Chapter 2. Multiple Particle Systems

2.1 Settling of A Suspension of Particles

Hindered Settling

입자가 모여 있으면 서로의 영향이 침강에 미친다. 이를 간섭침강이라 한다.

U_p , U_f: 계 안에서 정지한 관찰자가 관찰한 실제 입자 및 유체의 움직이는 속도

U_{ps} , U_{fs}: 계 밖에 있는 관찰자가 관찰한 입자 및 유체의 겉보기 속도 "superficial velocity"

$$U_{ps} = U_p(1-\varepsilon)$$

$$U_{fs} = U_f\varepsilon$$

한편으로 $1-\varepsilon$은 유체내 입자의 부피농도(cm^3 solid/cm^3 혼합물, C_v)라 할 수 있다. 이렇게 보면 U_{ps}는 유속과 농도의 곱, 즉 입자의 부피 flux(cm^3 solid/cm^2/s)로 볼 수 있고, 같은 맥락에서 U_{fs}는 유체의 부피 flux이다.

☞ U_{ps}와 U_{fs}는 superficial velocity임과 동시에 부피 flux 임!

2.2 Batch Settling

1) Settling Flux as a Function of Suspension Concentration

Corrected terminal settling velocity

$$U_{rel_T} \equiv U_p - U_f$$

$$\neq U_T$$

$$\equiv U_T\varepsilon f(\varepsilon)$$

Because of no net flow

$$U_{ps} + U_{fs} = 0$$

$$\therefore\ U_p(1-\varepsilon) + U_f\varepsilon = 0$$

$$\therefore\ U_p(1-\varepsilon)+[U_p-U_T\varepsilon f(\varepsilon)]\varepsilon=0$$

$$\therefore\ U_p=U_T\varepsilon^2 f(\varepsilon)$$

e.g. Richardson and Zaki(1954)

$$U_p=U_T\varepsilon^n$$

where

$$\frac{4.8-n}{n-2.4}=0.043Ar^{0.57}\left[1-2.4\left(\frac{d_p}{D}\right)^{0.27}\right]$$

Superficial solid velocity or volumetric solid flux(m/s)

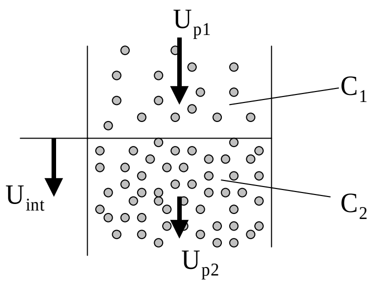
$$U_{ps}=U_p(1-\varepsilon)=U_T\varepsilon^2 f(\varepsilon)(1-\varepsilon)$$

$$=U_T(1-\varepsilon)\varepsilon^n$$

Settling flux curve (U_{ps} vs. C_v): Figure 2.1

2) Sharp Interfaces in Sedimentation: Movement of Interfaces and Layers

Material Balance over the interface



$$(U_{p1}-U_{int})C_1=(U_{p2}-U_{int})C_2$$

where $C=1-\varepsilon$, solids fraction

$$\therefore\ U_{int}=\frac{U_{ps_1}-U_{ps2}}{C_1-C_1}$$

As $\Delta C\rightarrow 0$,

$$U=\frac{dU_{ps}}{dC}$$

The velocity of layer of concentration C

3) Batch Settling test

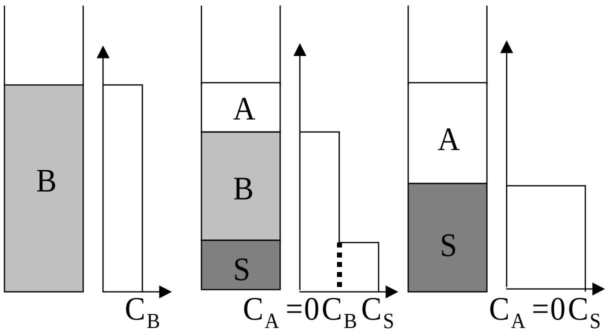
Depending on the initial concentration of the suspension

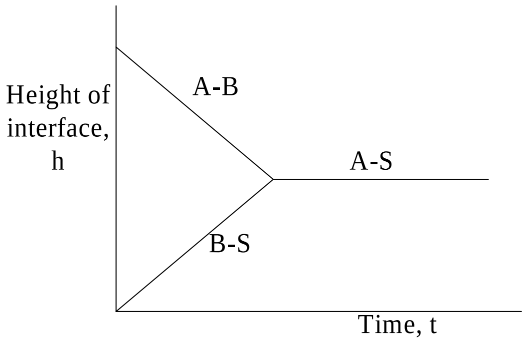
Type I Settling

A: clear liquid

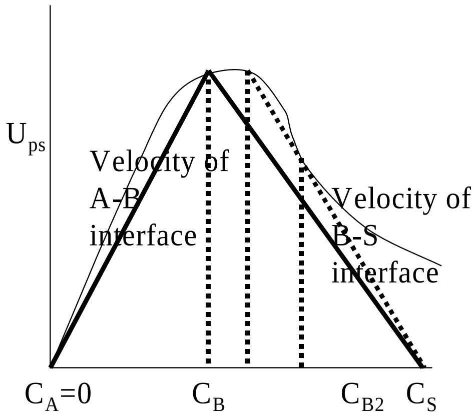
B: initial concentration of the suspension

C: Sediment



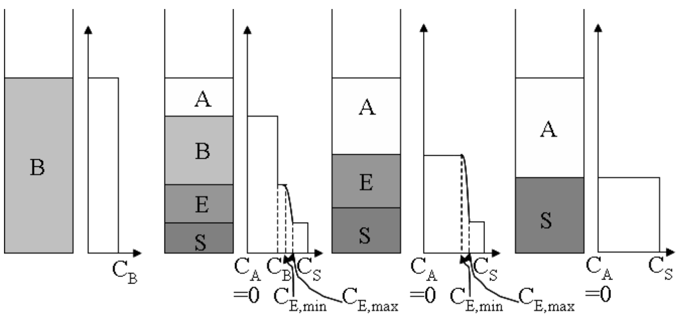


Variation of heights of interfaces with respect to time



Flux vs. concentration

Type II Settling



Zone E: concentrations variable with respect to position

$C_{E,min}$(fixed) ~$C_{E,max}$(fixed)

Recommended web site :

http://www.aem.umn.edu/Solid-Liquid_Flows/video.html

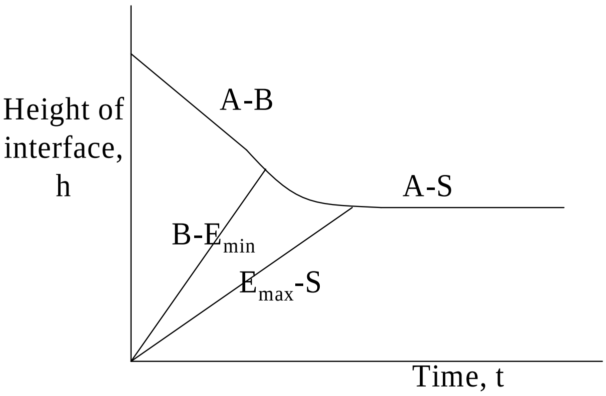
http://www-ec.njit.edu/ec_info/image2/ptc/

즉 두 type의 침강은 초기농도에 의존하는 것으로서 후자의 경우 하나의 층이 더 나타나는 것은 C_B의 위치에서 C_S가 변곡점 때문에 가려서 생기는 문제이다. 따라서 농도가 진하여 변곡점 아주 가까이에 있을 때 type II의 침강이 생긴다.

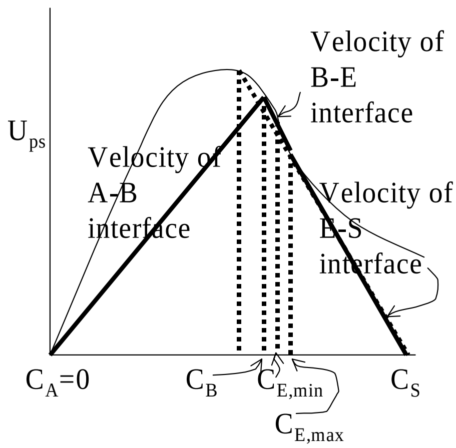
2.3 Continuous Settling: Liquid Sedimentation

Thickener vs. Clarifier

1) Settling of a Suspension in a Flowing Fluid



Variation in heights of interfaces with respect to time



Flux vs. concentration for particle slurry

Imposing a net fluid flow on (2.20)   Figure 2.11

$$U_{ps} = \frac{Q(1-\varepsilon)}{A} + U_T\varepsilon^2 f(\varepsilon)$$

Total solid flux   *Flux due to bulk flow*   *Flux due to settling*

Define $C_v \equiv 1-\varepsilon$, *particle volume concentration*

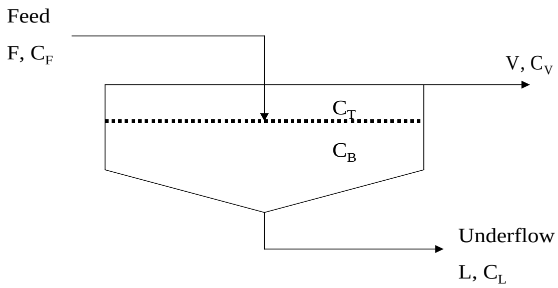
2) A Real Thickener: Thickener(Clarifier) Design

Material balance

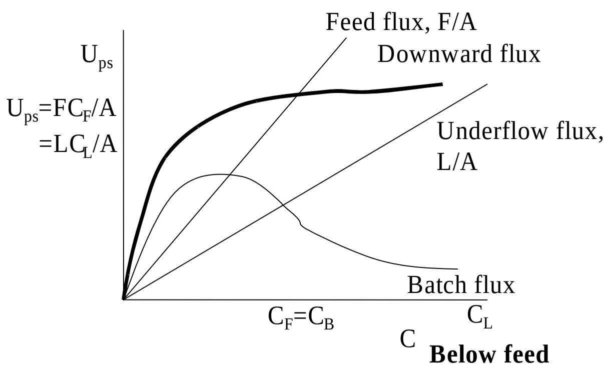
$$F = V + L$$

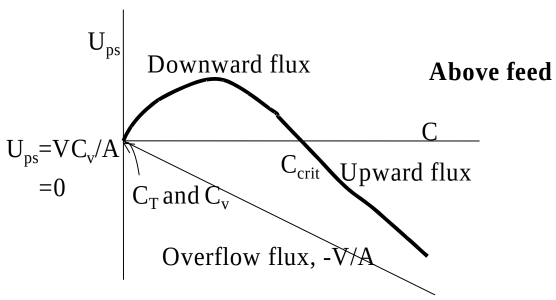
$$FC_F = VC_V + LC_L$$

Define $C_{crit} \equiv C$ **at** $U_{p,\,upward} = 0$



C_{crit}은 입자의 이동이 상향류에 포함되지 않는 한계 농도..Thickener 설계에서 중요한 값..





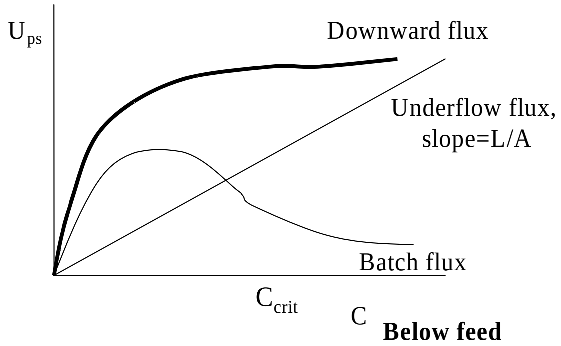
Critical loaded

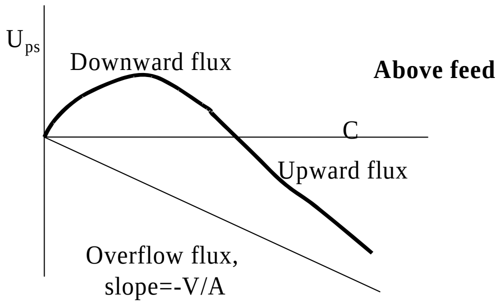
$$C_F = C_{crit}$$

$$\therefore \quad U_{p,\,upward} = 0, \quad C_B = C_F, \quad C_T = C_V = 0 \text{ and}$$

$$U_{ps,\,downflow} = \frac{FC_F}{A} = \frac{LC_L}{A}$$

Underloaded

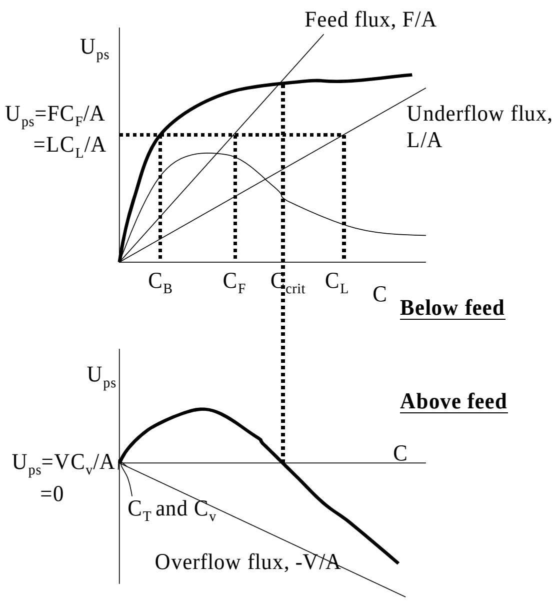




$$C_F < C_{crit}$$

$$\therefore \quad U_{p,\,upward} = 0, \quad C_B < C_F \ \text{ and } \ C_T = C_V = 0$$

$$U_{ps,\,downflow} = \frac{FC_F}{A} = \frac{LC_L}{A}$$

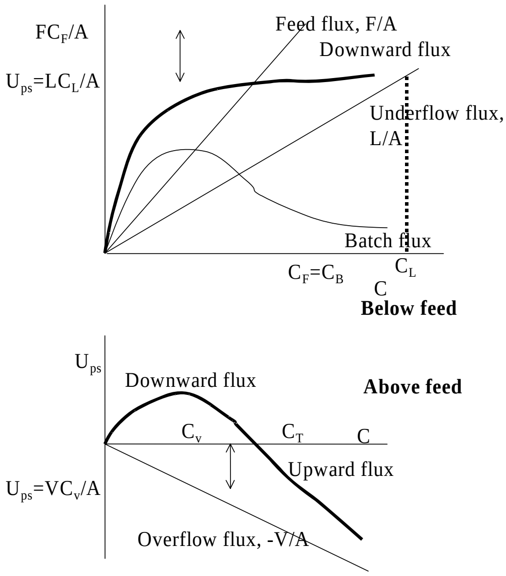


Underloaded Thickener

Overloaded

$C_F > C_{crit}$

$$\therefore \quad U_{p,\,upward} = \frac{FC_F}{A} - U_{ps,\,downflow}, \quad C_B = C_F \text{ and } C_T > C_V \neq 0$$



Overloaded thickener

$$U_{ps,\,downflow} = \frac{LC_L}{A}$$

Worked Example 2.4