

Polymath를 이용한 Diffusion Equation 풀기



# Diffusion Equation

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$$\frac{du}{dt} = \alpha \frac{d^2u}{dx^2}$$

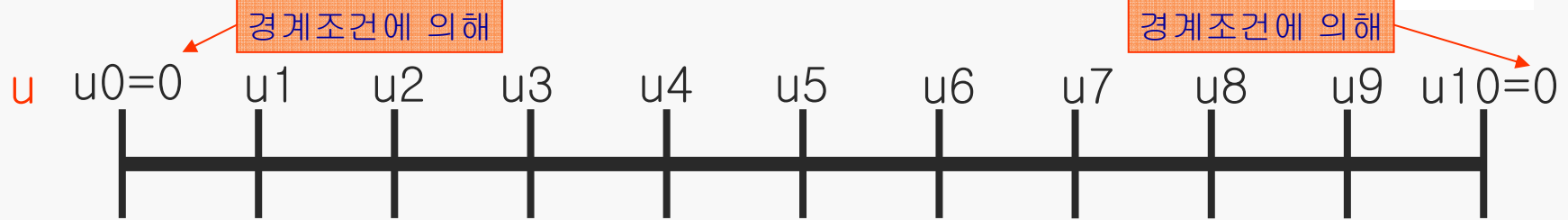
- ◇ 초기조건  $u(x,0) = \sin x \quad (0 \leq x \leq \pi)$
- ◇ 경계조건  $u(0,t) = u(\pi,t) = 0 \quad (0 \leq t \leq 1)$
- ◇ 정확해  $u(x,t) = \sin x e^{-0.5t}$

$$\Delta x = 0.1\pi \quad \alpha = 0.5 \quad \Delta t = 0.02$$

# modeling

U값을 구하기 위한 모델링을 해준다

$$\Delta x = 0.1\pi$$



x  $x_0=0$   $x_1=0.1\pi$   $x_2=0.2\pi$   $x_3=0.3\pi$   $x_4=0.4\pi$   $x_5=0.5\pi$   $x_6=0.6\pi$   $x_7=0.7\pi$   $x_8=0.8\pi$   $x_9=0.9\pi$   $x_{10}=\pi$

초기값

$\sin 0.1\pi$   $\sin 0.2\pi$   $\sin 0.3\pi$   $\sin 0.4\pi$   $\sin 0.5\pi$   $\sin 0.6\pi$   $\sin 0.7\pi$   $\sin 0.8\pi$   $\sin 0.9\pi$

각각의 u에 대한  
식을 차분화 시켜줌

$$u_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta x)^2}$$

Diffusion equation을 차분화 해주고, 경계값과 초기값을 대입해준 후, 시간t에 대한 변화를 알아본다.

# 데이터 입력

**POLYMATH 6.0 Professional Release - [Ordinary Differential Equations Solver: Enter Differential Equation]**

File Program Edit Format Problem Examples Window Help

del \*x= [RKF45]  Table  Graph  Report

Differential Equations: 9 Auxiliary Equations: 22  Ready for solution

**차분식 초기값**

```

d(u1)/d(t) = a * (u2 - 2 * u1 + u0) / deltax ^ 2
u1(0) = 0.309016994
d(u2)/d(t) = a * (u3 - 2 * u2 + u1) / deltax ^ 2
u2(0) = 0.587785252
d(u3)/d(t) = a * (u4 - 2 * u3 + u2) / deltax ^ 2
u3(0) = 0.809016994
d(u4)/d(t) = a * (u5 - 2 * u4 + u3) / deltax ^ 2
u4(0) = 0.951056516
d(u5)/d(t) = a * (u6 - 2 * u5 + u4) / deltax ^ 2
u5(0) = 1
d(u6)/d(t) = a * (u7 - 2 * u6 + u5) / deltax ^ 2
u6(0) = 0.951056516
d(u7)/d(t) = a * (u8 - 2 * u7 + u6) / deltax ^ 2
u7(0) = 0.809016994
d(u8)/d(t) = a * (u9 - 2 * u8 + u7) / deltax ^ 2
u8(0) = 0.587785252
d(u9)/d(t) = a * (u10 - 2 * u9 + u8) / deltax ^ 2
u9(0) = 0.309016994

```

**상수값**

```

a = 0.5
deltax = 0.1 * 3.141592654
u10 = 0
u0 = 0

```

**시간t의 초기값**

```

t(0) = 0
t(f) = 1

```

**시간t의 최종값**

```

z1 = sin(x1) * exp(-a * t)
x1 = 0.1 * 3.141592654
z2 = sin(x2) * exp(-a * t)
x2 = 0.2 * 3.141592654
z3 = sin(x3) * exp(-a * t)
x3 = 0.3 * 3.141592654
z4 = sin(x4) * exp(-a * t)
x4 = 0.4 * 3.141592654
z5 = sin(x5) * exp(-a * t)
x5 = 0.5 * 3.141592654
z6 = sin(x6) * exp(-a * t)
x6 = 0.6 * 3.141592654
z7 = sin(x7) * exp(-a * t)
x7 = 0.7 * 3.141592654
z8 = sin(x8) * exp(-a * t)
x8 = 0.8 * 3.141592654
z9 = sin(x9) * exp(-a * t)
x9 = 0.9 * 3.141592654

```

**정확해 변수분리 식**

**변수 X값 고정**

**Tip**  
 도구상자를 이용하여 하나의 식만 입력한 후 복사-붙여넣기를 하고 값들만 변경해 주면 손쉽게 데이터를 입력할 수 있음

**Differential Equations Solver: Enter Differential Equation**

Enter the differential equation:  
 $d(u1) = a * (u2 - 2 * u1 + u0) / deltax ^ 2$

Set the initial value:  
 $u1(0) = 0.309016994$

**Differential Equations Solver: Enter Explicit Equation**

Enter the explicit equation:  
 $a = 0.5$

**Polymath Guide**  
 Step #1 of 2  
 Enter <b> initial value.

**Polymath Guide**  
 Step #2 of 2  
 Enter <b> final value.

**Differential Equations Solver: Enter Explicit Equation**

Enter the explicit equation:  
 $z1 = sin(x1) * exp(-a * t)$

**Differential Equations Solver: Enter Explicit Equation**

Enter the explicit equation:  
 $x1 = 0.1 * 3.141592654$

**Annotations:**

- 각각의 차분화된 식을 입력 초기값 계산 후 입력 (다음페이지 엑셀을 통한 편리한 계산)
- 상수값이나 정해진 값들을 직접 입력
- 변수 t의 초기값과 최종값 직접 입력
- 정확해를 변수분리(x고정) 하여 입력 (정확해를 따로 구분해 주기 위해 z로 놓았음)
- 고정된 변수 x값 입력

# 초기값 엑셀을 이용한 계산

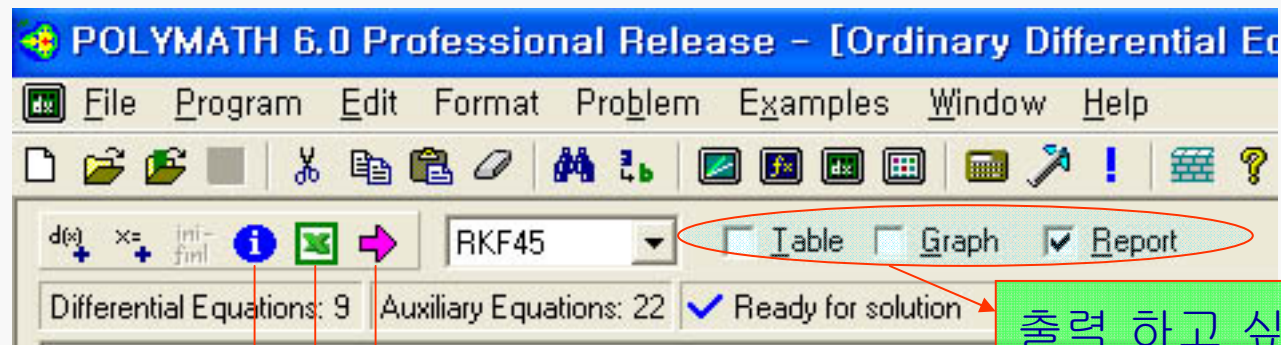
Polymath에선 초기값을 방정식으로 넣지 못함  
 데이터를 입력할 때 초기값을 일일이 계산해서 넣어줘야 함  
 ->엑셀을 통해 초기값을 쉽게 구할 수 있음

급해줌

deltax=0.1*pi일 때					
	x값	x값	초기값		
	pi	3.141592654	sinx		
0.1	0.1*pi	0.314159265	sin	0.309016994	u1일때의 초기값
0.2	0.2*pi	0.628318531		0.587785252	u2일때의 초기값
0.3	0.3*pi	0.942477796		0.809016994	u3일때의 초기값
0.4	0.4*pi	1.256637061		0.951056516	u4일때의 초기값
0.5	0.5*pi	1.570796327		1	u5일때의 초기값
0.6	0.6*pi	1.884955592		0.951056516	u6일때의 초기값
0.7	0.7*pi	2.199114858		0.809016994	u7일때의 초기값
0.8	0.8*pi	2.513274123		0.587785252	u8일때의 초기값
0.9	0.9*pi	2.827433388		0.309016994	u9일때의 초기값

계산된 초기값을 데이터에 입력

# 데이터 입력(몇가지 Tip)



출력 하고 싶은 항목 클릭

모든 데이터를 입력한 후 이 버튼을 누르면 결과 출력됨  
(입력데이터 값이 모두 타당할 경우에만 활성화 됨)

엑셀로 데이터 값들을 전환시킬 수 있음

데이터 입력상태를 체크 할 수 있는 기능



# 결과 출력 (Report)

## POLYMATH Report Ordinary Differential Equations

### Calculated values of DEQ variables

	Variable	Initial value	Minimal value	Maximal value	Final value
1	t	0	0	1.	1.
2	u1	0.309017	0.1881981	0.309017	0.1881981
3	u2	0.5877853	0.357974	0.5877853	0.357974
4	u3	0.809017	0.492709	0.809017	0.492709
5	u4	0.9510565	0.5792142	0.9510565	0.5792142
6	u5	1.	0.6090218	1.	0.6090218
7	u6	0.9510565	0.5792142	0.9510565	0.5792142
8	u7	0.809017	0.492709	0.809017	0.492709
9	u8	0.5877853	0.357974	0.5877853	0.357974
10	u9	0.309017	0.1881981	0.309017	0.1881981
11	a	0.5	0.5	0.5	0.5
12	deltax	0.3141593	0.3141593	0.3141593	0.3141593
13	u10	0	0	0	0
14	u0	0	0	0	0
15	x1	0.3141593	0.3141593	0.3141593	0.3141593
16	z1	0.309017	0.1874283	0.309017	0.1874283
17	x2	0.6283185	0.6283185	0.6283185	0.6283185
18	z2	0.5877853	0.3565098	0.5877853	0.3565098
19	x3	0.9424778	0.9424778	0.9424778	0.9424778
20	z3	0.809017	0.4906936	0.809017	0.4906936
21	x4	1.256637	1.256637	1.256637	1.256637
22	z4	0.9510565	0.5768449	0.9510565	0.5768449
23	x5	1.570796	1.570796	1.570796	1.570796
24	z5	1.	0.6065307	1.	0.6065307
25	x6	1.884956	1.884956	1.884956	1.884956
26	z6	0.9510565	0.5768449	0.9510565	0.5768449
27	x7	2.199115	2.199115	2.199115	2.199115
28	z7	0.809017	0.4906936	0.809017	0.4906936
29	x8	2.513274	2.513274	2.513274	2.513274
30	z8	0.5877853	0.3565098	0.5877853	0.3565098
31	x9	2.827433	2.827433	2.827433	2.827433
32	z9	0.309017	0.1874283	0.309017	0.1874283

### Differential equations

- $d(u_1)/d(t) = a * (u_2 - 2 * u_1 + u_0) / \text{deltax} ^ 2$
- $d(u_2)/d(t) = a * (u_3 - 2 * u_2 + u_1) / \text{deltax} ^ 2$
- $d(u_3)/d(t) = a * (u_4 - 2 * u_3 + u_2) / \text{deltax} ^ 2$
- $d(u_4)/d(t) = a * (u_5 - 2 * u_4 + u_3) / \text{deltax} ^ 2$
- $d(u_5)/d(t) = a * (u_6 - 2 * u_5 + u_4) / \text{deltax} ^ 2$
- $d(u_6)/d(t) = a * (u_7 - 2 * u_6 + u_5) / \text{deltax} ^ 2$
- $d(u_7)/d(t) = a * (u_8 - 2 * u_7 + u_6) / \text{deltax} ^ 2$
- $d(u_8)/d(t) = a * (u_9 - 2 * u_8 + u_7) / \text{deltax} ^ 2$
- $d(u_9)/d(t) = a * (u_{10} - 2 * u_9 + u_8) / \text{deltax} ^ 2$

### Explicit equations

- $a = 0.5$
- $\text{deltax} = 0.1 * 3.141592654$
- $u_{10} = 0$
- $u_0 = 0$
- $x_1 = 0.1 * 3.141592654$
- $z_1 = \sin(x_1) * \exp(-a * t)$
- $x_2 = 0.2 * 3.141592654$
- $z_2 = \sin(x_2) * \exp(-a * t)$
- $x_3 = 0.3 * 3.141592654$
- $z_3 = \sin(x_3) * \exp(-a * t)$
- $x_4 = 0.4 * 3.141592654$
- $z_4 = \sin(x_4) * \exp(-a * t)$
- $x_5 = 0.5 * 3.141592654$
- $z_5 = \sin(x_5) * \exp(-a * t)$
- $x_6 = 0.6 * 3.141592654$
- $z_6 = \sin(x_6) * \exp(-a * t)$
- $x_7 = 0.7 * 3.141592654$
- $z_7 = \sin(x_7) * \exp(-a * t)$
- $x_8 = 0.8 * 3.141592654$
- $z_8 = \sin(x_8) * \exp(-a * t)$
- $x_9 = 0.9 * 3.141592654$
- $z_9 = \sin(x_9) * \exp(-a * t)$

u1과 u9, u2와 u8, u3와 u7, u4와 u6의 값이 같음  
u5를 중심으로 대칭된 값을 갖음(양쪽 경계값이 같고, 초기값이 sin함수를 따르기 때문)

Numerical solution과 Analytical solution이 거의 비슷하게 나옴

z1과 z9, z2와 z8, z3와 z7, z4와 z6의 값이 같음  
z5를 중심으로 대칭된 값을 갖음

# 결과 출력(Table)

POLYMATH 6.0 Professional Release - [ODE Results: RKF45, Solution #2]

File Program Edit Row Column Format Analysis Examples Window Help

R001 : C001 C01

	t	u1	u2	u3	u4	u5	u6	u7	u8	u9	a	deltax	u10
1	0	0.309017	0.5877853	0.809017	0.9510565	1.	0.9510565	0.809017	0.5877853	0.309017	0.5	0.3141593	
2	0.0243473	0.3053084	0.5807311	0.7993078	0.9396427	0.9879988	0.9396427	0.7993078	0.5807311	0.3053084	0.5	0.3141593	
3	0.0323473	0.3040996	0.5784318	0.796143	0.9359223	0.9840869	0.9359223	0.796143	0.5784318	0.3040996	0.5	0.3141593	
4	0.0403473	0.3028955	0.5761416	0.7929908	0.9322166	0.9801906	0.9322166	0.7929908	0.5761416	0.3028955	0.5	0.3141593	
5	0.0563473	0.3005018	0.5715883	0.7867238	0.9248493	0.9724441	0.9248493	0.7867238	0.5715883	0.3005018	0.5	0.3141593	
6	0.0643473	0.299312	0.5693252	0.7836089	0.9211875	0.9685939	0.9211875	0.7836089	0.5693252	0.299312	0.5	0.3141593	
7	0.0723473	0.2981269	0.567071	0.7805063	0.9175402	0.9647589	0.9175402	0.7805063	0.567071	0.2981269	0.5	0.3141593	
8	0.0803473	0.2969465	0.5648258	0.777416	0.9139073	0.960939	0.9139073	0.777416	0.5648258	0.2969465	0.5	0.3141593	
9	0.0963473	0.2945997	0.560362	0.7712721	0.9066847	0.9533447	0.9066847	0.7712721	0.560362	0.2945997	0.5	0.3141593	
10	0.1043473	0.2934333	0.5581433	0.7682183	0.9030948	0.9495701	0.9030948	0.7682183	0.5581433	0.2934333	0.5	0.3141593	
11	0.1123473	0.2922715	0.5559334	0.7651767	0.8995191	0.9458104	0.8995191	0.7651767	0.5559334	0.2922715	0.5	0.3141593	
12	0.1203473	0.2911143	0.5537323	0.7621471	0.8959576	0.9420566	0.8959576	0.7621471	0.5537323	0.2911143	0.5	0.3141593	
13	0.1363473	0.2888136	0.5493561	0.7561238	0.8888769	0.9346204	0.8888769	0.7561238	0.5493561	0.2888136	0.5	0.3141593	
14	0.1443473	0.2876701	0.547181	0.7531301	0.8853575	0.93092	0.8853575	0.7531301	0.547181	0.2876701	0.5	0.3141593	
15	0.1523473	0.2865311	0.5450145	0.7501482	0.881852	0.9272341	0.881852	0.7501482	0.5450145	0.2865311	0.5	0.3141593	
16	0.1603473	0.2853966	0.5428566	0.7471781	0.8783605	0.9235629	0.8783605	0.7471781	0.5428566	0.2853966	0.5	0.3141593	
17	0.1763473	0.2831411	0.5385664	0.7412731	0.8714188	0.9162639	0.8714188	0.7412731	0.5385664	0.2831411	0.5	0.3141593	
18	0.1843473	0.2820201	0.5364341	0.7383381	0.8679685	0.9126361	0.8679685	0.7383381	0.5364341	0.2820201	0.5	0.3141593	
19	0.1923473	0.2809035	0.5343101	0.7354148	0.8645319	0.9090227	0.8645319	0.7354148	0.5343101	0.2809035	0.5	0.3141593	
20	0.2003473	0.2797913	0.5321946	0.732503	0.861109	0.9054235	0.861109	0.732503	0.5321946	0.2797913	0.5	0.3141593	
21	0.2163473	0.2775801	0.5279887	0.726714	0.8543036	0.8982679	0.8543036	0.726714	0.5279887	0.2775801	0.5	0.3141593	
22	0.2243473	0.276481	0.5258982	0.7238367	0.8509211	0.8947114	0.8509211	0.7238367	0.5258982	0.276481	0.5	0.3141593	
23	0.2323473	0.2753863	0.5238159	0.7209708	0.847552	0.8911689	0.847552	0.7209708	0.5238159	0.2753863	0.5	0.3141593	
24	0.2403473	0.274296	0.521742	0.7181162	0.8441962	0.8876405	0.8441962	0.7181162	0.521742	0.274296	0.5	0.3141593	
25	0.2563473	0.2721282	0.5176186	0.7124409	0.8375245	0.8806254	0.8375245	0.7124409	0.5176186	0.2721282	0.5	0.3141593	
26	0.2643473	0.2710508	0.5155692	0.7096201	0.8342085	0.8771387	0.8342085	0.7096201	0.5155692	0.2710508	0.5	0.3141593	
27	0.2723473	0.2699776	0.5135279	0.7068105	0.8309056	0.8736658	0.8309056	0.7068105	0.5135279	0.2699776	0.5	0.3141593	
28	0.2803473	0.2689086	0.5114946	0.704012	0.8276157	0.8702067	0.8276157	0.704012	0.5114946	0.2689086	0.5	0.3141593	
29	0.2963473	0.2667835	0.5074523	0.6984482	0.8210751	0.8633294	0.8210751	0.6984482	0.5074523	0.2667835	0.5	0.3141593	
30	0.3043473	0.2657272	0.5054431	0.6956828	0.8178241	0.8599112	0.8178241	0.6956828	0.5054431	0.2657272	0.5	0.3141593	
31	0.3123473	0.2646751	0.5034419	0.6929283	0.8145861	0.8565065	0.8145861	0.6929283	0.5034419	0.2646751	0.5	0.3141593	
32	0.3203473	0.2636271	0.5014486	0.6901848	0.8113608	0.8531153	0.8113608	0.6901848	0.5014486	0.2636271	0.5	0.3141593	
33	0.3363473	0.2615437	0.4974856	0.6847302	0.8049486	0.8463731	0.8049486	0.6847302	0.4974856	0.2615437	0.5	0.3141593	
34	0.3443473	0.2605081	0.4955159	0.6820191	0.8017616	0.843022	0.8017616	0.6820191	0.4955159	0.2605081	0.5	0.3141593	
35	0.3523473	0.2594767	0.493554	0.6793188	0.7985871	0.8396842	0.7985871	0.6793188	0.493554	0.2594767	0.5	0.3141593	
36	0.3603473	0.2584493	0.4915998	0.6766291	0.7954252	0.8363596	0.7954252	0.6766291	0.4915998	0.2584493	0.5	0.3141593	
37	0.3763473	0.2564068	0.4877147	0.6712817	0.789139	0.8297498	0.789139	0.6712817	0.4877147	0.2564068	0.5	0.3141593	
38	0.3843473	0.2553916	0.4857837	0.6686239	0.7860145	0.8264645	0.7860145	0.6686239	0.4857837	0.2553916	0.5	0.3141593	
39	0.3923473	0.2543804	0.4838603	0.6659765	0.7829024	0.8231923	0.7829024	0.6659765	0.4838603	0.2543804	0.5	0.3141593	
40	0.4003473	0.2533732	0.4819445	0.6633397	0.7798026	0.819933	0.7798026	0.6633397	0.4819445	0.2533732	0.5	0.3141593	
41	0.4163473	0.2513708	0.4781357	0.6580973	0.7736398	0.813453	0.7736398	0.6580973	0.4781357	0.2513708	0.5	0.3141593	
42	0.4243473	0.2503755	0.4762426	0.6554917	0.7705767	0.8102323	0.7705767	0.6554917	0.4762426	0.2503755	0.5	0.3141593	

Regression | Analysis | Graph

Report  Graph  Store Model

Residuals

Linear & Polynomial | Multiple linear | Nonlinear

Dependent Variable: u1

Independent Variable: t

Polynomial Degree: 1 Linear, 2, 3

Through origin

Polynomial Integration

Polynomial Derivative

No File No Title

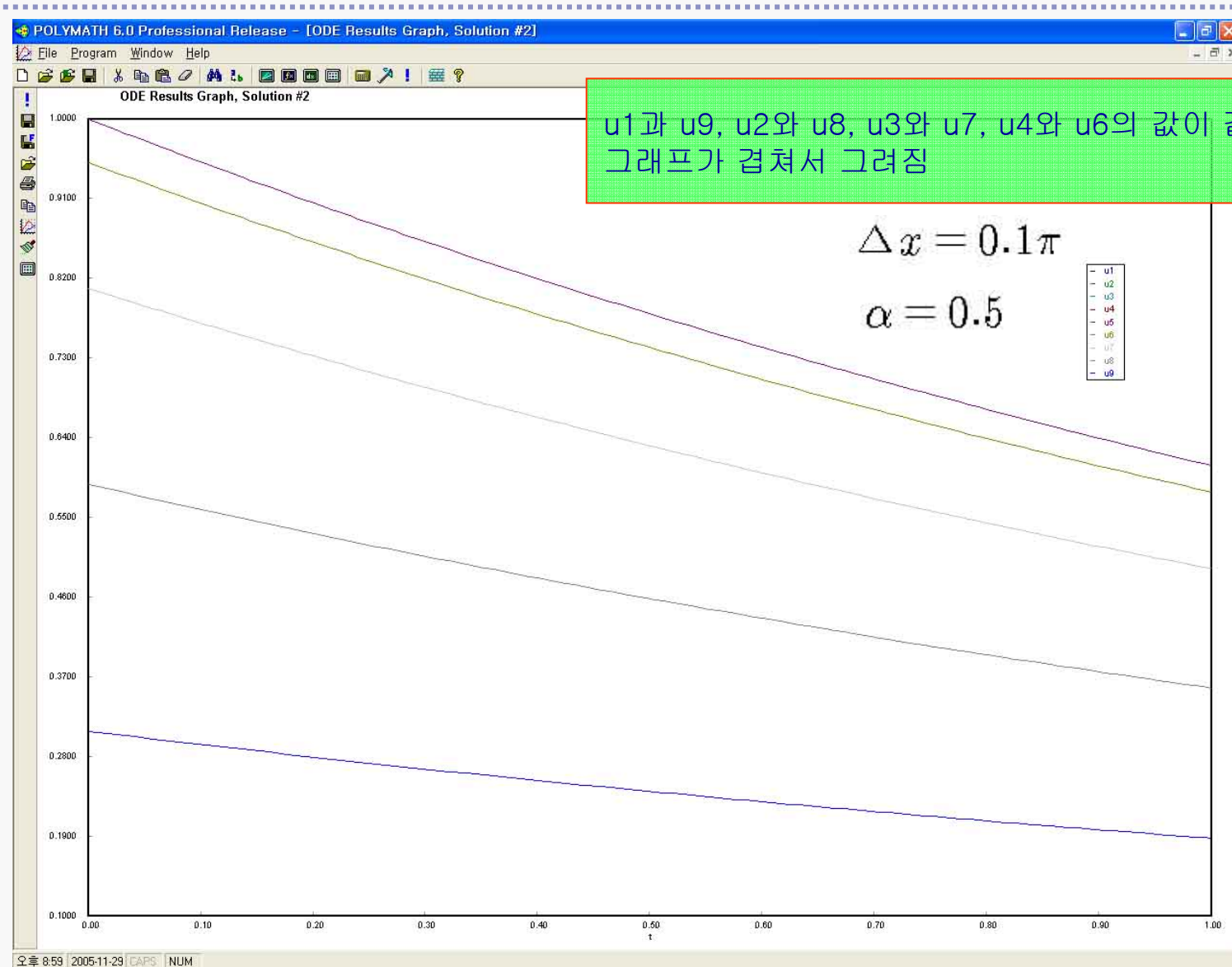
오류 9:02 2005-11-29 CAPS NUM

시작 POLYMATH 6.0 Pr... Adobe Photoshop

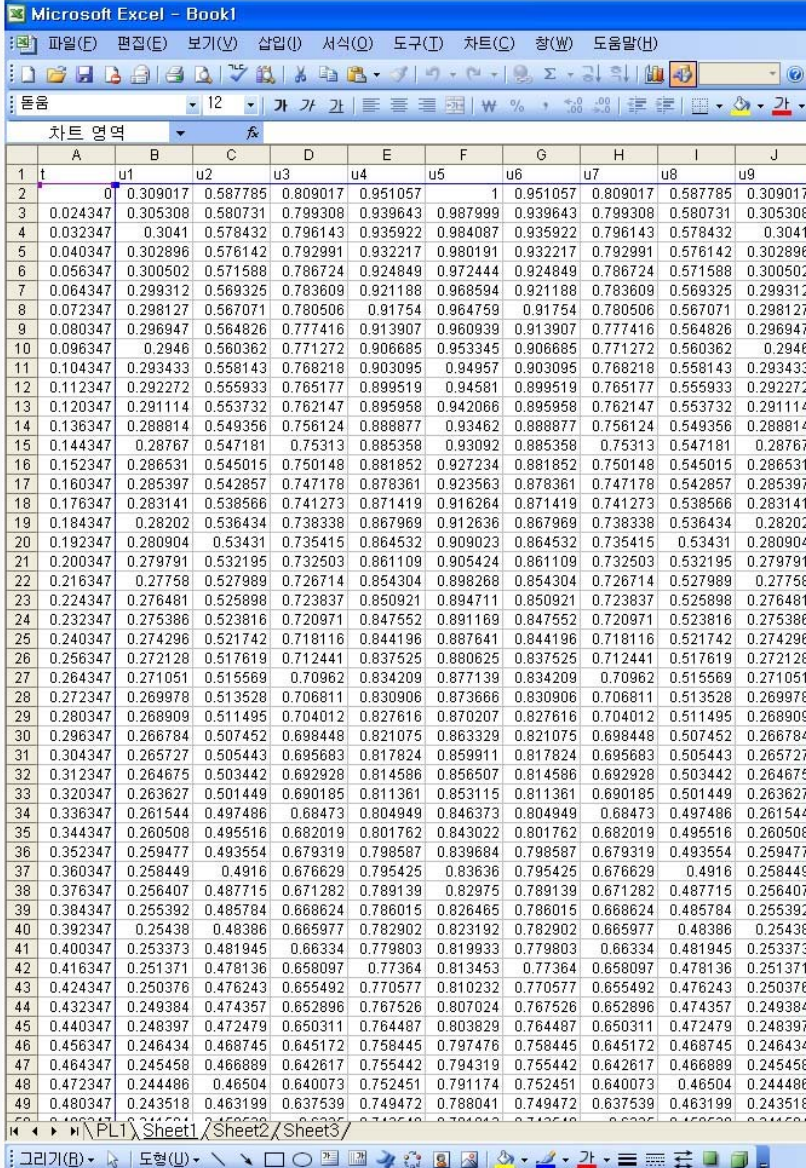
오류 9:02



# 결과 출력 (Numerical solution Graph)



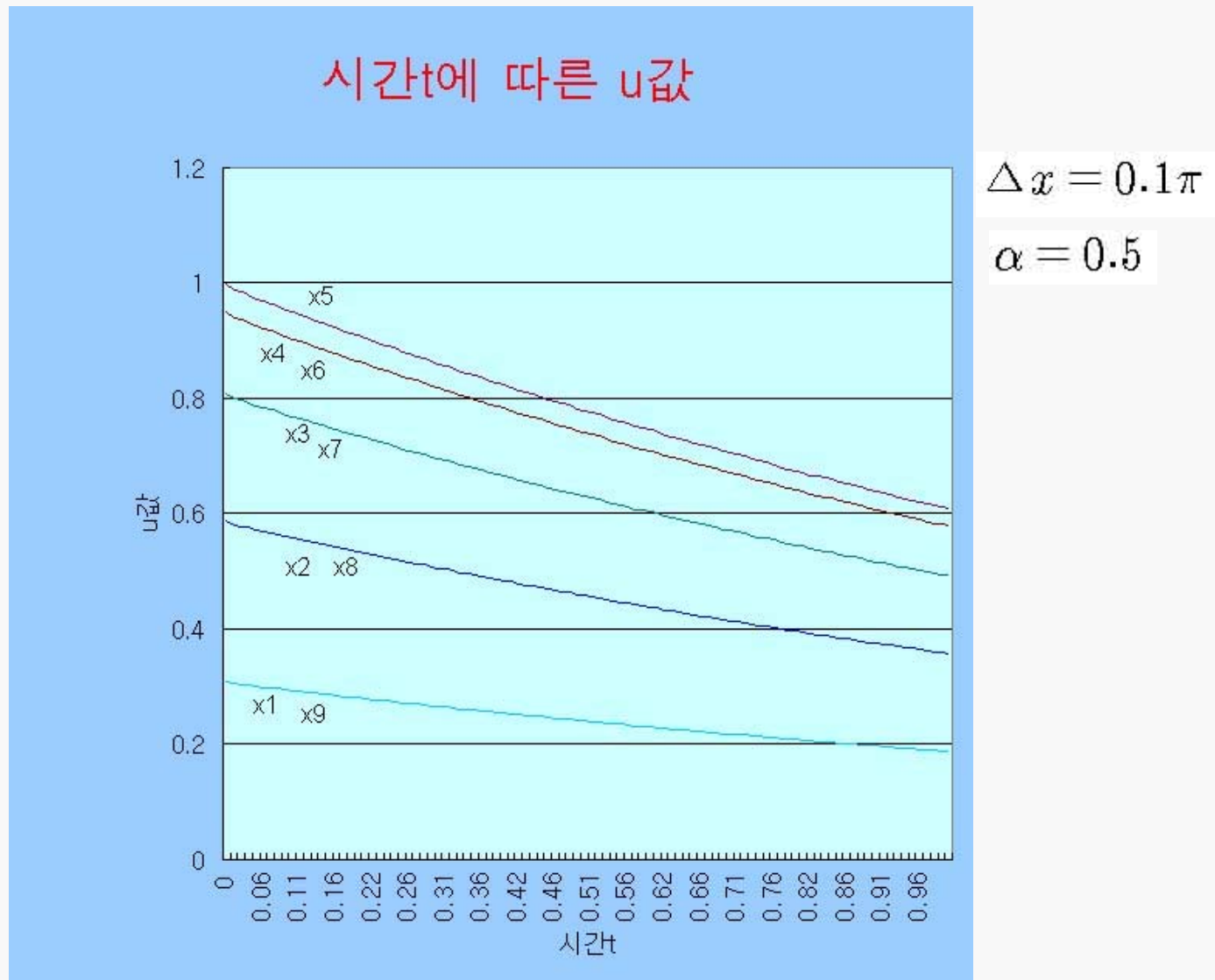
# Excel로의 결과 출력(Numerical solution table)



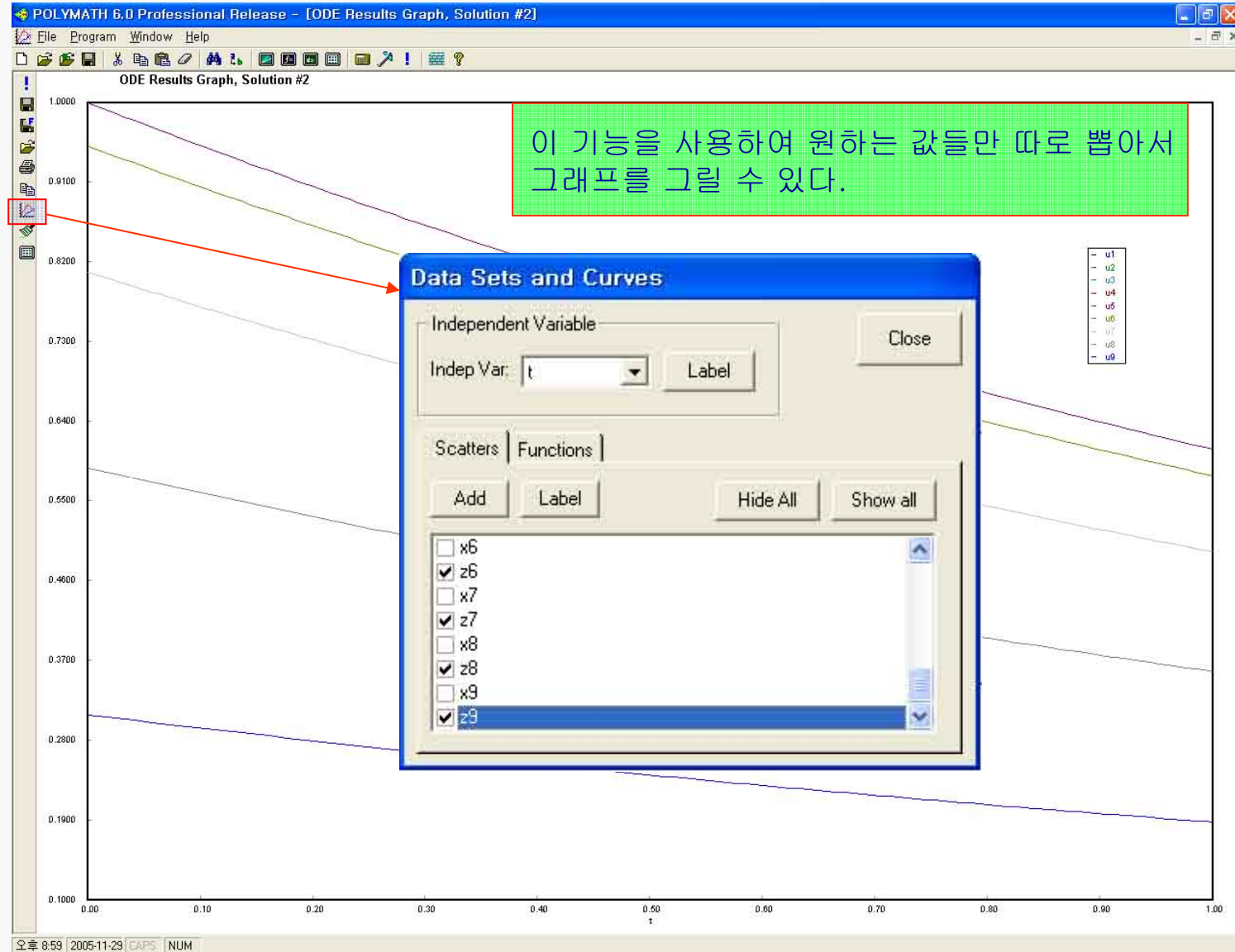
The screenshot displays a Microsoft Excel spreadsheet titled "Microsoft Excel - Book1". The spreadsheet contains a numerical solution table with the following structure:

t	u1	u2	u3	u4	u5	u6	u7	u8	u9
0	0.309017	0.587785	0.809017	0.951057	1	0.951057	0.809017	0.587785	0.309017
0.024347	0.305308	0.580731	0.799308	0.939643	0.987999	0.939643	0.799308	0.580731	0.305308
0.032347	0.3041	0.578432	0.796143	0.935922	0.984087	0.935922	0.796143	0.578432	0.3041
0.040347	0.302896	0.576142	0.792991	0.932217	0.980191	0.932217	0.792991	0.576142	0.302896
0.056347	0.300502	0.571588	0.786724	0.924849	0.972444	0.924849	0.786724	0.571588	0.300502
0.064347	0.299312	0.569325	0.783609	0.921188	0.968594	0.921188	0.783609	0.569325	0.299312
0.072347	0.298127	0.567071	0.780506	0.91754	0.964759	0.91754	0.780506	0.567071	0.298127
0.080347	0.296947	0.564826	0.777416	0.913907	0.960939	0.913907	0.777416	0.564826	0.296947
0.096347	0.2946	0.560362	0.771272	0.906685	0.953345	0.906685	0.771272	0.560362	0.2946
0.104347	0.293433	0.558143	0.768218	0.903095	0.94957	0.903095	0.768218	0.558143	0.293433
0.112347	0.292272	0.555933	0.765177	0.899519	0.94581	0.899519	0.765177	0.555933	0.292272
0.120347	0.291114	0.553732	0.762147	0.895958	0.942066	0.895958	0.762147	0.553732	0.291114
0.136347	0.288814	0.549356	0.756124	0.888877	0.93462	0.888877	0.756124	0.549356	0.288814
0.144347	0.28767	0.547181	0.75313	0.885358	0.93092	0.885358	0.75313	0.547181	0.28767
0.152347	0.286531	0.545015	0.750148	0.881852	0.927234	0.881852	0.750148	0.545015	0.286531
0.160347	0.285397	0.542857	0.747178	0.878361	0.923563	0.878361	0.747178	0.542857	0.285397
0.176347	0.283141	0.538566	0.741273	0.871419	0.916264	0.871419	0.741273	0.538566	0.283141
0.184347	0.28202	0.536434	0.738338	0.867969	0.912636	0.867969	0.738338	0.536434	0.28202
0.192347	0.280904	0.53431	0.735415	0.864532	0.909023	0.864532	0.735415	0.53431	0.280904
0.200347	0.279791	0.532195	0.732503	0.861109	0.905424	0.861109	0.732503	0.532195	0.279791
0.216347	0.27758	0.527989	0.726714	0.854304	0.898268	0.854304	0.726714	0.527989	0.27758
0.224347	0.276481	0.525898	0.723837	0.850921	0.894711	0.850921	0.723837	0.525898	0.276481
0.232347	0.275386	0.523816	0.720971	0.847552	0.891169	0.847552	0.720971	0.523816	0.275386
0.240347	0.274296	0.521742	0.718116	0.844196	0.887641	0.844196	0.718116	0.521742	0.274296
0.256347	0.272128	0.517619	0.712441	0.837525	0.880625	0.837525	0.712441	0.517619	0.272128
0.264347	0.271051	0.515569	0.70962	0.834209	0.877139	0.834209	0.70962	0.515569	0.271051
0.272347	0.269978	0.513528	0.706811	0.830906	0.873666	0.830906	0.706811	0.513528	0.269978
0.280347	0.268909	0.511495	0.704012	0.827616	0.870207	0.827616	0.704012	0.511495	0.268909
0.296347	0.266784	0.507452	0.698448	0.821075	0.863329	0.821075	0.698448	0.507452	0.266784
0.304347	0.265727	0.505443	0.695683	0.817824	0.859911	0.817824	0.695683	0.505443	0.265727
0.312347	0.264675	0.503442	0.692928	0.814586	0.856507	0.814586	0.692928	0.503442	0.264675
0.320347	0.263627	0.501449	0.690185	0.811361	0.853115	0.811361	0.690185	0.501449	0.263627
0.336347	0.261544	0.497486	0.68473	0.804949	0.846373	0.804949	0.68473	0.497486	0.261544
0.344347	0.260508	0.495516	0.682019	0.801762	0.843022	0.801762	0.682019	0.495516	0.260508
0.352347	0.259477	0.493554	0.679319	0.798587	0.839684	0.798587	0.679319	0.493554	0.259477
0.360347	0.258449	0.4916	0.676629	0.795425	0.83636	0.795425	0.676629	0.4916	0.258449
0.376347	0.256407	0.487715	0.671282	0.789139	0.82975	0.789139	0.671282	0.487715	0.256407
0.384347	0.255392	0.485784	0.668624	0.786015	0.826465	0.786015	0.668624	0.485784	0.255392
0.392347	0.25438	0.48386	0.665977	0.782902	0.823192	0.782902	0.665977	0.48386	0.25438
0.400347	0.253373	0.481945	0.66334	0.779803	0.819933	0.779803	0.66334	0.481945	0.253373
0.416347	0.251371	0.478136	0.658097	0.77364	0.813453	0.77364	0.658097	0.478136	0.251371
0.424347	0.250376	0.476243	0.655492	0.770577	0.810232	0.770577	0.655492	0.476243	0.250376
0.432347	0.249384	0.474357	0.652896	0.767526	0.807024	0.767526	0.652896	0.474357	0.249384
0.440347	0.248397	0.472479	0.650311	0.764487	0.803829	0.764487	0.650311	0.472479	0.248397
0.456347	0.246434	0.468745	0.645172	0.758445	0.797476	0.758445	0.645172	0.468745	0.246434
0.464347	0.245458	0.466889	0.642617	0.755442	0.794319	0.755442	0.642617	0.466889	0.245458
0.472347	0.244486	0.46504	0.640073	0.752451	0.791174	0.752451	0.640073	0.46504	0.244486
0.480347	0.243518	0.463199	0.637539	0.749472	0.788041	0.749472	0.637539	0.463199	0.243518

# Excel로의 결과 출력(Numerical solution Graph)

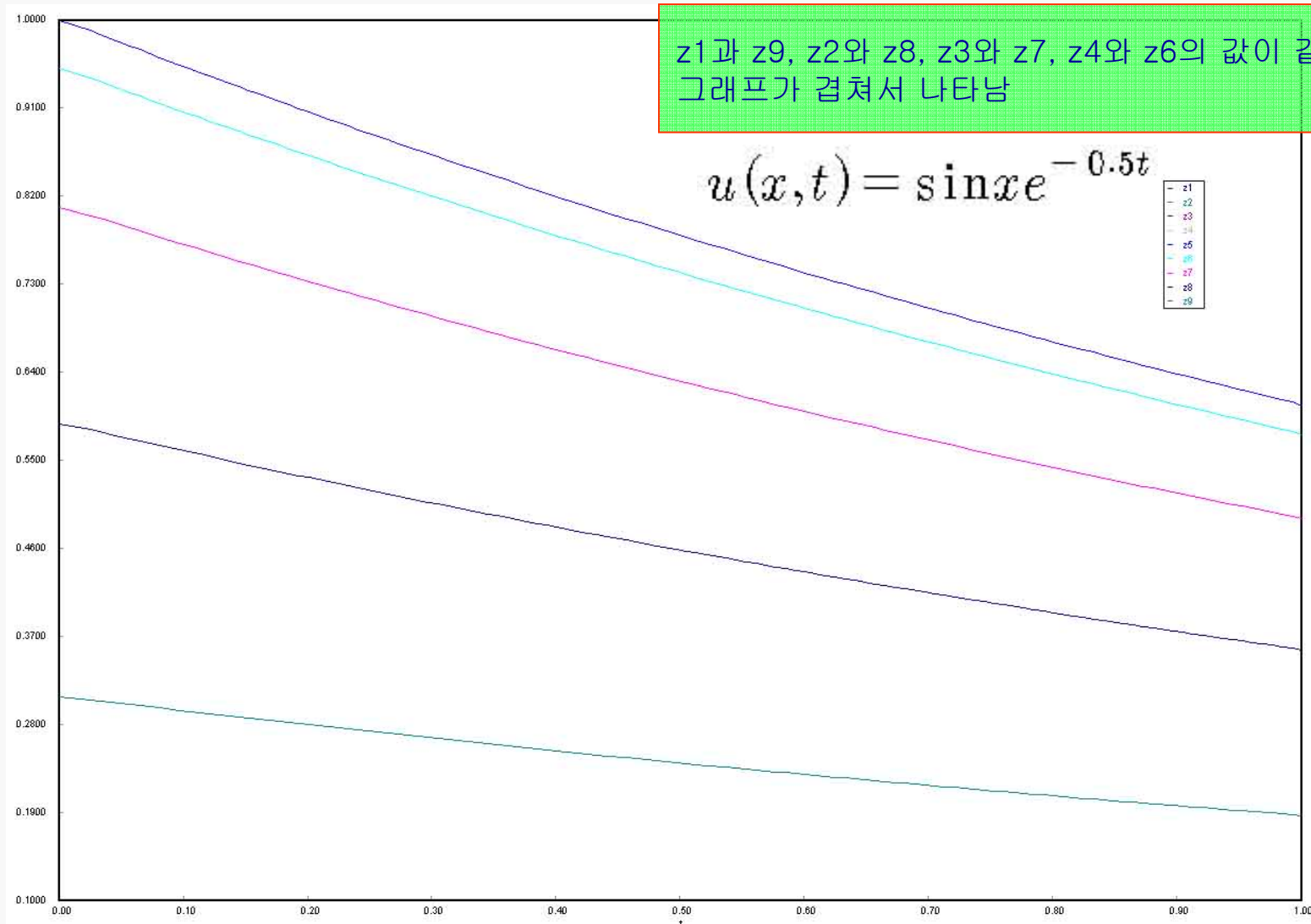


# 선택적 그래프 그리기





# 결과 출력 (Analytical solution Graph)







# $\Delta x = 0.2\pi$ 일때의 입력데이터

```

d(u1)/d(t) = a * (u2 - 2 * u1 + u0) / deltax ^ 2
u1(0) = 0.587785252
d(u2)/d(t) = a * (u3 - 2 * u2 + u1) / deltax ^ 2
u2(0) = 0.951056516
d(u3)/d(t) = a * (u4 - 2 * u3 + u2) / deltax ^ 2
u3(0) = 0.951056516
d(u4)/d(t) = a * (u5 - 2 * u4 + u3) / deltax ^ 2
u4(0) = 0.587785252
a = 0.5
deltax = 0.2 * 3.141592654
u5 = 0
u0 = 0
t(0) = 0
t(f) = 1
    
```

```

z1 = sin(x1) * exp(-a * t)
x1 = 0.2 * 3.141592654
z2 = sin(x2) * exp(-a * t)
x2 = 0.4 * 3.141592654
z3 = sin(x3) * exp(-a * t)
x3 = 0.6 * 3.141592654
z4 = sin(x4) * exp(-a * t)
x4 = 0.8 * 3.141592654
    
```

$\Delta x$ 의 변화에 따른 u1,u2,u3,u4값 새로 정리

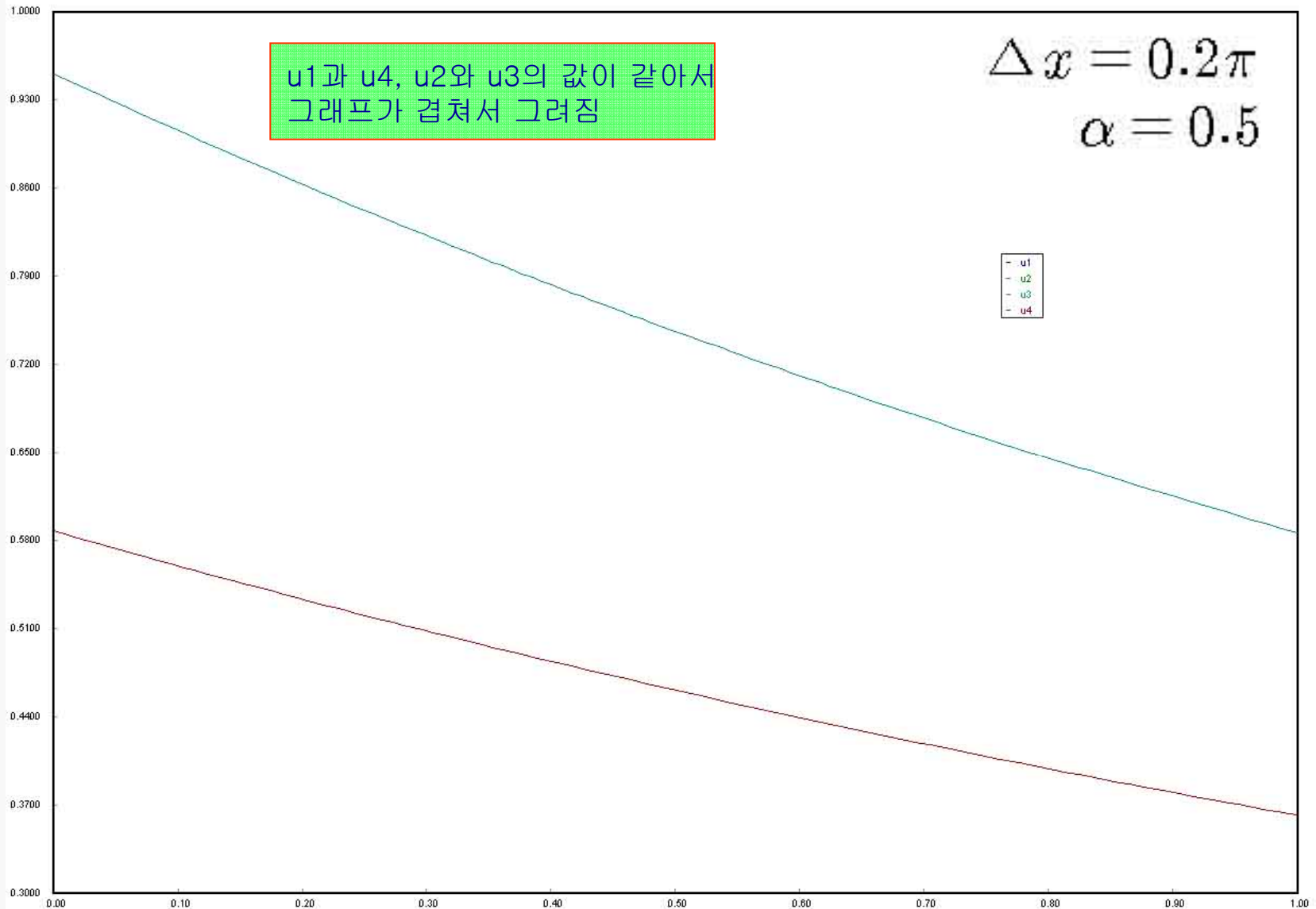
엑셀을 통한 초기값 입력

$\Delta x$ 의 변화에따른 x1,x2,x3,x4값 새로 지정해준 후 변수분리 된 정확해 입력

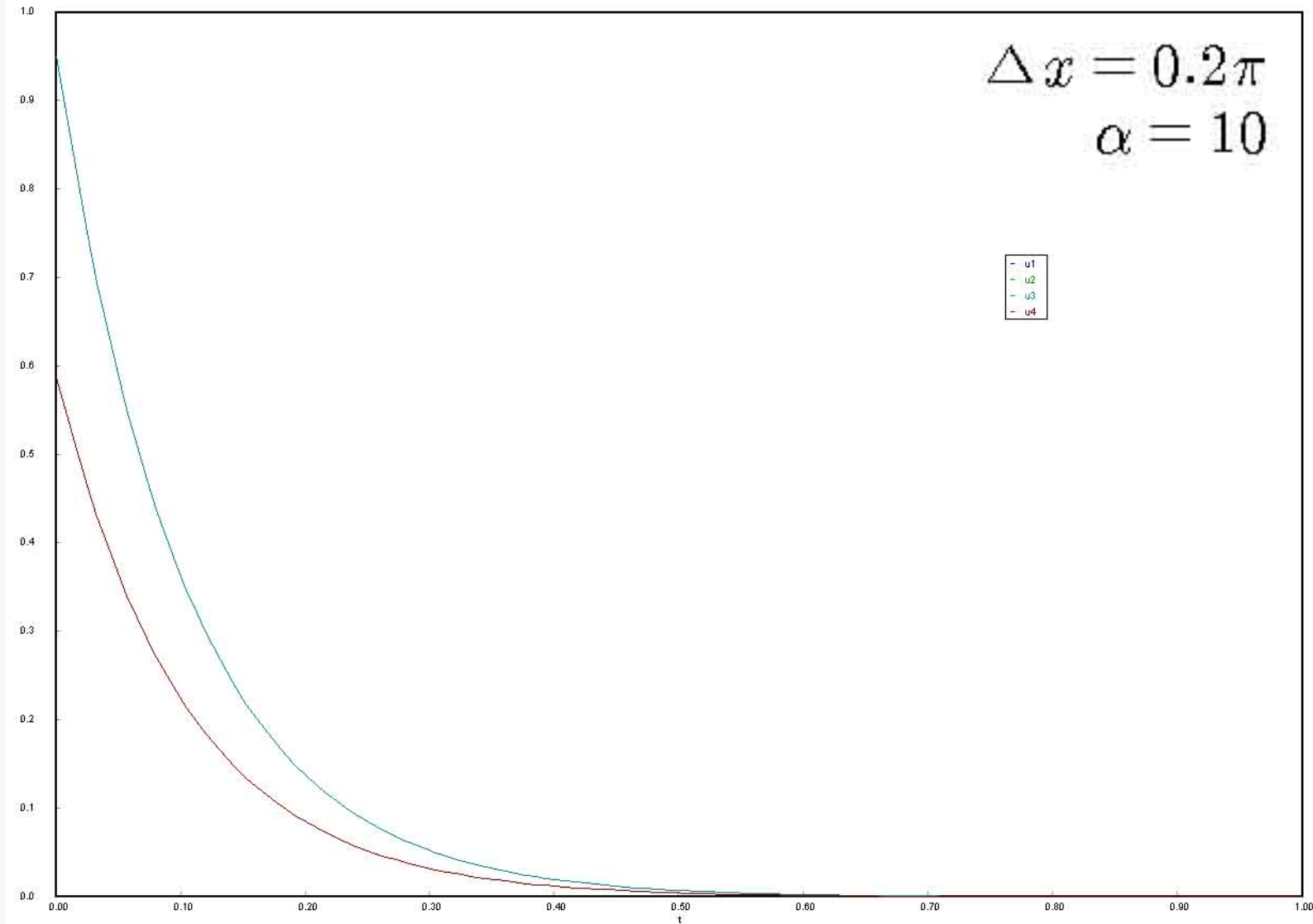
deltax=0.2*pi일때				
	x값	x값	초기값	
	pi	3.141592654	sinx	
0.2	0.2*pi	0.628318531	0.587785252	u1일때의 초기값
0.4	0.4*pi	1.256637061	0.951056516	u2일때의 초기값
0.6	0.6*pi	1.884955592	0.951056516	u3일때의 초기값
0.8	0.8*pi	2.513274123	0.587785252	u4일때의 초기값



# 값 변화에 따른 그래프



# 값 변화에 따른 그래프



## 토론 및 검토

---

- ◆ Numerical solution과 Analytical solution의 값이 거의 동일하게 나옴.
- ◆ U값은 가운데를 중심으로 대칭됨.
- ◆  $\Delta x$ 값을 작게 해주면 더 조밀한 간격의 u값의 변화를 알 수 있음.
- ◆  $\alpha$ 값의 변화에 따른 데이터 값은  $\alpha$ 값이 클수록 u값이 급격히 감소하는 경향을 보임.