

Chapter 2. Size and Shapes of Nanoparticles

2.1 Size and Shape of Single Particles

(1) Particle shapes and size description

**** Particle shapes***

Spherical/cube/angular/cylindrical/acicular/chainlike/fibrous/dendrite etc.

**** Primary particles***

Secondary particles:



-Also called agglomerates or aggregates

-Represented by fractal dimension

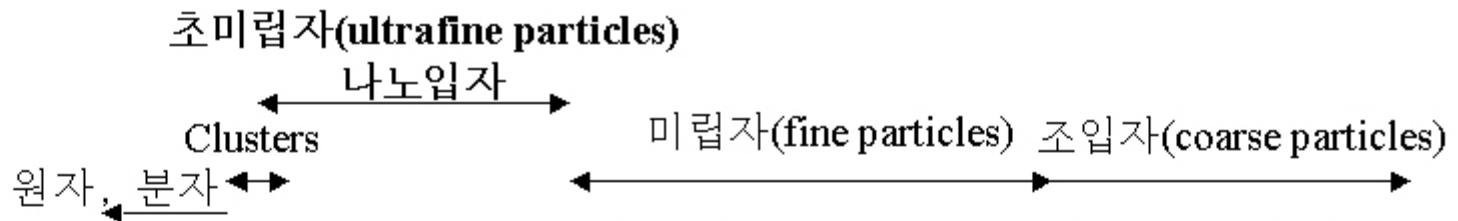
**** Particle size***

-Sphere: diameter

-Non-sphere: equivalent diameter

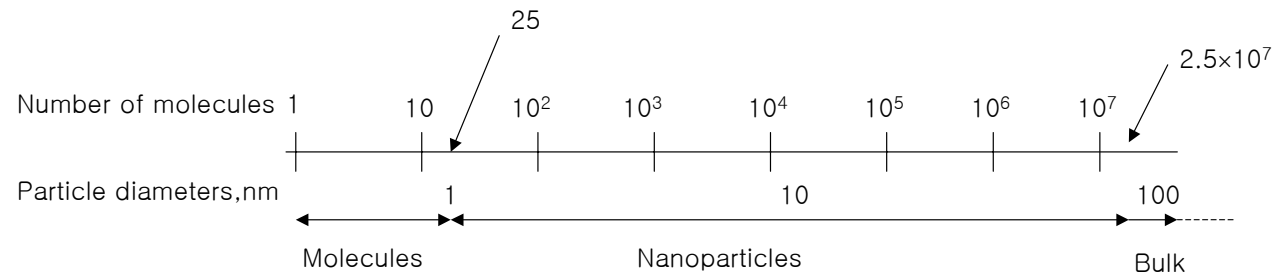
<i>Equivalent diameters</i>	<i>Description</i>	<i>Formula</i>	<i>Remarks</i>
<i>Equivalent volume (sphere) diameter</i>	<i>the diameter of the hypothetical sphere having the same <u>volume</u> as the particle volume</i>	$d_{p,v} = \left(\frac{6V}{\pi} \right)^{1/3}$	
<i>Equivalent surface-volume diameter</i>	<i>the diameter of the hypothetical sphere having <u>the same surface-to-volume ratio</u> as the particle</i>	$d_{p,sv} = \frac{6V}{S}$	
<i>Stokes diameter</i>	<i>the diameter of the hypothetical sphere having the same <u>terminal settling velocity</u> and density as the particle</i>		 <i>Later</i>
<i>Aerodynamic diameter</i>	<i>the diameter of the hypothetical unit-density sphere having the same <u>terminal settling velocity</u> as the particle</i>		 <i>Later</i>

"Which diameter we use depends on the end use of the information."



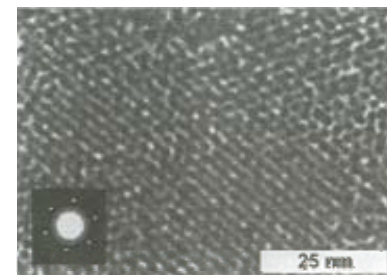
		Particle diameter, microns (μ)																																																																													
		0.0001				(1m μ) 0.001				0.01				0.1				1				10				100				(1 mm) 1000				(1 cm) 10000																																													
		2	3	4	68	2	3	4	68	2	3	4	68	2	3	4	68	2	3	4	68	2	3	4	68	2	3	4	68	2	3	4	68	2	3	4	68																																										
Equivalent sizes	Ångström Units (Å)																	10000	2500	625	400	300	270	250	200	150	100	80	Tyler Screen Mesh																48	28	14	8	4	3/8"	3/4"																												
	Theoretical mesh (used very infrequently)																	100	200	400	600	800	1000	1400	2000	2700	3500	4500	6000	8000	10000	14000	20000	27000	35000	45000	60000	80000	100000	U.S. Screen Mesh																10	20	40	60	80	100	120	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950
Electromagnetic waves		X-rays																Ultraviolet				Visible				Near infrared				Far infrared				Microwaves (radar, etc)																																													
																		Solar radiation																																																													
Technical definitions	Gas dispersoids																	Fume				Dust																																																									
	Solid:																	Mist				Spray																																																									
	Liquid:																																																																														
Common atmospheric dispersoids	Soil	Atterberg or International Skt. Classification System adopted by Internat. Soc. Soil Sci. Since 1934																Clay				Silt				Fine sand				Coarse sand				Gravel																																													
																		Smog				Clouds and fog				Mist				Drizzle				Rain																																													
Typical particles and gas dispersoids																		Rosin smoke				Oil smoke				Tobacco smoke				Mineralurgical dusts and fumes				Ammonium chloride fume				Sulfuric concentrator mist				Beach sand																																					
		Gas molecules																Carbon black				Contact sulfuric mist				Paint pigments				Insecticide dusts				Ground talc				Spray dried milk				Alkali fume				Milled flour				Atmospheric dust				Sea salt nuclei				Combustion nuclei				Viruses																	
																		Fly ash				Coal dust				Cement dust				Pulverized coal				Flotation gas				Plant poisons				Pollens				Nebulizer drops				Pneumatic nozzle drops				Hydraulic nozzle drops																									
																		Fertilizer				ground limestone				Plant poisons				Human hair				Bacteria				Human hair																																									

(2) Particle size and constituent atoms (molecules)



** Full shell clusters*

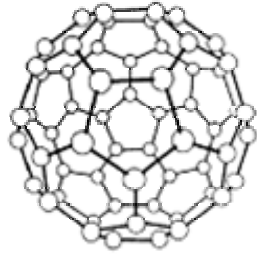
Full-shell Clusters	Total Number of Atoms	Surface Atoms (%)
1 Shell	13	92
2 Shells	55	76
3 Shells	147	68
4 Shells	309	62
5 Shells	561	56
7 Shells	1419	48



Hexagonal monolayer of Au55 nanoparticles on Poly(vinylpyrrolidone)

(3) Size and shape of specific particles

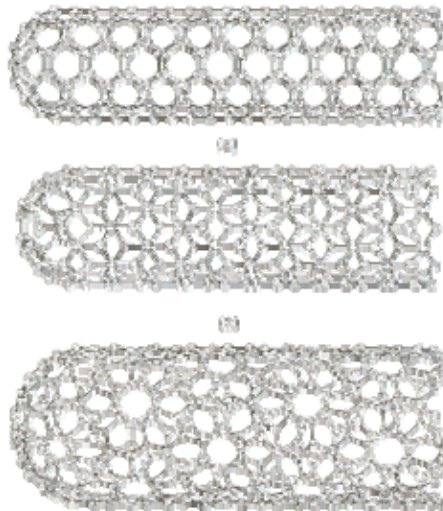
* *Fullerene*- named after the architect, Buckminster Fuller, who designed "Geodesic dome"



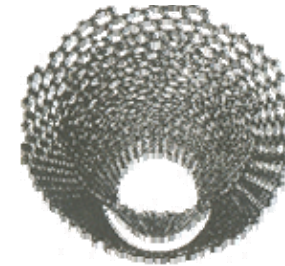
C60- buckminsterfullerene



* *Carbon nanotube*



Single-walled CNT
(a) armchair; (b) zigzag; (c) chiral structures



Multi-walled CNT

* *Polymers*

$$V = 0.001661 \frac{M_w}{\rho} \text{ nm}^3 \quad \text{where } M_w, \rho \text{ in cgs unit}$$

$$\therefore d_v = 0.1469 \left(\frac{M_w}{\rho} \right)^{1/3} \text{ nm}$$

* *Biological substance*

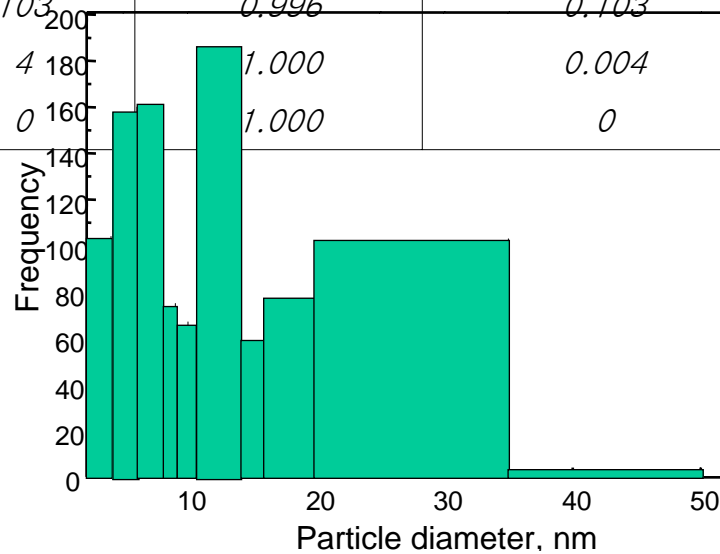
Class	Material	M_w (Da)	Size d (nm)
Amino acids	Glycine (smallest amino acid)	75	0.42
	Tryptophan (largest amino acid)	246	0.67
Nucleotides	Cytosine monophosphate (smallest DNA nucleotide)	309	0.81
	Guanine monophosphate (largest DNA nucleotide)	361	0.86
	Adenosine triphosphate (ATP, energy source)	499	0.95
Other molecules	Steric acid $C_{17}H_{35}CO_2H$	284	0.87
	Chlorophyll, in plants	720	1.1
Proteins	Insulin, polypeptide hormone	6,000	2.2
	Hemoglobin, carries oxygen	68,000	7.0
	Albumin, in white of egg	69,000	9.0
	Elastin, cell-supporting material	72,000	5.0
	Fibrinogen, for blood clotting	400,000	50
	Lipoprotein, carrier of cholesterol (globular shape)	1,300,000	20
	Ribosome (where protein synthesis occurs)		30
Viruses	Glycogen granules of liver		150
	Influenza		60
	Tobacco mosaic, length		120
	Bacteriophage T_2		140

Class	Material	Size d (μm)
Organelles (structures in cells outside nucleus)	Mitochondrion, where aerobic respiration produces ATP molecules	$0.5 \times 0.9 \times 3$
	Chloroplast, site of photosynthesis, length	4
	Lysosome (vesicle with enzymes for digesting macromolecules)	0.7
	Vacuole of amoeba	10
Cells	<i>Escherichia coli</i> (<i>E. coli</i>) bacterium, length	8
	Human blood platelet	3
	Leukocytes (white blood cells), globular shape	8–15
	Erythrocytes (red blood cells), disk shape	1.5×8
Miscellaneous	Human chromosome	9
	Fascicle in tendon	50–300

2.2 Particle size distribution functions

Data on particle size measurement

Size Range, nm., $d_{p,i} \sim d_{p,i+1}$	Count (Frequency)	Cumulative Fraction, $F_{c,i}$	Fraction, $F_{c,i+1} - F_{c,i}$	$\frac{F_{c,i+1} - F_{c,i}}{d_{p,i+1} - d_{p,i}}$
0-4	104	0.104	0.104	0.026
4-6	160	0.264	0.160	0.080
6-8	161	0.425	0.161	0.0805
8-9	75	0.500	0.075	0.075
9-10	67	0.567	0.067	0.067
10-14	186	0.753	0.186	0.0465
14-16	61	0.814	0.061	0.0305
16-20	79	0.893	0.079	0.0197
20-35	103	0.996	0.103	0.0034
35-50	4	1.000	0.004	0.0001
> 50	0	1.000	0	0.0



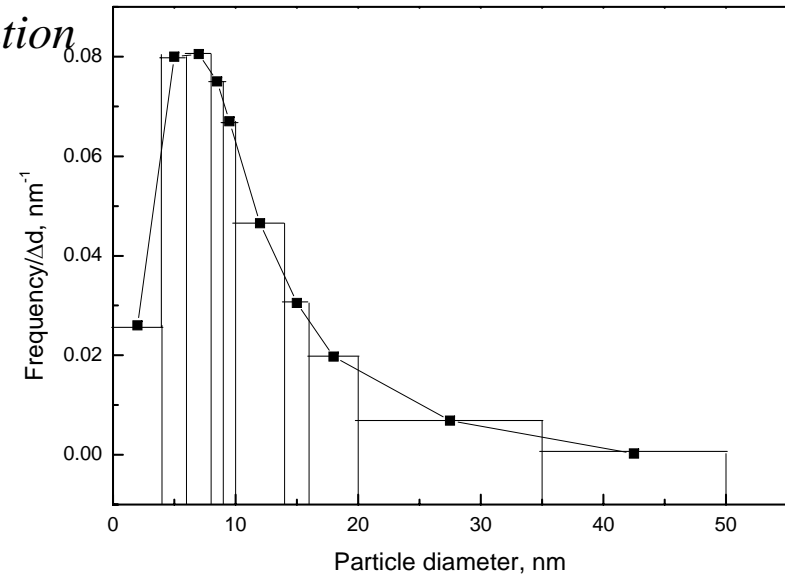
(1) Size distribution function

* Count size distribution function

$$\frac{F_{c,i+1} - F_{c,i}}{d_{p,i+1} - d_{p,i}} \text{ vs. } d_{p,i} \text{ Discrete size distribution}$$

- Sample data gives size distribution as \rightarrow

See the Table on p19 for details

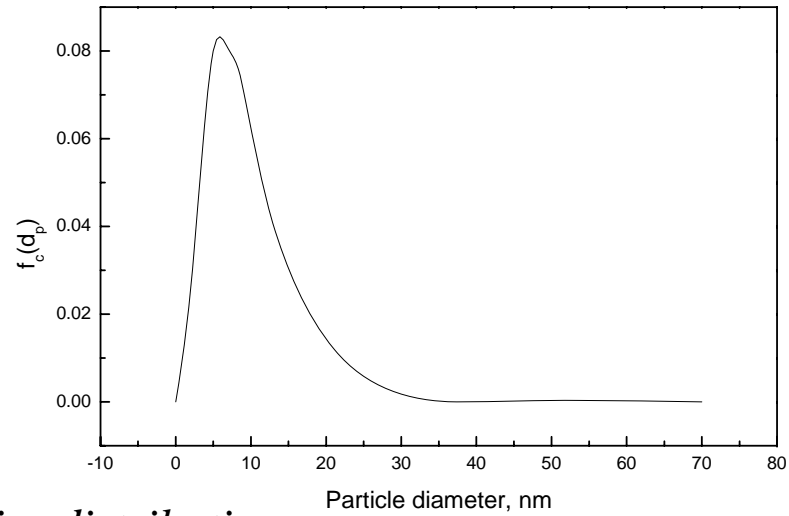


$$\lim_{d_{p,i+1} \rightarrow d_{p,i}} \frac{F_{c,i+1} - F_{c,i}}{d_{p,i+1} - d_{p,i}} = \frac{dF_c}{dd_p} = f_c(d_p) \text{ vs. } d_{p,i} \text{ Continuous size distribution}$$

where $f_c(d_p)$: count(number) size distribution function

$f_c(d_p)dd_p$: fraction of particle counts(numbers) with the diameters

between d_p and $d_p + dd_p$

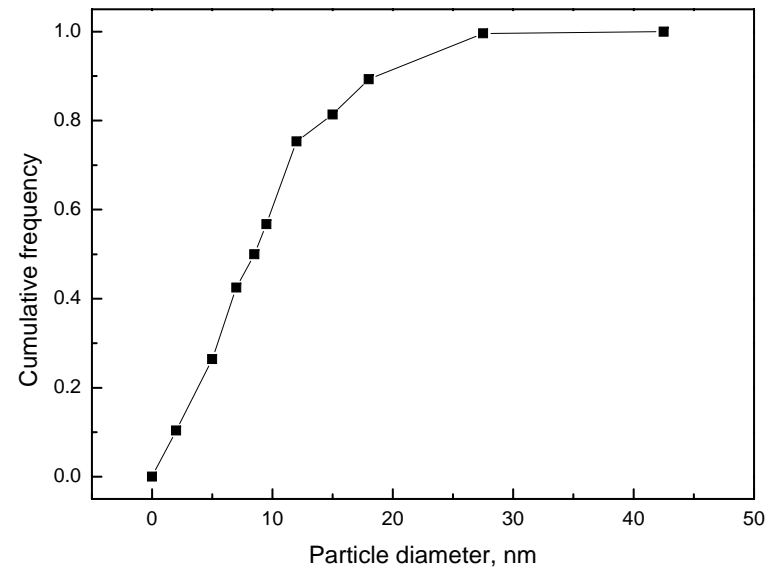


- *Cumulative count size distribution*

$$F_c(d_p) = \int_0^{d_p} f_c(d_p) dd_p$$

- *Sample data gives size distribution as →*

See the Table on p19 for details



* *Mass size distribution function* $f_m(d_p)$

$$\lim_{d_{p,i+1} \rightarrow d_{p,i}} \frac{F_{m,i+1} - F_{m,i}}{d_{p,i+1} - d_{p,i}} = \frac{dF_m}{dd_p} = f_m(d_p)$$

$f_m(d_p)dd_p$: *fraction of particle mass with the diameters between d_p and $d_p + dd_p$*

$$f_m(d_p) = \frac{\frac{\pi}{6} \rho_p d_p^3 f_c(d_p)}{\int_0^\infty \frac{\pi}{6} \rho_p d_p^3 f_c(d_p) dd_p} = \frac{\frac{\pi}{6} d_p^3 f_c(d_p)}{\int_0^\infty \frac{\pi}{6} d_p^3 f_c(d_p) dd_p} \equiv f_v(d_p)$$

Volume size distribution function

- *also called third moment distribution*

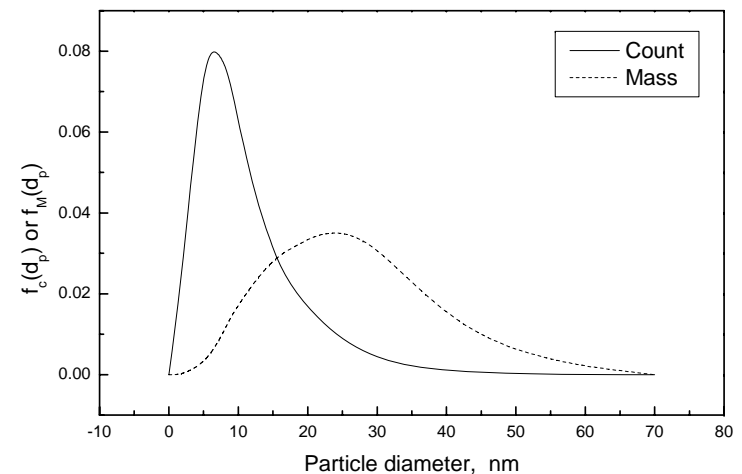
- *Sample data gives mass size distribution as \rightarrow*

See the Table on p19 for details

cf. Second moment distribution function

= surface distribution function

- *Cumulative mass size distribution function*



(2) *Mass distribution functions*

$$\lim_{d_{p,i+1} \rightarrow d_{p,i}} \frac{F_{M,i+1} - F_{M,i}}{m_{p,i+1} - m_{p,i}} = \frac{dF_M}{dm_p} = f_M(m)$$

$f_M(m)dm$: mass fraction of particles having mass between m and $m+dm$

$$f_M(m)dm = f_M(m)d\left(\frac{\rho_p \pi d_p^3}{6}\right) = f_M(m) \frac{\rho_p \pi d_p^2}{2} dd_p$$
$$\therefore f_m(d_p) = f_M(m) \frac{\rho_p \pi d_p^2}{2}$$

* *surface area distribution functions*

or others: distributions of charge, concentration, shape, price etc.

(3) Averages and Dispersion

Average diameters

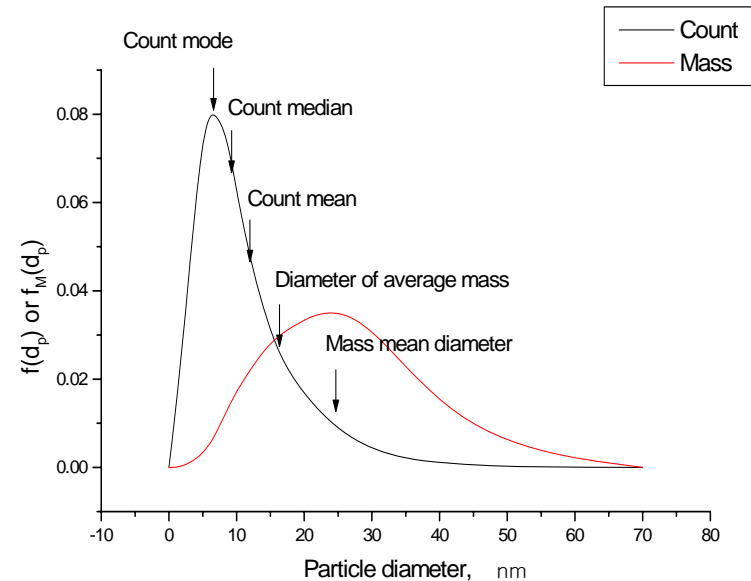
For any size distribution function, $f_I(d_p)$, where $I=c, m...$

Mean diameter: $\bar{d}_p = \int_0^{\infty} d_p f_I(d_p) dd_p$
 e.g. count mean, mass mean...

Medium diameter: d_p at $F(d_p)=0.5$

Mode diameter: most frequent size

- From sample data, count mean diameter = 11.8nm
 median diameter = 9.0nm
 mode diameter = 6.0nm



The differences in averages come from *skewed distribution with long tail*.

* mass (average diameter) > surface (average diameter) > count (average diameter)

Standard deviation

$$\sigma = \left(\int_0^{\infty} (\bar{d}_p - d_p)^2 f(d_p) dd_p \right)^{1/2}, \quad nm$$

(4) Diameter of average properties

Diameter of average mass

$$d_{p,\bar{m}} = \left(\frac{6}{\pi\rho_p} \frac{M}{N} \right)^{1/3} = \left(\frac{6}{\pi\rho_p} \frac{\sum m_i}{N} \right)^{1/3} = \left(\frac{6}{\pi\rho_p} \frac{\sum \frac{\pi}{6} \rho_p d_p^3}{N} \right)^{1/3} = \left(\frac{\sum d_p^3}{N} \right)^{1/3}$$

where M: total mass or total mass concentration

N: total number or total number concentration

** In general, for any property $Q \sim d_p^q$*

Then diameter of average Q

$$d_{p,\bar{Q}} = \left(\frac{\sum d_p^q}{N} \right)^{1/q}$$

(5) Lognormal size distribution function

- Standard form of distribution

* Normal(Gaussian) distribution function

$$f_G(x)dx = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\bar{x} - x)^2}{2\sigma^2}\right) dx$$

where $\sigma = x_{50\%} - x_{16\%} = x_{84\%} - x_{50\%} = \frac{1}{2}(x_{84\%} - x_{16\%})$

* Lognormal size distribution function

Replacing x with $\ln d_p$ and σ with $\ln \sigma_g$

$$f_{LN}(d_p)d(\ln d_p) = \frac{1}{\ln \sigma_g \sqrt{2\pi}} \exp\left(-\frac{(\overline{\ln d_p} - \ln d_p)^2}{2(\ln \sigma_g)^2}\right) d(\ln d_p)$$

where $\overline{\ln d_p} = \int_0^{\infty} (\ln d_p) f_{LN}(d_p) d(\ln d_p) = \ln d_{pg}$ d_{pg} : geometric mean diameter

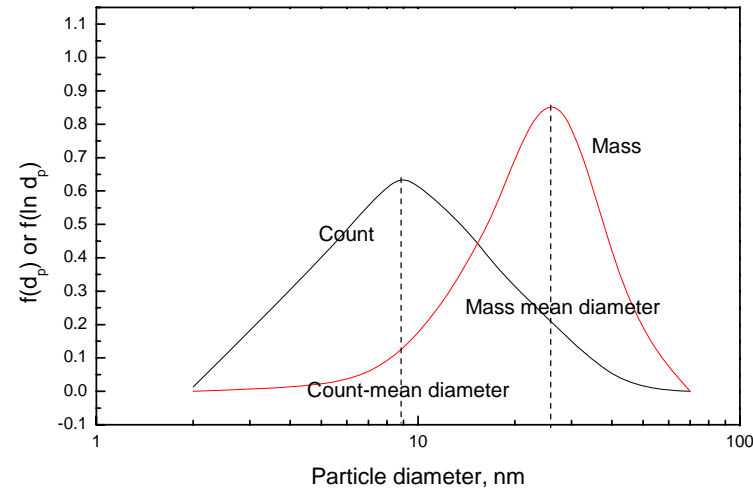
$$\ln \sigma_g = \left(\int_0^{\infty} (\overline{\ln d_p} - \ln d_p)^2 f_{LN}(d_p) d(\ln d_p) \right)^{1/2} \sigma_g$$

σ_g : geometric standard deviation

where
$$\sigma_g = \frac{x_{50\%}}{x_{16\%}} = \frac{x_{84\%}}{x_{50\%}} = \left(\frac{x_{84\%}}{x_{16\%}} \right)^{1/2}$$

- Sample data can be plotted on log scale abscissa ↓

See Table on p19 for details



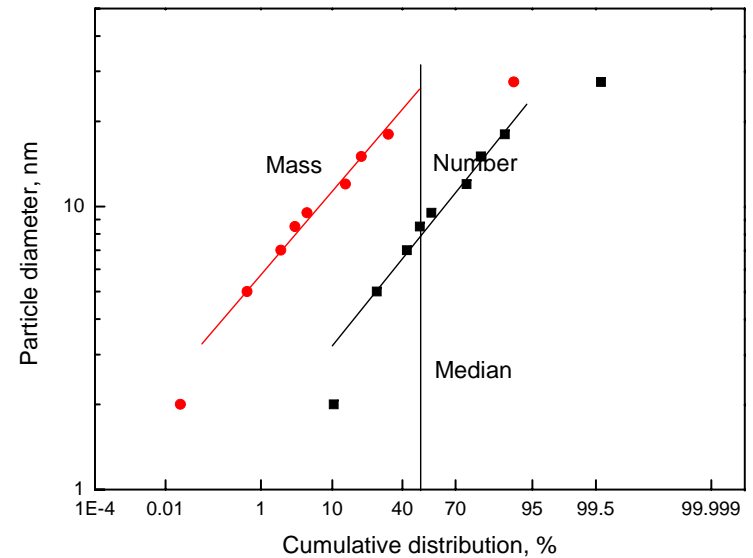
* *Log probability diagram*

Cumulative lognormal size distribution

$$F_{LN}(d_p) = \int_{-\infty}^{\ln d_p} f_{LN}(d_p) d(\ln d_p)$$

- Sample data can be plotted on log-probability coordinates →

See Table on p19 for details



* *Criterion for monodispersity: $\sigma_g = 1.2 \sim 1.4$.*

* *Relations of average diameters (Hatch-Choate equation)*

- *From CMD to other average diameter*

$$d_A = \text{CMD} \exp(b \ln^2 \sigma_g)$$

where $b \rightarrow$

- *From CMD to diameter of average*

$$d_q = \text{CMD} \exp\left(\frac{p}{2} \ln^2 \sigma_g\right)$$

where $p=1$ for length, 2 for surface, 3 for volume or mass

<i>To convert from CMD to</i>	<i>For the value of b</i>
<i>Mode</i>	<i>-1</i>
<i>Count mean diameter</i>	<i>0.5</i>
<i>Diameter of average mass</i>	<i>1.5</i>
<i>Mass median diameter</i>	<i>3</i>
<i>Mass mean diameter</i>	<i>3.5</i>

Table. Calculation of various distribution functions

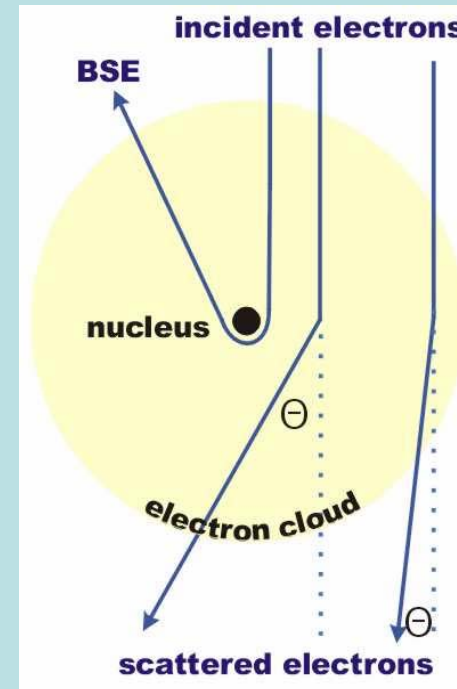
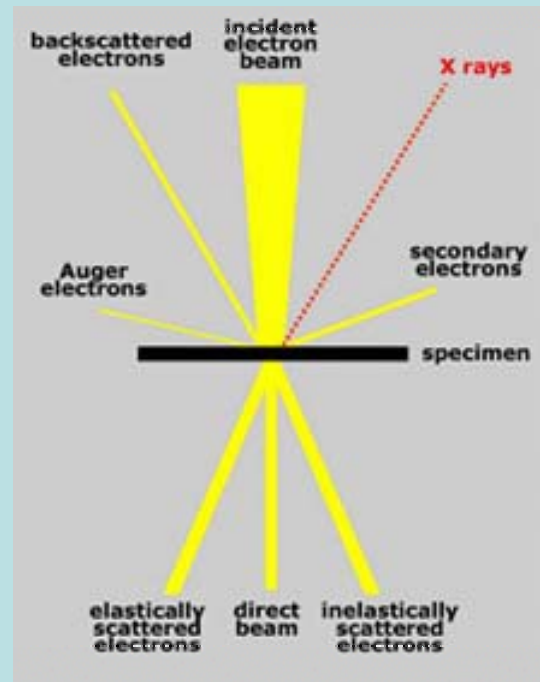
avg dia	frequency	cum frac	fraction/ln d	volume	vol*freq	vol fraction	v fr/del d	largest d	v cum frac	v fr/ln d
0	0	0	0	0	0	0	0	0	0	0
2	104	0.104	0.012539	4.188787	435.6338	0.00023	5.75E-05	4	0.00023	2.77359E-05
5	160	0.264	0.394609	65.44979	10471.97	0.00553	0.002765	6	0.00576	0.013638381
7	161	0.425	0.559646	179.5942	28914.67	0.015269	0.007634	8	0.021029	0.053075427
8.5	75	0.5	0.636764	321.5548	24116.61	0.012735	0.012735	9	0.033764	0.108123873
9.5	67	0.567	0.635912	448.9201	30077.65	0.015883	0.015883	10	0.049647	0.150748857
12	186	0.753	0.552794	904.7779	168288.7	0.088868	0.022217	14	0.138514	0.264115413
15	61	0.814	0.456821	1767.144	107795.8	0.056923	0.028462	16	0.195438	0.426291361
18	79	0.893	0.354032	3053.625	241236.4	0.127389	0.031847	20	0.322826	0.570882282
27.5	103	0.996	0.184055	10889.21	1121589	0.592273	0.039485	35	0.915099	1.058355682
42.5	4	1	0.011215	40194.35	160777.4	0.084901	0.00566	50	1	0.238034813
70	0	1	0	179594.2	0	0	0	70	1	0
					1893703	1				

2.3 Observation and size measurement of particles

(1) Electron Microscopy

http://davinci.ethz.ch/solid/_research/elmi/methods.htm

* *Electron-matter interaction*



* *Transmission electron microscope*

- *Energy: 100KeV ~ 1MeV*

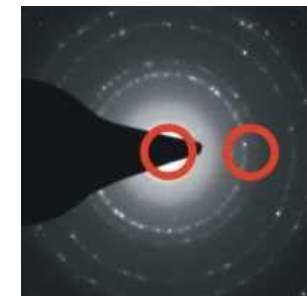
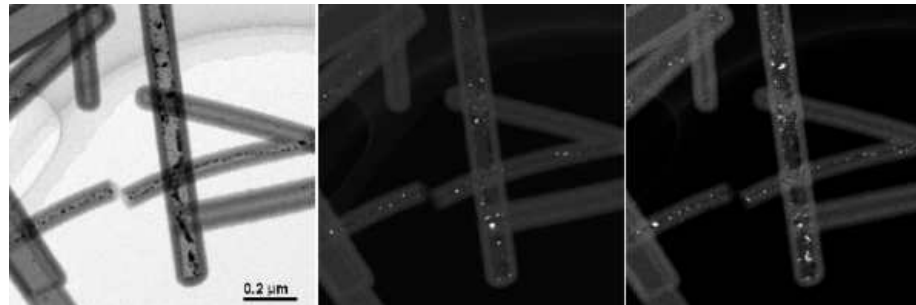
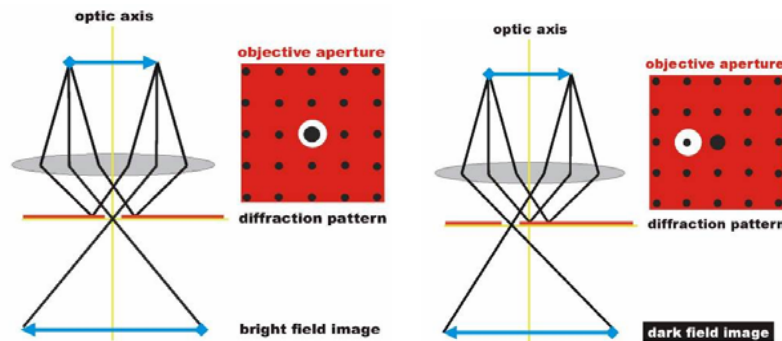
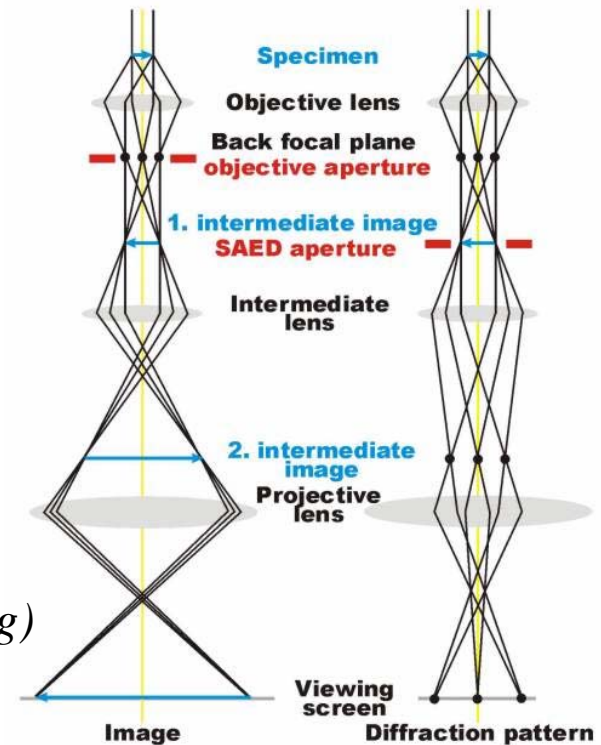
- *Requires thin specimen(<200nm)*

- *Increases with the electron energy*

- *High magnification: ~ 10⁶*

Resolution ~ $\lambda^{3/4}$ where $\lambda = \frac{h}{\sqrt{2mqV}}$

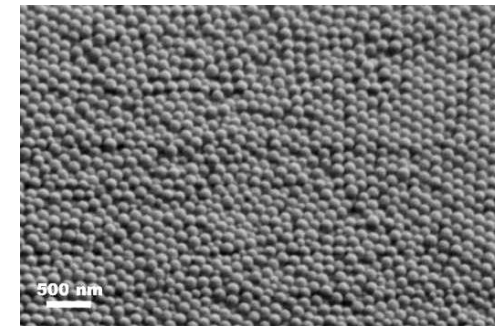
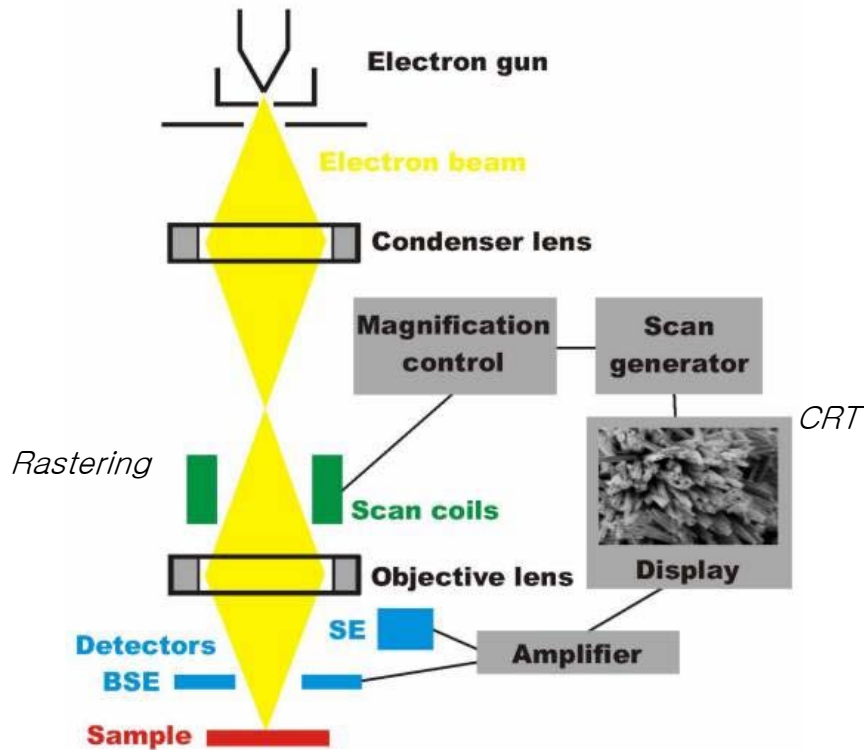
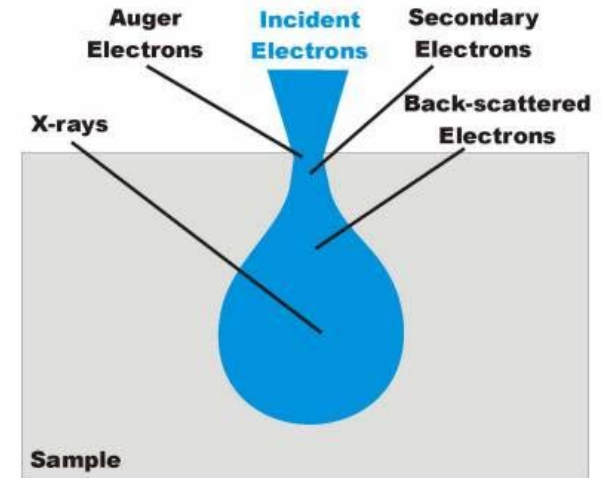
- *Gives images and diffraction patterns(elastic scattering)*



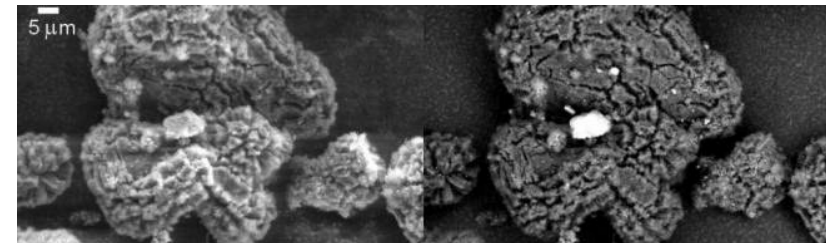
BF, DF and conical DF image of Pt particles inside of SiO₂-NTs

* *Scanning electron microscope*

- *Imaging from secondary electron (<50eV) imaging (SE)*
back-scattered electron imaging (BSE)
- *Resolution: 2-3nm; magnification: 10~>30,000*
- *Spot size : 5nm; energy: 2~200eV to 50KeV*
- *Information on chemical composition as well as topography*



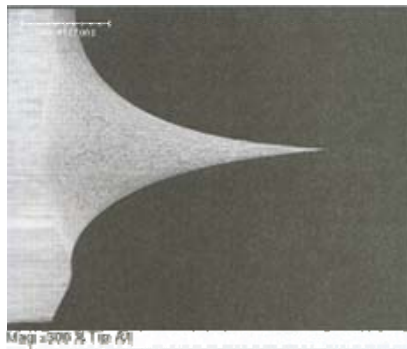
Latex balls, image from secondary electrons



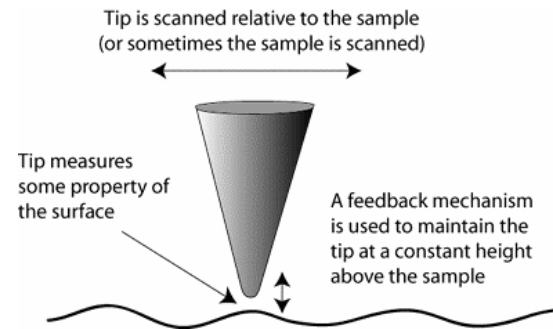
Fe particles in carbon, left: from secondary electrons; Right: from back scattered

* *Scanning probe microscopy*

- *Used to obtain surface property of the nanostructured materials e.g. films.*
- *3-D real space imaging and localized measurement of structure and properties*
- *Obtain topographical data of surface by measuring interaction between tip and surface*



*STM tip
made of
tungsten*



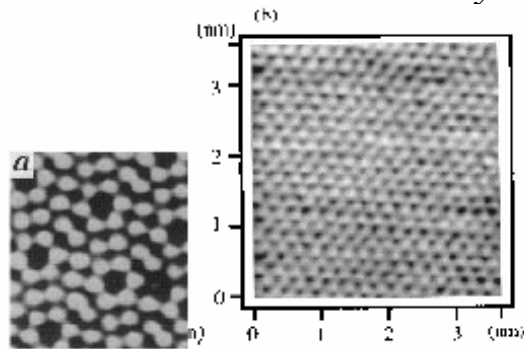
- *From atoms to $>250\mu\text{m} \times 250\mu\text{m}$;*

- *Vertical ranges of $\sim 15\mu\text{m}$*

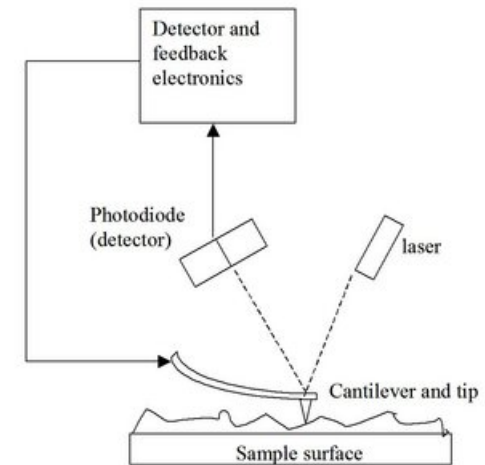
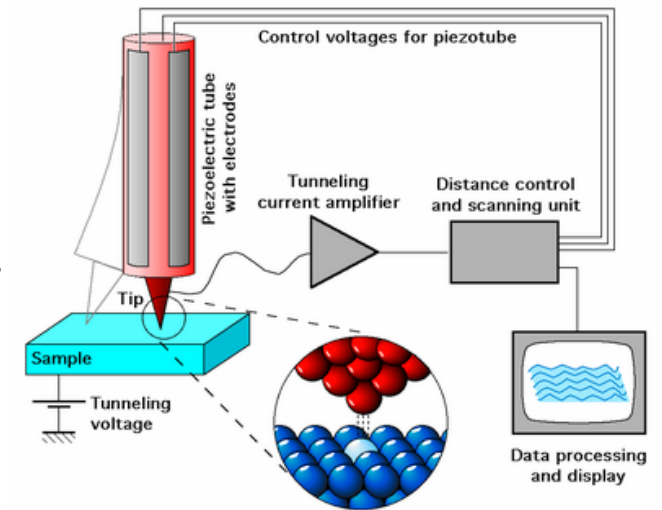
http://www.iap.tuwien.ac.at/www/surface/STM_Gallery/stm_animated.gif

Scanning tunneling microscope

- Constant current mode
- Constant height mode
- STM: restricted to electrically conductive sample surface



a.) STM of silicon [Si (111) 7x7 reconstruction; 2x2 nm]; b.) AFM of graphite



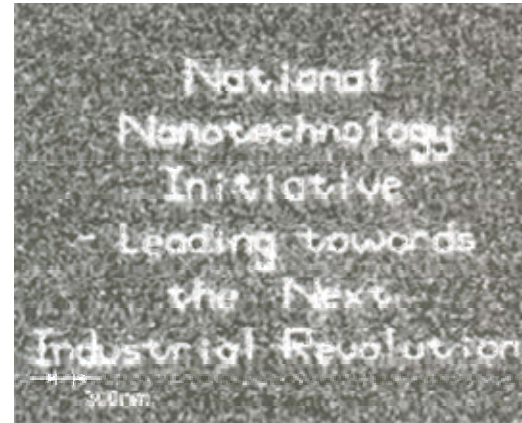
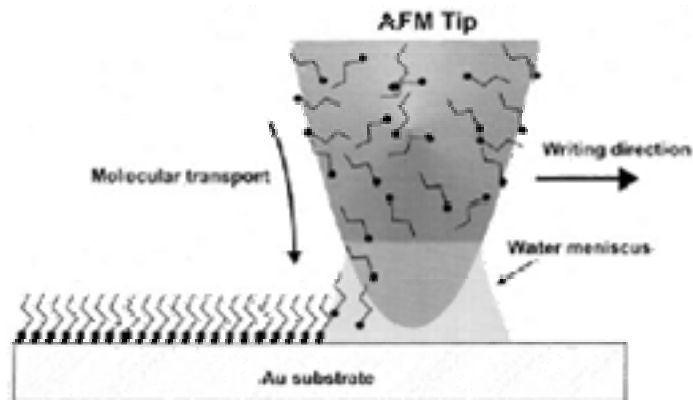
Atomic force microscope

- Constant force mode
- Electrostatic interaction, current-induced, static magnetic interaction, capillary forces

-SPM using other probe and sample-surface interactions

- *Magnetic force microscope*
- *Electrostatic force spectroscopy*
- *Scanning voltage microscope*
- *Kelvin probe microscope*
- *Scanning thermal microscope*
- *Near-field scanning optical microscopy*
- *Scanning capacitance microscope*
- *Force modulation microscope*

- Also used for nanomanipulation, nanolithography and nanodevice (sensor)



Use of “nano-pen”

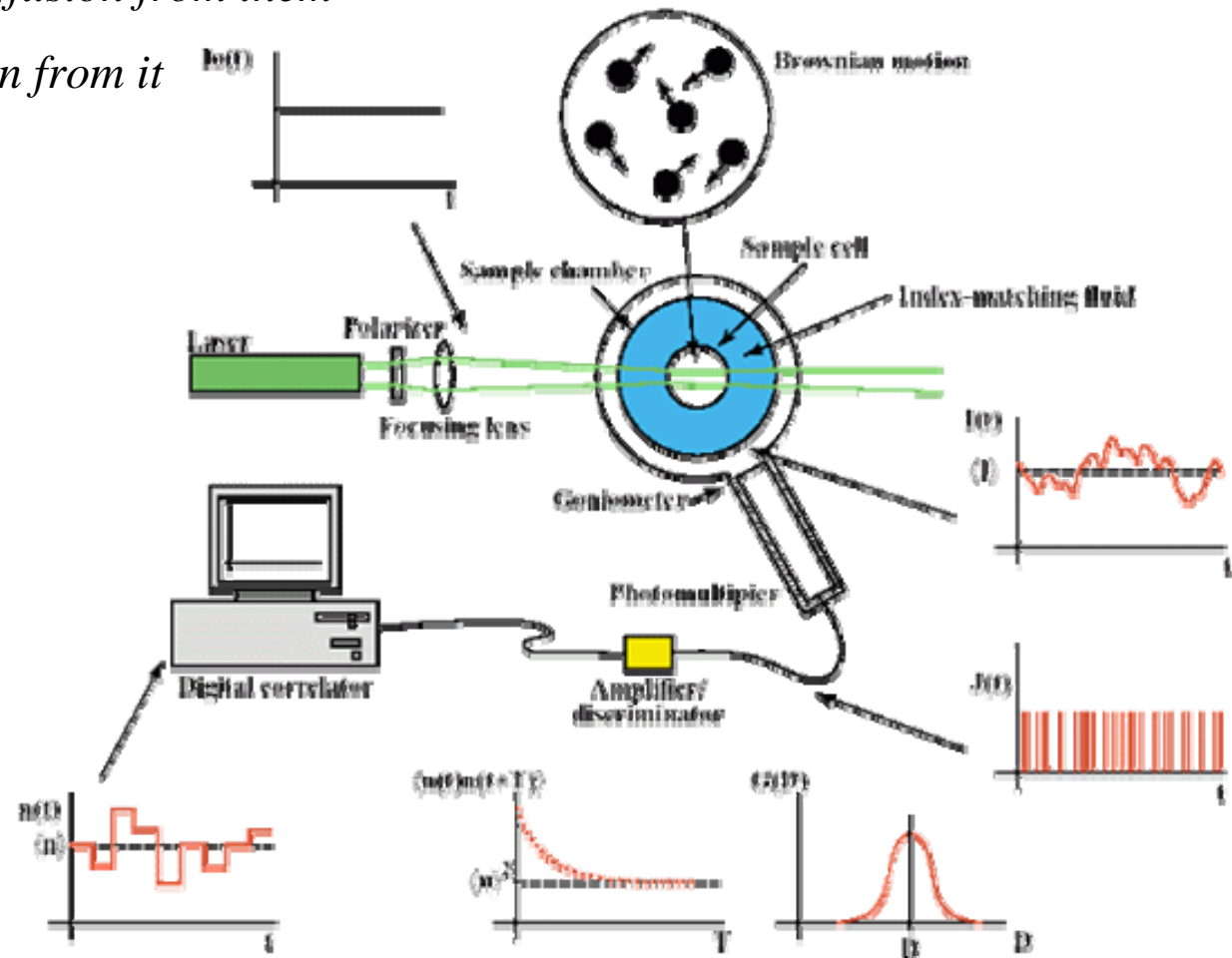
(2) Light scattering: laser particle sizer

* *Dynamic light scattering*

<http://www.bic.com/90oper.htm>

http://www.ap-lab.com/light_scattering.htm#Dynamic_Light_Scattering

- *Data on scattered light intensity vs. time*
- *Quantifying Brownian diffusion from them*
- *Deducing size distribution from it*



(3) *Electrical migration*

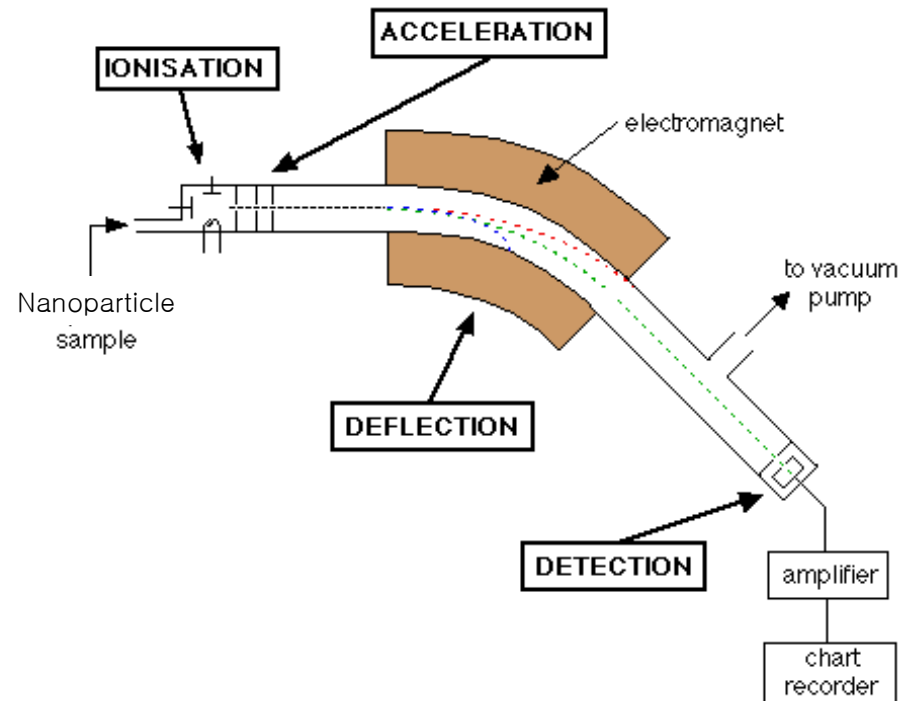
* *Mass spectrometry*

- *Movement of charged particles in magnetic field*

$$\frac{m}{q} = \frac{B^2 r^2}{2V}$$

where *m*: mass of particle; *q*: charge; *B*: field strength; *r*: bending radius of the duct; *V*: applied voltage

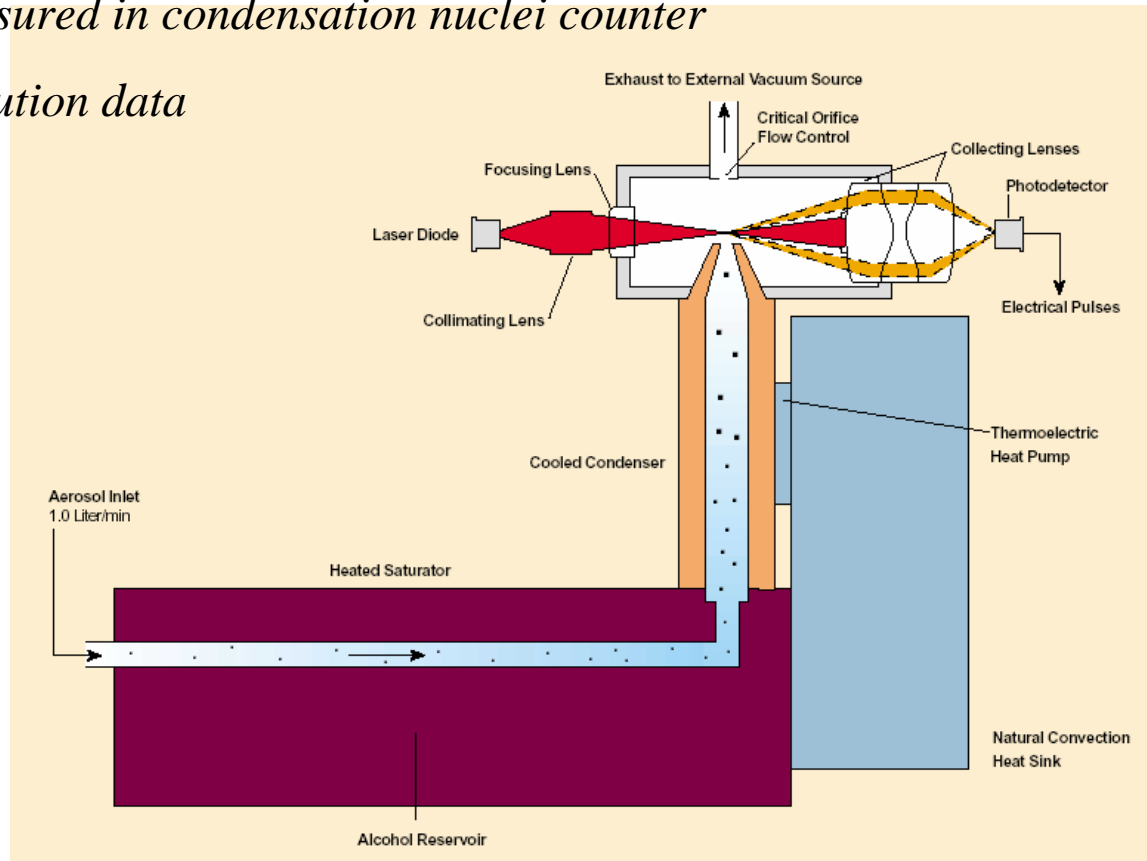
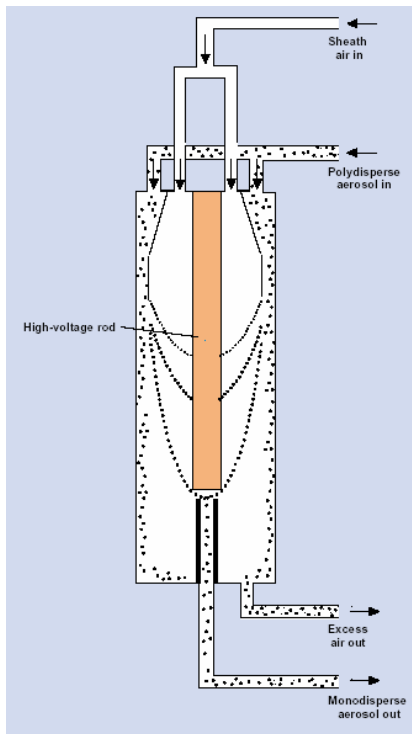
- *Size data from m*



* *Differential mobility analyzer*

$$Z = \frac{U_e}{E} = \frac{eC}{3\pi\mu d_p} \quad \text{for singly charged particles}$$

- *Small particles deposit on the central rod*
- *Only particles in very narrow size range can pass through the exit tube*
- *Number concentration measured in condensation nuclei counter*
- *Change E gives size distribution data*



Condensation nuclei (particle) counter