Chapter 1. Single Particles In a Fluid

1.1 Motion of Solid Particles in a Fluid

1) ***Drag Force***, F_D:

Net force exerted by the fluid on the spherical particle(diameter x) in the direction of flow

$$F_D = C_D\left(\frac{\pi}{4}x^2\right)\frac{\rho_f U^2}{2}$$

Area exerted friction	Kinetic by energy of unit mass fluid

where C_D : Drag coefficient

cf. for pipe flow

$$\tau_w = f\frac{\rho_f U^2}{2} \quad \rightarrow \quad F_w = f(\pi DL)\frac{\rho_f U^2}{2}$$

where f: Fanning friction factor

C_D vs. Re_p - Figure 1.3

$$Re_p = \frac{xU\rho_f}{\mu}$$

where U : the relative velocity of particle with respect to fluid

Four regions in drag curve

- Stokes' law range: $Re_p < 1$ (creeping flow region)

$$F_D = 3\pi x\mu U$$

Stokes' law

$$= \left(\frac{24}{Re_p}\right)\left(\frac{\pi}{4}x^2\right)\frac{\rho_f U^2}{2}$$

$$\therefore C_D = \frac{24}{Re_p}$$

cf. $f = \frac{16}{Re}$

Intermediate range

$$C_D = \frac{24}{Re_p}(1 + 0.15 Re_p^{0.687})$$

or

Table 1.1

Newton's law range: $500 < Re_p < 2 \times 10^5$

$$C_D \approx 0.44$$

Newton's law

* inertial flow

2) ***Non-continuum Effect***

Mean-free path of fluid

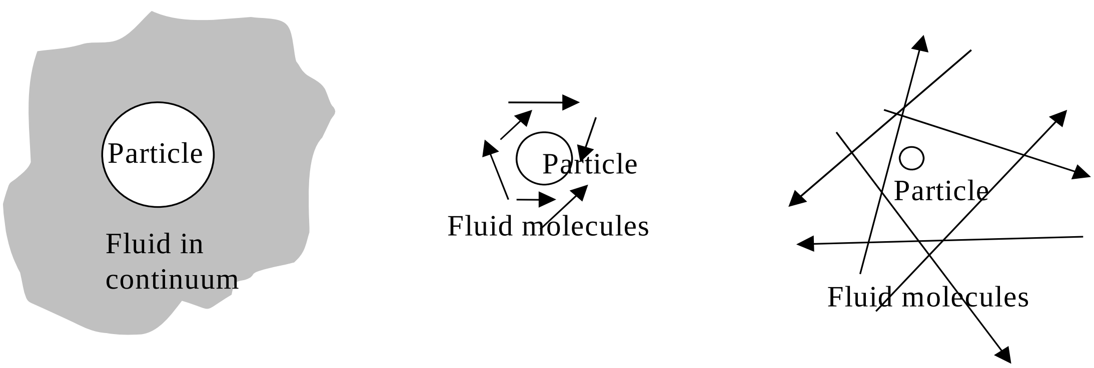
$$\lambda = \frac{1}{\sqrt{2}\, n_m \pi d_m^2}$$

where

n_m : number concentration of molecules

d_m : diameter of molecules

For air at 1 atm and 25℃ λ = 0.0651 μm



Continuum regime *transition regime* *free-molecule regime*

Knudsen number, Kn

$$Kn = \frac{\lambda}{x}$$

- Continuum regime : $Kn \sim 0$ (<0.1)
- Transition regime : $Kn \sim 1$ (0.1∼10)
- Free molecule regime : $Kn \sim \infty$ (>10)

Corrected drag force

$$F_D = \frac{3\pi x \mu U}{C_c}$$

where C_c : Cunningham correction factor

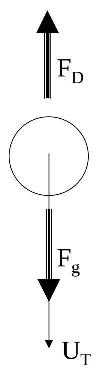
$$C_c = 1 + Kn[2.514 + 0.8\exp(-0.55/Kn)]$$

x, μm	C_c
0.01	22.7
0.05	5.06
0.1	2.91
1.0	1.168
10	1.017

In air at 1atm and 25℃

3.2 Particle Falling Under Gravity Through a Fluid

1) ***Terminal Settling Velocity***, U_T



The velocity of free falling particle when

$$F_D = F_g - F_B$$

$$\therefore\ C_D\left(\frac{\pi}{4}x^2\right)\frac{\rho_f U^2}{2} = \frac{\pi}{6}(\rho_p - \rho_f)x^3 g$$

$$\therefore\ U_T = \left[\frac{4}{3}\frac{gx}{C_D}\left(\frac{\rho_p - \rho_f}{\rho_f}\right)\right]^{1/2}$$

For Stokes law regime

$$\frac{3\pi x \mu U}{C_c} = \frac{\pi}{6}(\rho_p - \rho_f)x^3 g$$

$$\therefore U_T = \frac{(\rho_p - \rho_f) g x^2 C_c}{18\mu}$$

d_p, μm	U_T, cm/s[a]
0.1	8.8×10^{-5}
0.5	1.0×10^{-3}
1.0	3.5×10^{-3}
5.0	7.8×10^{-2}
10.0	0.31

[a]For unit density particle in air at 1 atm and 25°C

Worked Example 1.1

Example. In 1883 the volcano Krakatoa exploded, injecting dust 32km up into the atmosphere. Fallout from this explosion continued for 15 months. If one assumes settling velocity was constant and neglects slip correction, what was the minimum particle size present? Assume particles are rock spheres with a specific gravity of 2.7.

$$\therefore U_T = \frac{(\rho_p - \rho_f) g x^2 C_c}{18\mu} = \frac{2.7 \cdot 980 \cdot x^2 \cdot 1}{18 \cdot 1.81 \cdot 10^{-4}}$$

$$= \frac{32 \cdot 10^3 \cdot 10^2}{15 \cdot 30 \cdot 24 \cdot 3600} = 0.0823\ cm/s$$

$$\therefore d_p = 3.19 \mu m$$

In Newton's regime

$$U_T = 1.74 \left(\frac{xp(\rho_p - \rho_f) g}{\rho_f} \right)^{1/2}$$

For intermediate regime : Trial and error(Numerical) or

$$C_D Re_p^2 \Rightarrow \frac{4}{3} \frac{x^3 \rho_f (\rho_p - \rho_f) g}{\mu^2} \equiv K \text{ (constant)}$$

$$\downarrow$$

$$\log C_D = \log K - 2 \log Re_p$$

From Re_p,

$$U_T = \frac{Re_p \mu}{x \rho_f}$$

* x? for given U_T

$$\frac{C_D}{Re_p} \Rightarrow \frac{4}{3}\frac{g\mu(\rho_p - \rho_f)}{U_T^3 \rho_f^2} \equiv K'$$

$$\log C_D = \log Re_p + \log K'$$

From the Figure above,

$$x = \frac{Re_p \mu}{U_T \rho_f}$$

Worked Example 1.5

Worked Example 1.6

http://www.processassociates.com/process/separate/termvel.htm ☞ U_T

http://www.processassociates.com/process/separate/termdiam.htm☞ d_p

2) Transient Response to Gravitational Field

In general,

Particle motion in gravity

$$m_p \frac{dU}{dt} = F_g - F_B - F_D$$

$$\rho_p \frac{\pi}{6} x^3 \frac{dU}{dt} = \frac{\pi}{6}(\rho_p - \rho_f)x^3 g - C_D\left(\frac{\pi}{4}x^2\right)\frac{\rho_f U^2}{2}$$

For Stokes' law regime, $F_D = 3\pi x \mu U$

Rearranging,

$$\tau \frac{dU}{dt} = \tau g - U$$

where $\tau \equiv \frac{(\rho_p - \rho_f) x^2 C_c}{18\mu}$

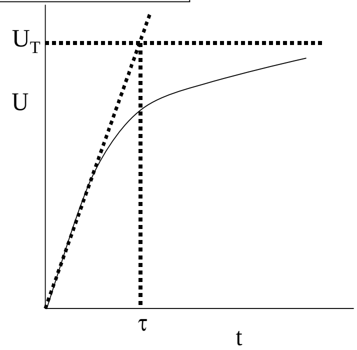
Relaxation time

Relaxation time for Unit Density Particles at Standard Conditions

Particle diameter, μm	Relaxation time, s
0.01	6.8×10^{-9}
0.1	8.8×10^{-8}
1.0	3.6×10^{-6}
10	3.1×10^{-4}
100	3.1×10^{-2}

Integration yields

$$U = U_T [1 - \exp(-t/\tau)]$$



* 여기서 τ는 외부의 변화에 대처하는 입자의 기민성과 관련한다.

1.3 Nonspherical particles

* C_D for nonspherical particles - Figure 1.3

* Sphericity

$$\Psi = \frac{\textit{surface area of a sphere having the volume of the particle}}{\textit{surface area of the particle}}$$

1.3 Effect of Boundaries on Terminal Velocity

So far $U_T \rightarrow U_{T,\infty}$, terminal velocity infinite fluid

Terminal velocity in pipe

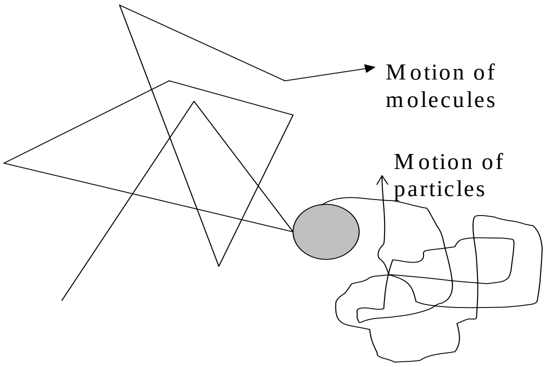
$$U_{T,D} = f_w U_{T,\infty}$$

f_w=(1.17)(1.18)(1.19)

1.4S Diffusion and Phoresis

1) Particle (Brownian) Diffusion

Brownian motion



: Random wiggling motion of particles by collision of fluid molecules on them

Brownian Diffusion :

Particle migration due to concentration gradient by Brownian motion

$$\vec{J} = -D_p \vec{\nabla} C$$

Fick's law

where D_p : diffusion coefficient of particles cm^2/s

C : particle concentration by number or mass

☞ phenomenological expression with collective properties

확산의 표시방법은 일반적인 migration 표시방법과 다르다.

* Coefficient of Diffusion, D_p

$$D_p = \frac{kTC_c}{3\pi\mu x}$$

액체분자의 확산계수 가 $10^{-5}cm^2/s$ 정도임에 유의

* x_{rms}: root-mean square diameter

$$x_{rms} = \sqrt{2D_p t}$$

Diffusion Coefficient of Unit-density sphere at 20°C in air

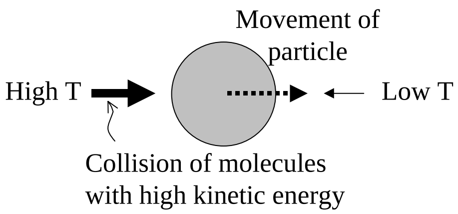
Particle diameter, μm	Diffusion coefficient, D_p(cm²/s)
0.00037(air molecule)	0.19
0.01	5.2×10^{-4}
0.1	6.7×10^{-6}
1.0	2.7×10^{-7}
10	2.4×10^{-8}

**Dynamic light scattering(photon correlation spectroscopy)*

: Brown 운동하는 입자의 빛산란을 추적하여 입자크기 측정

2) *Thermophoresis*

- Discovered by Tyndall in 1870



실제 예 : radiator의 벽이나 인근 벽에 먼지가 쓸지 않는 현상
담배연기가 차가운 벽 또는 창문 쪽으로 이동해 가는 현상
차가운 쪽에 면한 벽이 먼저 더러워지는 현상

In free molecular regime

$$\overrightarrow{F_{th}} = -p\lambda x^2 \frac{\overrightarrow{\nabla T}}{T}$$

Waldmann and Schmidt(1966)

From $\overrightarrow{F_{th}} = \overrightarrow{F_D}$ in Stokes' regime

$$\therefore \overrightarrow{U_{th}} = -\frac{3\nu \overrightarrow{\nabla T}}{4\left(1+\frac{\pi\alpha}{8}\right)T} = \sim 0.55\,\nu \frac{\overrightarrow{\nabla T}}{T}$$

- independent of x

Correction for continuum fluid-particle interaction

$$\overrightarrow{F_{th}} = \frac{-9\pi\mu^2 x H \overrightarrow{\nabla T}}{2\rho_G T}$$

Brock(1962)

$$H \sim \frac{1}{1+6Kn}\left(\frac{\frac{k_G}{k_p}+4.4Kn}{1+2\frac{k_G}{k_p}+8.8Kn}\right)$$

$$\therefore \overrightarrow{U_{th}} = \frac{-3\mu C_c H \overrightarrow{\nabla T}}{2\rho_G T}$$

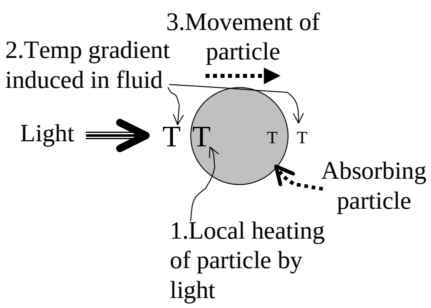
Terminal settling and thermophoretic velocities in a temperature gradient of 1°C/cm at 293K

Particle diameter(μm)	Terminal settling velocity(m/s)	Thermophoretic velocities in a temperature gradient of 1°C/cm at 293K[a]
0.01	6.7×10^{-8}	2.8×10^{-6}
0.1	8.6×10^{-7}	2.0×10^{-6}
1.0	3.5×10^{-5}	1.3×10^{-6}
10.0	3.1×10^{-3}	7.8×10^{-7}

[a] $k_p = 10k_a$

3) Phoresis by Light

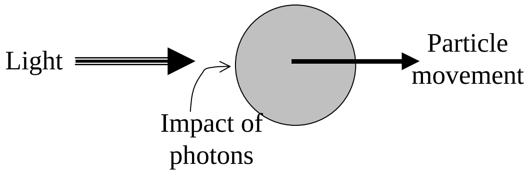
Photophoresis



- Where to be heated depends on the refractive index of the particle

e.g. submicron particles in the upper atmosphere

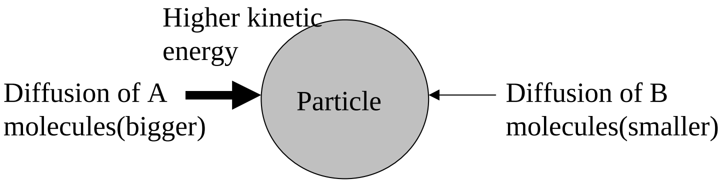
Radiation pressure



e.g. tails of comet, laser-lift of particles

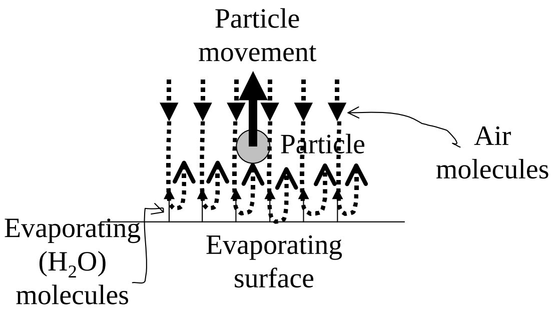
4) Diffusion of medium

Diffusiophoresis

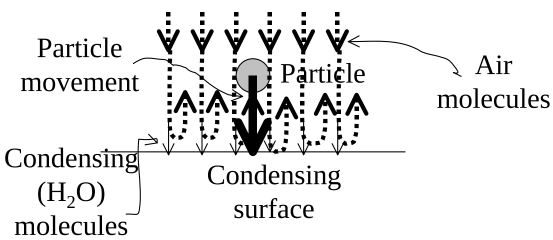


Stefan flow

For evaporating surface



For condensing surface



e.g. *Venturi scrubber*

1.5S Inertial Motion and Impact of Particles

1) ***Stop Distance***

For *Stokesian* particles

Momentum(force) balance for a single sphere

$$m_p \frac{dU}{dt} = -\frac{3\pi\mu x U}{C_c}$$

Integrating once

$$U = U_0 e^{-t/\tau}$$

$$\text{where} \quad \tau = \frac{m_p C_c}{3\pi\mu x} = \frac{\rho_p x^2 C_c}{18\mu}$$

relaxation time

Integrating twice

$$x = U_0 \tau (1 - e^{t/\tau})$$

As $\frac{t}{\tau} \to \infty$,

$$x \sim U_0 \tau = \frac{\rho_p x^2 U_0 C_c}{18\mu} \equiv s$$

stop distance

Net displacement in 1s due to Brownian motion and gravity for satandard-density spheres at standard conditions

Particle diameter, μm	Re_0	s at U_0=10m/s	time to travel 95% of s
0.01	0.0066	7.0×10^{-5}	2.0×10^{-8}
0.1	0.066	9.0×10^{-4}	2.7×10^{-7}
1.0	0.66	0.035	1.1×10^{-5}
10	6.6	2.3*	8.5×10^{-4}*
100	66	127*	0.065*

* out of Stokes' range

2) ***Simiulitude Law* for Impaction** : Stokesian Particles

For Re < 1

Force balance around a particle (equation of particle motion)

$$m_p \frac{d\vec{U}}{dt} = -3\pi\mu x(\vec{U} - \vec{U_p})$$

Defining dimensionless variables

$$\vec{U_1} \equiv \frac{\vec{U}}{L}, \quad \vec{U_{f1}} \equiv \frac{\vec{U_f}}{L} \quad \text{and} \quad \Theta \equiv \frac{tU}{L}$$

where U, L: *characteristic velocity and length* of the system

$$\therefore St\frac{d\overrightarrow{U_1}}{d\Theta} = -(\overrightarrow{U_1} - \overrightarrow{U_{f1}})$$

or

In terms of displacement,

$$St\frac{d^2\overrightarrow{r_1}}{d\Theta^2} + \frac{d\overrightarrow{r_1}}{d\Theta} = \overrightarrow{U_{f1}}$$

where $\vec{r}$: dispacement vector

$$\overrightarrow{r_1} \equiv \frac{\vec{r}}{L}$$

$$St \equiv \frac{\rho_p x^2 U}{18\mu L} = \frac{\tau U}{L} \equiv \frac{\text{particle persistence}}{\text{size of obstacle}}$$

Stokes number

$$\therefore \overrightarrow{r_1} = f(St, Re, R) \sim \eta_R$$

↑ particle trajectory; ↑ $\overrightarrow{U_{f1}}$; ↑ B.C.; ↑ collection efficiency

* *For the two particle systems*
If Re, St and B.C. are the same, particle trajectories are the same.

* 이와 같은 관성현상은 운동방정식을 풀어 입자의 시간에 따른 변위(궤적)을 추적하여 이들의 거동을 해석한다.

* Applications
- Cyclone
- Particle impactor
- Filter

3.4 Migration of Particles by Other External Force Fields

In Stokes' law regime

$$F_{ext} = F_D\left(= \frac{3\pi\mu x}{C_c} U_{mig}\right)$$

where U_{mig} : migration or drift velocity in the fields

(1) Centrifugal migration

$$F_c = m_p\left(1 - \frac{\rho_f}{\rho_p}\right)\frac{U_t^2}{r} = m_p\left(1 - \frac{\rho_f}{\rho_p}\right)r\omega^2$$

↑ ↑

Acceleration of centrifugation,

cf. g

$$\therefore U_{cf} = \frac{(\rho_p - \rho_f)x^2 U_t^2 C_c}{18\mu r}$$

(2) Electrical Migration

$$\vec{F} = q\vec{E} = n_e e E$$

where q : charge of particles

E : strength of electric field

e : charge of electron (elementary unit of charge)

n_e : number of the units

$$U_e = \frac{n_e e E C_c}{3\pi\mu x}$$

전기적 방법에 의한 입도 측정이나 전기집진기의 원리가 된다.