastic vs. deterministic*
- *Including Brownian motion term in the motion equation*

$$m_p \frac{d\vec{U}}{dt} = -\frac{3 \quad x(\vec{U} - \vec{U_f})}{C_c} + \sum_i \overrightarrow{F_{ei}} + m_p \ (t)$$

Langevin equation...

where $\alpha(t)$*: random acceleration caused by bombardment by molecules*

For pure Brownian motion, all the $\overline{F_{ei}} = 0$ *and* $\overline{U_f} = 0$

The mean square displacement of the Brownian motion $(\overline{s}^2)^{1/2}$ *is given*

$$(\overline{x^2}) = (\overline{y^2}) = (\overline{z^2}) = \frac{2kTC_c t}{3\pi\mu x} \quad (1)$$

<u>*Brownian Diffusion*</u> *:*

- *Particle migration due to concentration gradient by Brownian motion*

$$\vec{J} = -D_p \vec{\nabla} C$$

1st Fick's law

where D_p *: diffusion coefficient of particles* cm^2/s

· *Phenomenological expression with collective properties*

- *The material balance caused by Brownian motion*

$$\frac{\partial C}{\partial t} = -\nabla \cdot \vec{J} = D_p \nabla^2 C$$

2nd Fick's law

C *: particle concentration by number or mass*

The solution gives

$$(\overline{x^2}) = (\overline{y^2}) = (\overline{z^2}) = 2D_p t \quad (2)$$

From (1) and (2)

$$D_p = \frac{kTC_c}{3 \quad x}$$

Stokes-Einstein relation

Example

Calculate the diffusion coefficient for a 1-um particle at standard conditions.

$$Kn := \frac{0.066 \cdot \mu m}{0.1 \mu m}$$

$$Cc := 1 + Kn \cdot \left(2.34 + 1.05 \exp\left(\frac{-0.39}{Kn}\right)\right) = 2.928$$

$$k := 1.38 \cdot 10^{-23} \frac{J}{K} \qquad T := 293K$$

$$x := 1 \mu m \qquad \mu := 18.1 \cdot 10^{-6} Pa \cdot s$$

$$\boxed{D := \frac{k \cdot T \cdot Cc}{3\pi \mu \cdot x} = 6.941 \times 10^{-11} \frac{m^2}{s}}$$

Diffusion Coefficient of Unit-density sphere at 20°C in air

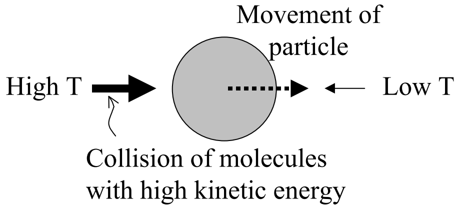
액체분자의 확산계수 $10^{-5} cm^2/s$ 정도

Particle diameter, m	*Diffusion coefficient, $D_p (cm^2/s)$*
0.00037(air molecule)	*0.19*
0.01	5.2×10^{-4}
0.1	6.7×10^{-6}
1.0	2.7×10^{-7}
10	2.4×10^{-8}

- *Determines behavior of nanoparticles:*
  - *Brownian coagulation*
  - *Brownian deposition...*
- *Applied to diffusion batteries*
  - *dynamic light scattering(photon correlation spectroscopy)*
  - *: Brown 운동하는 입자의 빛산란을 추적하여 입자크기 측정*

2) Thermophoresis

- *Discovered by Tyndall in 1870*



실제 예 : radiator의 벽이나 인근 벽에 먼지가 쓸지 않는 현상
담배연기가 차가운 벽 또는 창문 쪽으로 이동해 가는 현상
차가운 쪽에 면한 벽이 먼저 더러워지는 현상

<u>*Thermophoretic force and velocity*</u>

- In free molecular regime

λ

$$\overrightarrow{F_{th}} = -p\ \ x^2 \frac{\overrightarrow{\nabla T}}{T}$$

Waldmann and Schmidt(1966)

From $\overrightarrow{F_{th}} = \overrightarrow{F_D}$ *in Stokes' regime*

ν
ν
α

$$\therefore\ \overrightarrow{U_{th}} = -\frac{3\ \overrightarrow{\nabla} T}{4\left(1+\frac{}{8}\right)T} = \sim 0.55\ \frac{\overrightarrow{\nabla} T}{T}$$

- independent of x

- Correction for continuum fluid-particle interaction

μ
ρ

$$\overrightarrow{F_{th}} = \frac{-9\ \ ^{2}xH\overrightarrow{\nabla} T}{2\ _{G}T}$$

Brock(1962)

$$H \sim \frac{1}{1+6Kn}\ \frac{\frac{k_G}{k_p}+4.4Kn}{1+2\frac{k_G}{k_p}+8.8Kn}$$

μ
ρ

$$\therefore \overrightarrow{U_{th}} = \frac{-3\ \ C_c H\overrightarrow{\nabla} T}{2\ _{G}T}$$

Terminal settling and thermophoretic velocities in a temperature gradient of 1°C/cm at 293K

$^{a}K_p = 10k_a$

Particle diameter(m)	*Terminal settling velocity(m/s)*	*Thermophoretic velocities in a temperature gradient of 1°C/cm at 293K[a]*
0.01	6.7×10^{-8}	2.8×10^{-6}
0.1	8.6×10^{-7}	2.0×10^{-6}
1.0	3.5×10^{-5}	1.3×10^{-6}
10.0	3.1×10^{-3}	7.8×10^{-7}

- U_{th}*: independent of particle size*

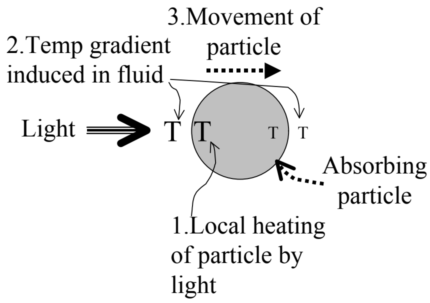
<u>*Applications*</u>

- Particle sampling: dust collection in UK mines

- Suppression of particle deposition of particles in fine industries and hospitals

3) Phoresis by Light

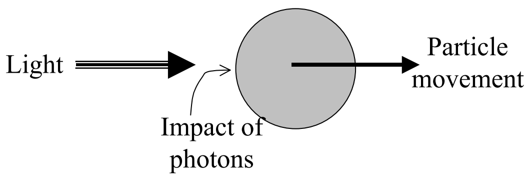
Photophoresis



- *Where to be heated depends on the refractive index of the particle*

  e.g. Movement of submicron particles in the upper atmosphere

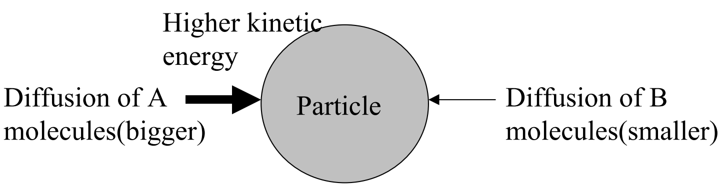
Radiation pressure



e.g. tails of comet, laser-lift of particles

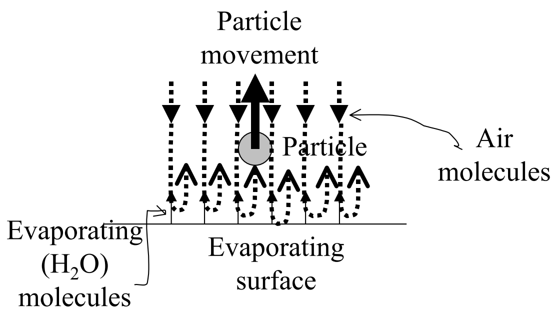
4) Diffusion of medium

Diffusiophoresis

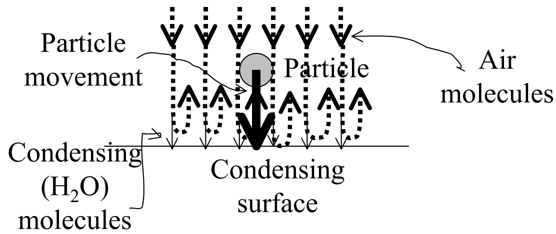


Stefan flow

- *For evaporating surface*



- *For condensing surface*



e.g. Venturi scrubber