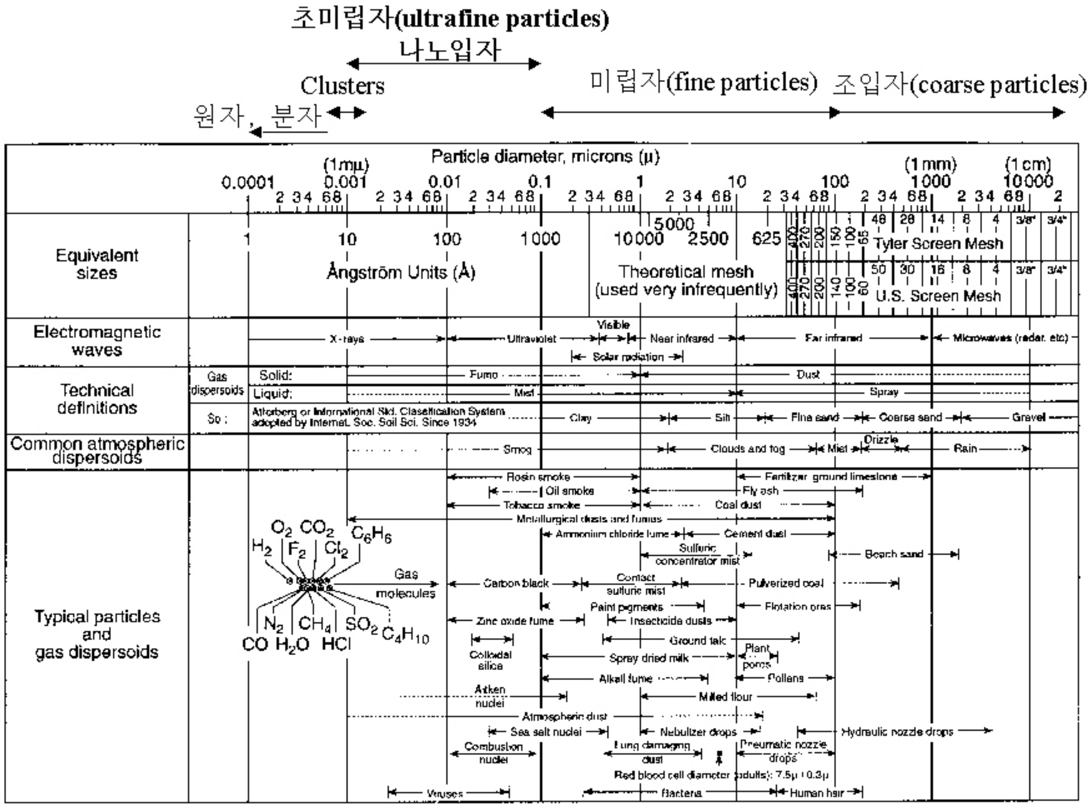
Chapter 3. Particle Size Analysis

3.1 Introduction

Particle size/Particle size distribution: a key role in determining the bulk properties of the powder...

Size Ranges of Particles

- Coarse particles : >10 ㎛
- Fine particles : ~1 ㎛
- Ultrafine(nano) particles : <0.1 ㎛ (100nm)



3.2 Describing the Size of a Single Particle

Description of regular-shaped particles: Table 3.1

Figure 3.1

- Equivalent circle diameter
- Martin's diameter
- Feret diameter

Equivalent (sphere) diameters   Figure 3.2

- *Equivalent volume (sphere)* diameters:

  the diameter of the hypothetical sphere having the same volume

$$d_{p,v} = \left(\frac{6V}{\pi}\right)^{1/3}$$

- *Equivalent surface* diameter:

  the diameter of the hypothetical sphere having the same surface area

$$d_{p,s} = \left(\frac{S}{\pi}\right)^{1/2}$$

- *Surface-volume* diameter:

  the diameter of the hypothetical sphere having the same surface-to-volume ratio

$$d_{p,sv} = \frac{6V}{S}$$

- *Stokes* diameter:

  the diameter of the hypothetical sphere having the same terminal settling velocity

- *Aerodynamic* diameter:

  the diameter of the hypothetical unit-density sphere having the same terminal settling velocity

"Which diameter we use depends on the end use of the information."

3.2 Description of Population of Particles

1) Introduction to Size Distribution of Particles

Particle size ~ diameter, $d_p(\mu m)$

Data on particle size measurement

Size Range $d_{p,i} \sim d_{p,i+1}$, μm	Count	Cumulative Fraction, F_i	Fraction, $F_{i+1}-F_i$	$\frac{F_{i+1}-F_i}{d_{p,i+1}-d_{p,i}}$
0-4	104	0.104	0.104	0.026
4-6	160	0.264	0.160	0.080
6-8	161	0.425	0.161	0.0805
8-9	75	0.500	0.075	0.075
9-10	67	0.567	0.067	0.067
10-14	186	0.753	0.186	0.0465
14-16	61	0.814	0.061	0.0305
16-20	79	0.893	0.079	0.0197
20-35	103	0.996	0.103	0.0034
35-50	4	1.000	0.004	0.0001
> 50	0	1.000	0	0.0

Count(number) size distribution: or frequency distribution by number

※ $\frac{F_{i+1}-F_i}{d_{p,i+1}-d_{p,i}}$ vs. d_p : *discrete size distribution*

※ $\lim_{\Delta d_p \to 0} \frac{F_{i+1}-F_i}{d_{p,i+1}-d_{p,i}} = \frac{dF_N(d_p)}{dd_p} \equiv f_N(d_p)$ vs. d_p

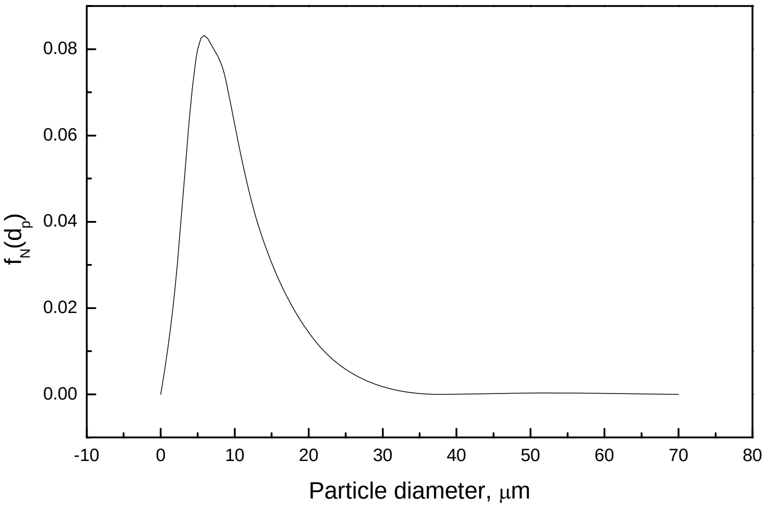
- *continuous size distribution* :

where $f_N(d_p)$, ($fraction/\mu m$): count(number) *size distribution function*

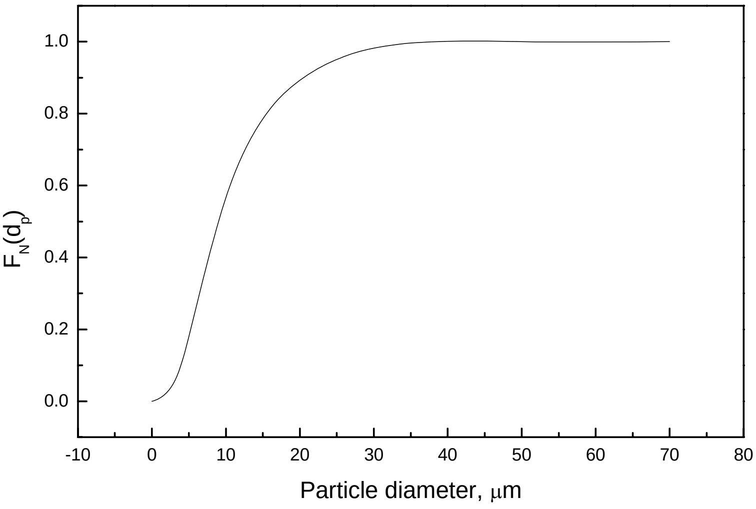
$f_N(d_p)dd_p$: fraction of particle counts(numbers) with diameters between d_p and $d_p + dd_p$

-*Cumulative* count *size distribution* : $F_N(a) = \int_0^a f_N(d_p)dd_p$, (fraction)

cf. $f_N(d_p) = \frac{dF_N(d_p)}{dd_p}$



Particle size distribution curve



Cumulative distribution curve

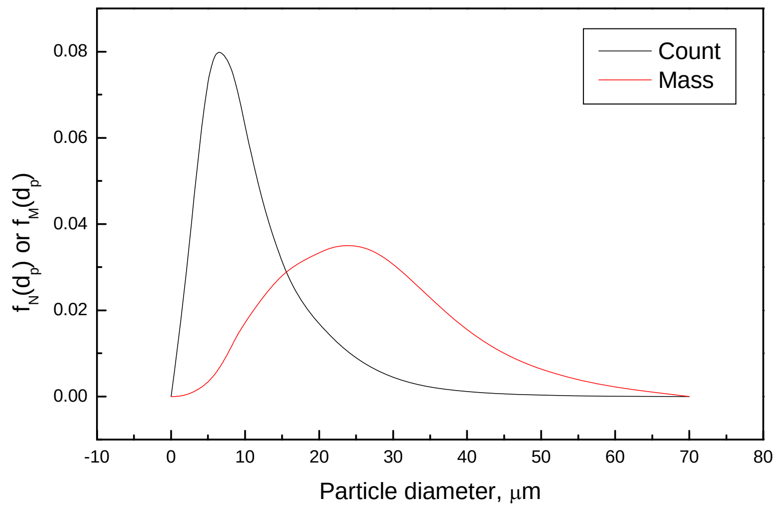
Mass(or volume) size distribution function

$f_M(d_p)$, ($mass\ fraction/\mu m$) : mass size distribution function

$f_M(d_p)dd_p$: fraction of particle mass with diameters between d_p and $d_p + dd_p$

$$f_M(d_p)dd_p = \frac{\rho_p \frac{\pi}{6} d_p^3 f(d_p) dd_p}{\int_0^\infty \rho_p \frac{\pi}{6} d_p^3 f(d_p) dd_p} = \frac{d_p^3 f(d_p) dd_p}{\int_0^\infty d_p^3 f(d_p) dd_p} = f_V(d_p)$$

Volume or mass size distribution function



Count and mass size distribution curves

Surface-area size distribution function

$$f_S(d_p)dd_p = \frac{\pi d_p^2 f(d_p)dd_p}{\int_0^\infty \pi d_p^2 f(d_p)dd_p} = \frac{d_p^2 f(d_p)dd_p}{\int_0^\infty d_p^2 f(d_p)dd_p}$$

Figure 3.4

Table 3.3

3.5 Describing the Population by a Single Number:

1) Averages

Averages

Based on count size distribution:

- Arithmetic Mean: $\overline{d_p} = \int_0^\infty d_p\ f(d_p)dd_p = \int_0^1 d_p\ dF(d_p)$ ☞ 11.8 µm
- Median : d_p at $F(d_p) = 0.5$ ☞ 9.0 µm
- Mode : most-frequent size ☞ 6.0 µm

The differences in averages come from *skewed distribution with* *long tail*.

Other Arithmetic Means

Mass mean diameter

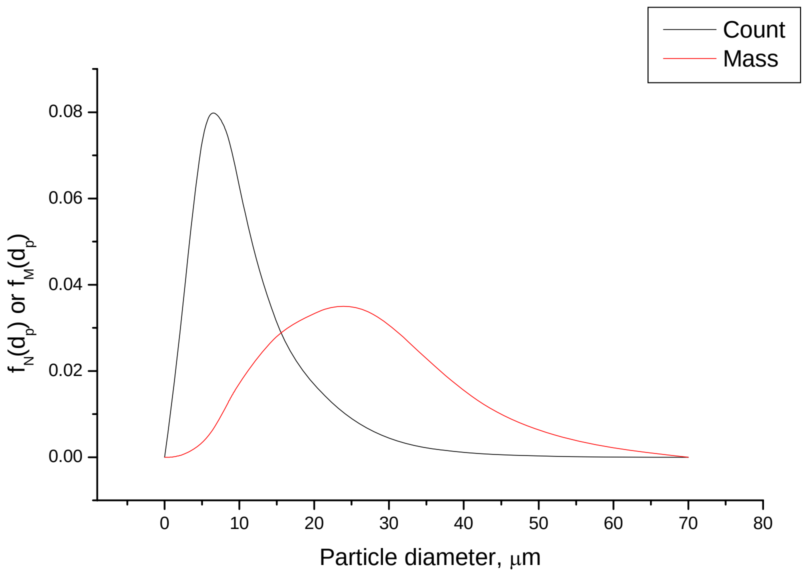
$$d_{p,\,mm} = \int_0^\infty d_p f_M(d_p)\, dd_p = \int_0^1 d_p\, dF_M$$

or

$$d_{p,\,mm} = \frac{\int_0^\infty d_p d_p^3 f_N(d_p)\, dd_p}{\int_0^\infty d_p^3 f_N(d_p)\, dd_p} = \frac{\int_0^1 d_p^4\, dF_N}{\int_0^1 d_p^3\, dF_N}$$

Surface-area mean diameter

$$d_{p,\,sm} = \int_0^\infty d_p f_S(d_p)\, dd_p = \int_0^1 d_p\, dF_S$$



Various average diameters
for skewed distribution with long tail

Moment average

First moment average: arithmetic count mean

$$\overline{d_p} = \left[\int_0^1 d_p\, dF \right]$$

Second moment average: diameter of average surface area or quadratic mean

$$d_{p,\bar{s}} = \left[\int_0^1 d_p^2 dF \right]^{1/2}$$

Third moment average: diameter of average mass or cubic mean

$$d_{p,\bar{m}} = \left[\int_0^1 d_p^3 dF \right]^{1/3}$$

위의 surface-mean, mass(volume)-mean 직경 및 second, third moment 평균

직경은 모두 직경의 지수배(1보다 큰)의 weight가 주어지므로 산술평균 직경보다 당연히 커진다.

Figure 3.6

Geometric mean

$$\overline{\log d_p} = \left[\int_0^1 \log d_p dF \right]$$

Harmonic mean

$$\frac{1}{d_{p,h}} = \left[\int_0^1 \frac{1}{d_p} dF \right]$$

2) *Standard deviation*

$$\sigma = \left[\int_0^\infty (d_p - \overline{d_p})^2 dF(d_p) \right]^{1/2} = \left[\int_0^\infty (d_p - \overline{d_p})^2 f(d_p) dd_p \right]^{1/2}$$

Degree of dispersion

입도의 분산(흩어짐)을 수치화한 값 ☞ Next section

3.7 Common Methods of Displaying Size Distribution: Standard Size Distribution Functions

1) Arithmetic Normal(Gaussian) distribution:

$$f(d_p)dd_p = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(d_p - \overline{d_p})^2}{2\sigma^2}\right]dd_p$$

and

$$\sigma = d_{p,84\%} - d_{p,50\%} = d_{p,50\%} - d_{p,16\%} = 0.5(d_{p,84\%} - d_{p,16\%})$$

- Hardly applicable to particle size distribution

∵ Particles : no negative diameter/distribution with long tail

2) *Lognormal distribution* :

위의 정규분포함수에서 d_p를 $\ln d_p$로 σ를 $\ln \sigma_g$로 바꾸면 얻어진다.

$$f(\ln d_p)d\ln d_p = \frac{1}{(\ln \sigma_g)\sqrt{2\pi}} \exp\left[-\frac{(\ln d_p - \overline{\ln d_p})^2}{2(\ln \sigma_g)^2}\right]d\ln d_p$$

where

$$\overline{\ln d_p} = \int_{-\infty}^{\infty} \ln d_p \; dF(d_p) = \int_{-\infty}^{\infty} \ln d_p \; dF(\ln d_p) = \int_{-\infty}^{\infty} \ln d_p \; f(\ln d_p)d\ln d_p = \ln d_{p,g}$$

$d_{p,g}$: geometric mean(median) diameter

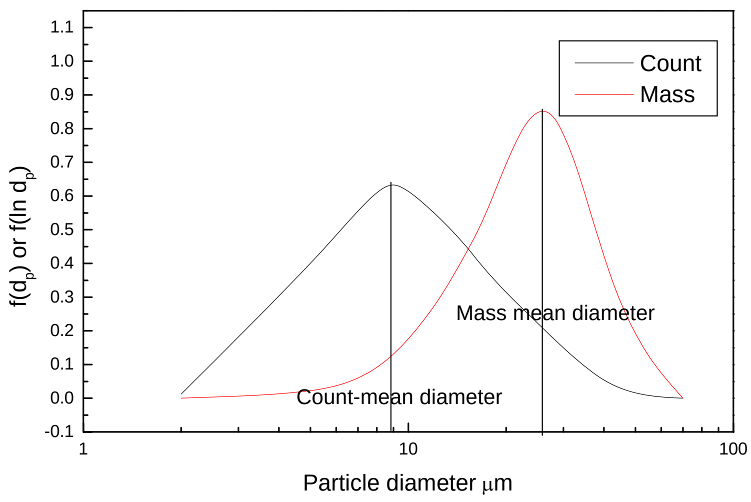
$$\ln \sigma_g = \left[\int_0^{\infty} (d_p - \overline{d_p})^2 dF(d_p)\right]^{1/2} = \left[\int_{-\infty}^{\infty} (\ln d_p - \ln d_{p,g})^2 f(\ln d_p)d\ln d_p\right]^{1/2}$$

σ_g : geometric standard deviation

$$\sigma_g = \frac{d_{p,84\%}}{d_{p,50\%}} = \frac{d_{p,50\%}}{d_{p,16\%}} = \left[\frac{d_{p,84\%}}{d_{p,16\%}}\right]^{1/2}$$

From the Table above our sample data can be plotted as follows:

위의 그림에서 우리 데이터는 대수정규분포함수에 가까움을 확인할 수 있다.



Expression of data as a logarithmic size distribution function

* <u>Log-probability diagram</u>

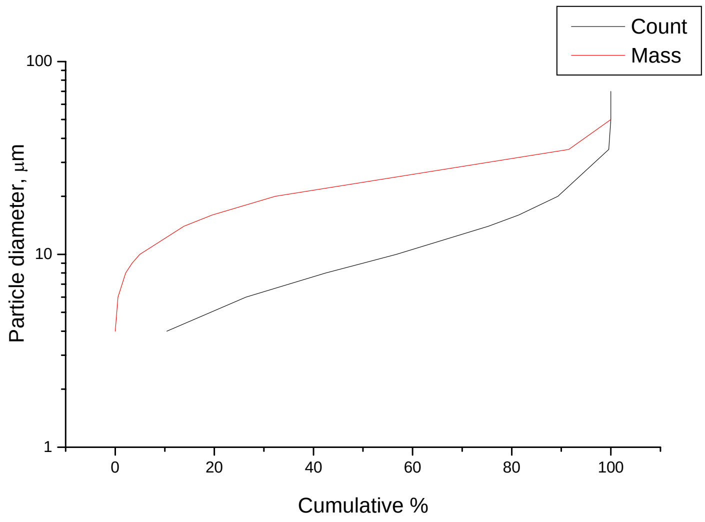
For cumulative size distribution

$$F(\ln a) = \int_0^{\ln a} f(\ln d_p) d\ln d_p$$

$$\downarrow$$

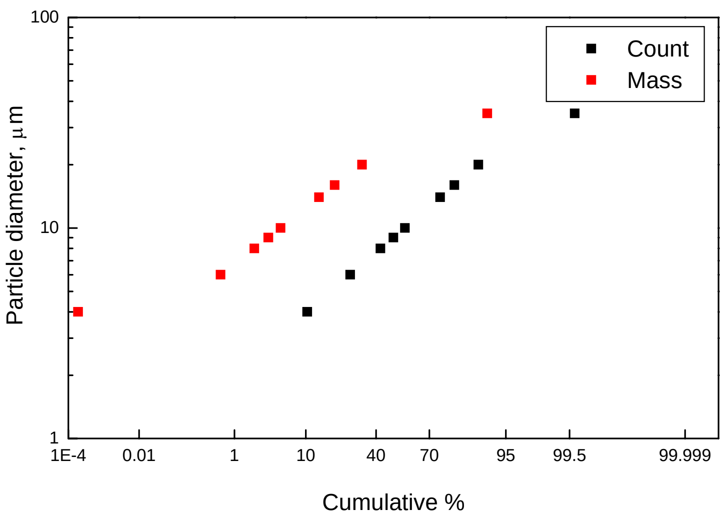
F vs. a

probability scale logarithmic scale



위 그림에서 보듯 대수정규분포함수에 가까움을 재확인할 수 있고, 두 직선이 나란하므로 두 분포함수에서 표준편차의 값은 같음을 알 수 있다.

* Dispersity criterion



Representation of cumulative % of the data
on log-probability graph

- *Monodisperse* : σ = 0 or σ_g =1.0, in actual $\sigma_g < 1.4$ (≈ 1.2)

- *Polydisperse*

여기서 굳이 1.4로 정한 것은 이 값이 모든 입자시스템을 자연스레 오랜 시간 방치하면 도달하는 입도분포의 값이기 때문이다. 1.2는 이에 비해 단분산의 기준을 좀더 엄격히 정한 값이다.

3.8 Methods of Particle Size Measurement

1) Sieving
2) Microscopy
3) Sedimentation
4) Permeametry
5) Electrozone Sensing
6) Laser Diffraction

3.9 Sampling

Avoiding segregation...

- The poeder should be in motion when sampled
- The whole of the moving stream should be taken for many short time increments...