Chapter 1

What is fluid dynamics?

Definition & Goal

- Fluid dynamics is a branch of mechanics, or physics, that seeks to describe or explain the nature of physical phenomena that involve the flow of liquids and/or gases.

- One of our primary goals will be to produce mathematical models that permit us to understand, describe, and design engineering systems and processes that involve the mechanics and dynamics of fluids.

- Thinking about fluid dynamics with some typical problems

Drop breakup in a stirred tank

Suppose we place a liquid in a tank and agitate it with a high speed rotating impeller

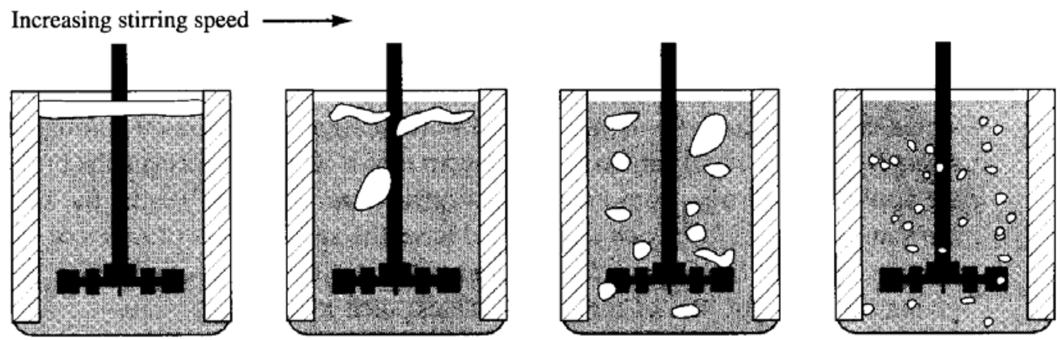


Figure 1.1.2 A light oil becomes increasingly dispersed as stirring speed increases.

Table 1.1.1 Important Physical Properties for the System Illustrated in Fig. 1.1.1

Parameter	Symbol	Dimensions[a,b]
Parameters that are physical properties		
Viscosity of phase 1 (water)	μ_1	m/Lt
Viscosity of phase 2 (oil)	μ_2	m/Lt
Densities of each phase	ρ_1, ρ_2	m/L^3
Interfacial tension between the phases	σ_{12}	m/t^2
What *operating* parameters do we expect to be important?		
Rotational speed	N	t^{-1}
Water temperature	T	T
What *design* parameters would affect the system?		
Geometrical parameters	H_T, D_T, D_I	L
Is the volume fraction ϕ expected to matter?		
If so, we add	ϕ	None

[a]The following conventional symbols are used: L, length; t, time; m, mass; T, temperature.
[b]It may not be intuitively obvious that the dimensions of viscosity and interfacial tension are as stated here. Accept these as fact for the moment. We define the terms explicitly, later.

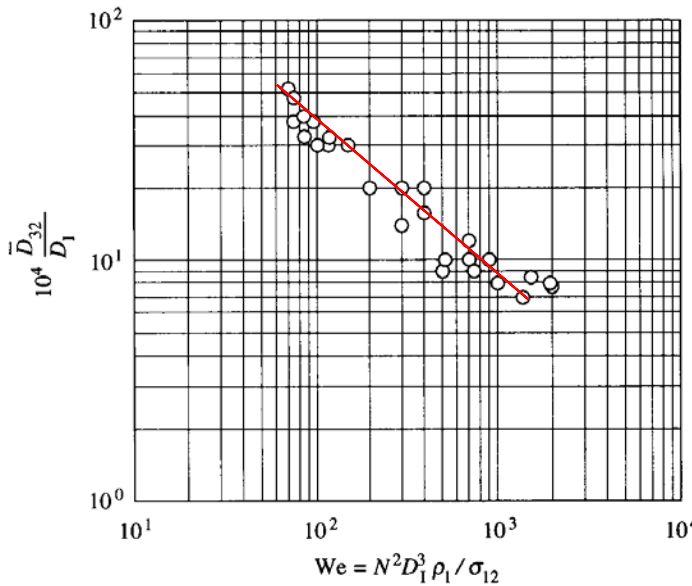
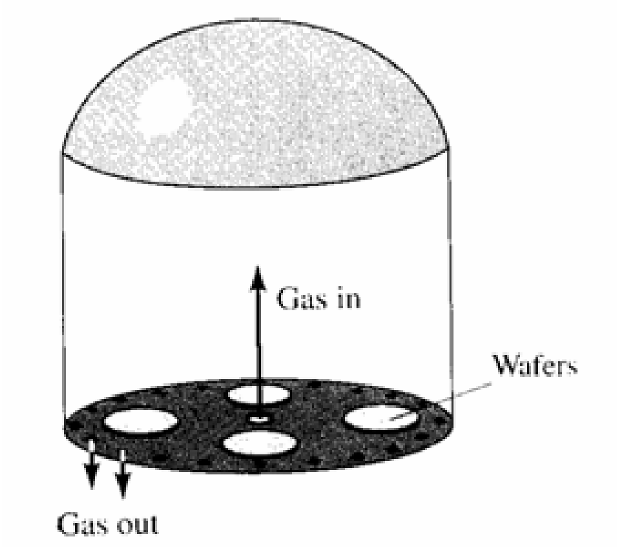


Figure 1.1.3 Data on the mean drop size in an agitated oil/water dispersion. Chen and Middleman, *AIChE J*, **13,** 989 (1967).

Flow field in a CVD reactor

In semiconductor device manufacturing, thin films are grown on silicon wafer
Uniformity of film growth is essential to the success of this process
Control of the flow field is very important

Predict the response of the flow field to changes in reactor geometry, gas flow rate, the positions of gas inlet and outlets, and operating conditions



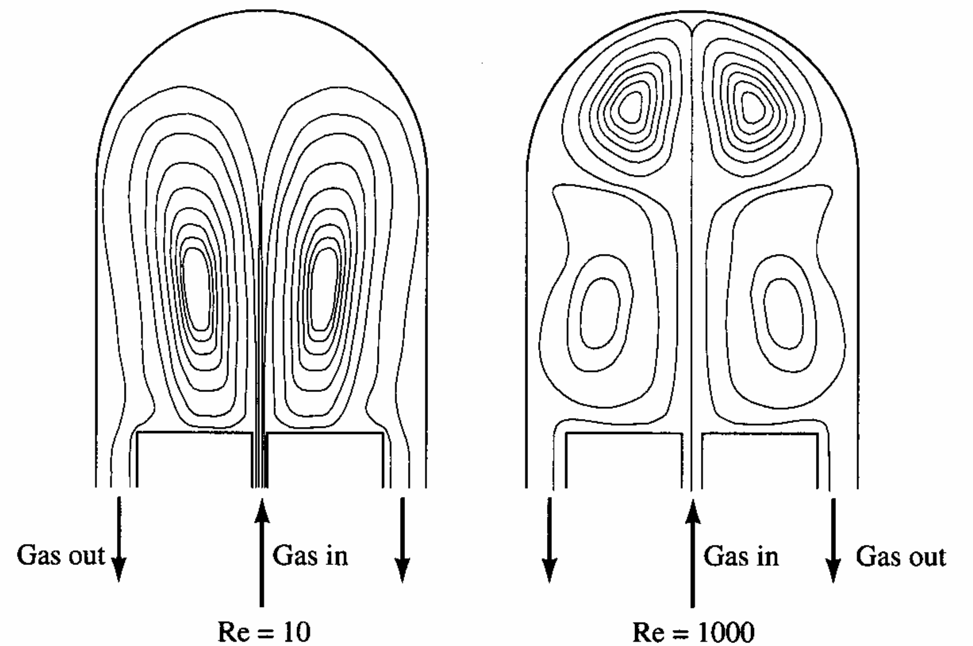
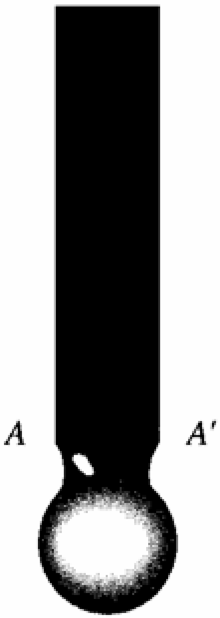


Figure 1.1.7 Streamlines for gas flow through an isothermal bell jar reactor.

Dimensional analysis

The problem of a drop of liquid formed at the lower end of a vertical capillary



Step 1: Make a list of the relevant parameters

requires a good sense of the physics of the process

Step 2: List the fundamental dimensions of each parameter

$$V_{\text{drop}}\ [=]\ L^3 \quad D_c\ [=]\ L \quad \rho\ [=]\ m/L^3 \quad \sigma\ [=]\ m/t^2$$

$$g\ [=]\ L/t^2 \quad \mu\ [=]\ m/Lt \quad U\ [=]\ L/t$$

The effect of temperature arises only through its effect on the physical properties

Step 3: Determine, from the Buckingham pi theorem, the number of dimensionless groups that characterize this problem

No. of independent dimensionless groups = No. of parameters – No. of dimensions (4 = 7 – 3)

Step 4: From the list of the independent parameters (all but V_{drop}) select a number equal to D (=3, in this case) that will be used as 'recurring parameters'.

It is wise to pick a set of parameters that include all the dimensions

$$D_{\mathrm{c}}\ , \rho\ , \sigma$$

Step 5: Form, in turn, dimensionless groups that are proportional to each of the remaining nonrecurring parameters

recurring parameters

$$\downarrow$$

$$V_{\mathrm{drop}}^{*} = V_{\mathrm{drop}}[D_{\mathrm{c}}^{a} \sigma^{b} \rho^{c}]$$

$\uparrow$ nonrecurring parameter

$$g^{*} = g D_{\mathrm{c}}^{a} \sigma^{b} \rho^{c}$$

$$U^{*} = U D_{\mathrm{c}}^{a} \sigma^{b} \rho^{c}$$

$$\mu^{*} = \mu D_{\mathrm{c}}^{a} \sigma^{b} \rho^{c}$$

a,b,c are different for each of these four equations

Step 6: For each of the four equations above, solve for the set of exponents

$$V_{\text{drop}}^{*} = V_{\text{drop}} D_{\text{c}}^{a} \sigma^{b} \rho^{c}$$

$$m^{0} L^{0} t^{0} = L^{3} L^{a} (m / t^{2})^{b} (m / L^{3})^{c}$$

$$m: \quad 0 = b + c$$

$$L: \quad 0 = 3 + a - 3c$$

$$t: \quad 0 = -2b$$

$$b = 0, c = 0, a = -3$$

$$V_{\text{drop}}^{*} = \frac{V_{\text{drop}}}{D_{\text{c}}^{3}}$$

$$m^{0} L^{0} t^{0} = (L / t^{2}) L^{a} (m / t^{2})^{b} (m / L^{3})^{c}$$

$$0 = b + c$$

$$0 = 1 + a - 3c$$

$$0 = -2 - 2b$$

$$b = -1 \quad c = 1 \quad a = 2$$

$$g^{*} = \frac{g \rho D_{\text{c}}^{2}}{\sigma}$$

$$m^{0} L^{0} t^{0} = (L / t) L^{a} (m / t^{2})^{b} (m / L^{3})^{c}$$

$$0 = .b + c$$

$$0 = 1 + a - 3a$$

$$0 = -1 - 2b$$

$$b = -\tfrac{1}{2} \quad c = \tfrac{1}{2} \quad a = \tfrac{1}{2}$$

$$U^{*} = U \left(\frac{D_{\text{c}} \rho}{\sigma} \right)^{1/2}$$

$$m^{0} L^{0} t^{0} = (m / Lt) L^{a} (m / t^{2})^{b} (m / L^{3})^{c}$$

$$0 = 1 + b + c$$

$$0 = -1 + a - 3c$$

$$0 = -1 - 2b$$

$$b = -\tfrac{1}{2} \quad c = -\tfrac{1}{2} \quad a = -\tfrac{1}{2}$$

$$V_{\text{drop}}^{*} \equiv \frac{V_{\text{drop}}}{\text{D}_{\text{c}}^{3}} = f\left[U \left(\frac{\rho D_{\text{c}}}{\sigma} \right)^{1/2}, \frac{\rho g D_{\text{c}}^{2}}{\sigma}, \frac{\mu}{(\rho \sigma D_{\text{c}})^{1/2}} \right]$$

$$\mu^{*} = \mu (D_{\text{c}} \rho \sigma)^{-1/2}$$

Dispersion of an oil stream in an aqueous pipe flow

Predict the mean droplet diameter as a function of the parameters that characterize this flow

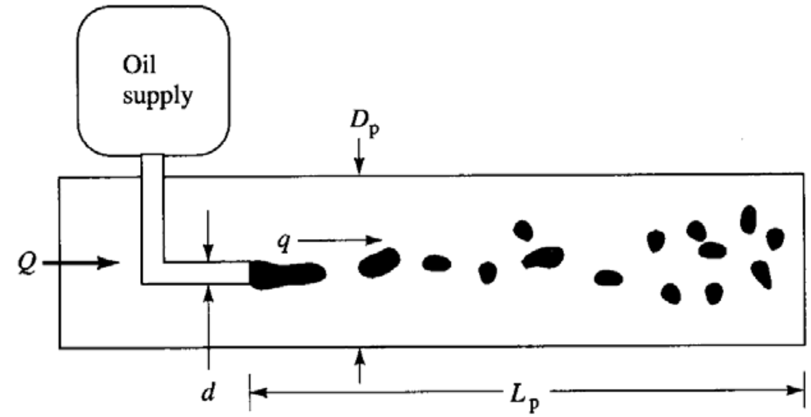


Figure 1.2.3 An oil stream is dispersed into droplets by a surrounding aqueous flow.

Step 1: Make a list of parameters.

Step 2: List the fundamental dimensions.

$$\overline{D}\,[=]\,L \quad D_p\,[=]\,L \quad L_p\,[=]\,L \quad d\,[=]\,L \quad \rho\,[=]\,m/L^3 \quad \sigma\,[=]\,m/t^2$$

$$\mu\,[=]\,m/Lt \quad Q\,[=]\,L^3/t \quad q\,[=]\,L^3/t \quad \rho'\,[=]\,m/L^3 \quad \mu'\,[=]\,m/Lt$$

Step 3: Use the Buckingham pi theorem.

The number of fundamental dimensionless groups = 11 – 3 = 8

Step 4: Select the recurring parameters. D_p, ρ, σ

Step 5: Form, in turn, dimensionless groups.

$$\overline{D}^* = \overline{D} D_p^a \sigma^b \rho^c \quad L_p^* = L_p D_p^a \sigma^b \rho^c \quad \mu^* = \mu D_p^a \sigma^b \rho^c \quad \mu'^* = \mu' D_p^a \sigma^b \rho^c$$

$$\rho'^* = \rho' D_p^a \sigma^b \rho^c \quad d^* = d D_p^a \sigma^b \rho^c \quad q^* = q D_p^a \sigma^b \rho^c \quad Q^* = Q D_p^a \sigma^b \rho^c$$

Step 6: Solve for the coefficients for each of the equations.

$$\overline{D}^* = \overline{D} D_p^a \sigma^b \rho^c$$

$$m^0 L^0 t^0 = L\, L^a (m/t^2)^b (m/L^3)^c$$

$$m: \quad 0 = b + c$$

$$L: \quad 0 = 1 + a - 3b$$

$$t: \quad 0 = -2b$$

$$b = 0 \quad c = 0 \quad a = -1$$

$$\overline{D}^* = \frac{\overline{D}}{D_p}$$

$$m^0 L^0 t^0 = (m/Lt) L^a (m/t^2)^b (m/L^3)^c$$

$$0 = 1 + b + c$$

$$0 = -1 + a - 3c$$

$$0 = -1 - 2b$$

$$b = -\tfrac{1}{2} \quad c = -\tfrac{1}{2} \quad a = -\tfrac{1}{2}$$

$$\mu^* = \mu \left(D_p \rho \sigma\right)^{-1/2}$$

$$\overline{D}^* = \frac{\overline{D}}{D_p} = F\left[\mu \left(D_p \rho \sigma\right)^{-1/2}, \mu' \left(D_p \rho \sigma\right)^{-1/2}, \frac{\rho'}{\rho}, \frac{d}{D_p}, \frac{L_p}{D_p}, Q\left(\frac{\rho}{\sigma D_p^3}\right)^{1/2}, q\left(\frac{\rho}{\sigma D_p^3}\right)^{1/2} \right]$$

$$\overline{D}^* = \frac{\overline{D}}{D_p} = F\left[\mu' \left(D_p \rho \sigma\right)^{-1/2}, \frac{\mu'}{\mu}, \frac{\rho'}{\rho}, \frac{d}{D_p}, \frac{L_p}{D_p}, Q\left(\frac{\rho}{\sigma D_p^3}\right)^{1/2}, \frac{q}{Q} \right]$$

Speculation about the physics of the process

- viscosity of oil is not significant if it is comparable to that of water
- inlet tube diameter is of no significance if it is large compared to the drop size
- most liquid densities lie in a narrow range -> no effect of density ratio
- as long as pipe length is large, drop size reaches equilibrium and does not change
- if *q/Q* is small, it does not affect the drop breakup

$$\overline{D}^* = \frac{\overline{D}}{D_p} = F\left[Q\left(\frac{\rho}{\sigma D_p^3}\right)^{1/2} \right]$$