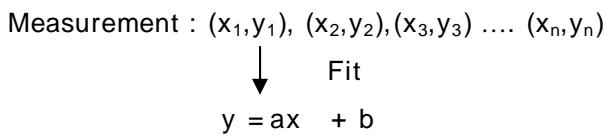


Least Square Method ()

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Principle

Error between calculated and measured variables

$$e_i = y_i - ax_i - b_i$$

Objective of the fit is to minimize sum of squared errors

$$S_r = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - ax_i - b)^2$$

At the minimum, two derivatives with respect to a and b have to be zero.

$$\frac{\partial S_r}{\partial a} = 0 \text{ and } \frac{\partial S_r}{\partial b} = 0$$

$$\frac{\partial S_r}{\partial a} = -2 \sum_{i=1}^n (y_i - ax_i - b)x_i = 0$$

$$\frac{\partial S_r}{\partial b} = -2 \sum_{i=1}^n (y_i - ax_i - b) = 0$$

Unknown a and b can be solved using two equations above .

$$\sum_{i=1}^n y_i - \sum_{i=1}^n ax_i - \sum_{i=1}^n b = 0$$

$$\sum_{i=1}^n y_i x_i - \sum_{i=1}^n ax_i^2 - \sum_{i=1}^n bx_i = 0$$

Final Solution:

$$a = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}, \quad b = \bar{y} - a \bar{x}$$

$$\bar{y} = \frac{\sum y_i}{n}, \quad \bar{x} = \frac{\sum x_i}{n}$$

Using the notation in the text book,

$$s_x = \frac{\sum x_i}{n} \quad s_y = \frac{\sum y_i}{n}$$

$$s_{xx} = \frac{\sum x_i^2}{n} \quad s_{xy} = \frac{\sum x_i y_i}{n}$$

$$a = \frac{s_{xy} - s_x s_y}{s_{xx} - (s_x)^2}, \quad b = \frac{s_{xx} s_y - s_{xy} s_x}{s_{xx} - (s_x)^2}$$

Representation for goodness of fit

(1) Sum of squares : Total sum of squared error

$$S_r = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - ax_i - b)^2$$

(2) Standard Deviations

$$S_{y/x} = \sqrt{\frac{S_r}{n-2}}$$

(3) Total sum of squared error around mean y

$$S_t = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (\bar{y} - ax_i - b)^2$$

(4) Correlation coefficient

$$r^2 = \frac{S_t - S_r}{S_t}$$

$r \rightarrow 1$ for good fit ($S_r \rightarrow 0$)

$r \rightarrow 0$ for bad fit ($S_t > S_r$)