

CHE302 LECTURE XI CONTROLLER DESIGN AND PID CONTROLLER TUNING

Professor Dae Ryook Yang

Fall 2001

Dept. of Chemical and Biological Engineering
Korea University

CHE302 Process Dynamics and Control

Korea University 11-1

GUIDELINES FOR COMMON CONTROL LOOPS

- **Flow and liquid pressure control**
 - Fast response with no time delay
 - Usually with small high-frequency noise
 - PI controller with intermediate controller gain
- **Liquid level control**
 - Noisy due to splashing and turbulence
 - High gain PI controller for integrating process
 - Conservative setting for averaging control when it is used for damping the fluctuation of the inlet stream
- **Gas pressure control**
 - Usually fast and self regulating
 - PI controller with small integral action (large reset time)

CHE302 Process Dynamics and Control

Korea University 11-3

CONTROLLER DESIGN

- **Performance criteria for closed-loop systems**
 - Stable
 - Minimal effect of disturbance
 - Rapid, smooth response to set point change
 - No offset
 - No excessive control action
 - Robust to plant-model mismatch

$$\min_{K_c, t_I, t_D} \int_0^{\infty} (w_1 e^2(t) + w_2 \Delta u^2(t)) dt$$

- **Trade-offs in control problems**
 - Set point tracking vs. disturbance rejection
 - Robustness vs. performance

CHE302 Process Dynamics and Control

Korea University 11-2

- **Temperature control**
 - Wide variety of the process nature
 - Usually slow response with time delay
 - Use PID controller to speed up the response
- **Composition control**
 - Similar to temperature control usually with larger noise and more time delay
 - Effectiveness of derivative action is limited
 - Temperature and composition controls are the prime candidates for advance control strategies due to its importance and difficulty of control

CHE302 Process Dynamics and Control

Korea University 11-4

TRIAL AND ERROR TUNING

- Step1: With P-only controller**

- Start with low K_C value and increase it until the response has a sustained oscillation (continuous cycling) for a small set point or load change. (K_{cu})
- Set $K_C = K_{cu}$.

- Step2: Add I mode**

- Decrease the reset time until sustained oscillation occurs. (t_{Ii})
- Set $t_I = 3t_{Ii}$
- If a further improvement is required, proceed to Step 3.

- Step3: Add D mode**

- Decrease the reset time until sustained oscillation occurs. (t_{Di})
- Set $t_D = 3t_{Di}$.

(The sustained oscillation should not be caused by the controller saturation)

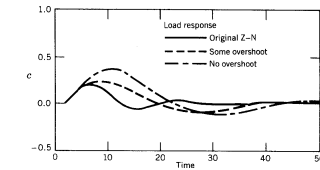
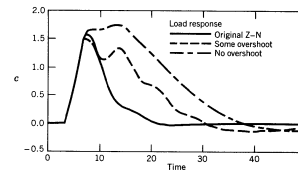
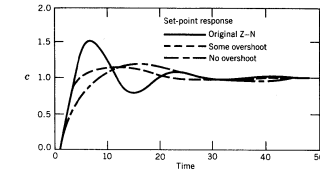
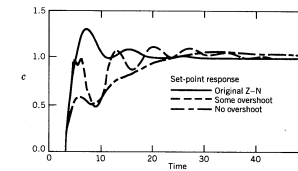
- Examples**

$$G_p(s) = \frac{4e^{-3.5s}}{7s+1} \quad K_{cu} = 0.95 \quad P_{cu} = 12$$

$$G_p(s) = \frac{2e^{-s}}{(10s+1)(5s+1)} \quad K_{cu} = 7.88 \quad P_{cu} = 11.6$$

Controller	K_C	t_I	t_D
Original	0.57	6.0	1.5
Some overshoot	0.31	6.0	4.0
No overshoot	0.19	6.0	4.0

Controller	K_C	t_I	t_D
Original	4.73	5.8	1.45
Some overshoot	2.60	5.8	3.87
No overshoot	1.58	5.8	3.87



CONTINUOUS CYCLING METHOD

- Also called as loop tuning or ultimate gain method**

- Increase controller gain until sustained oscillation
- Find ultimate gain (K_{cu}) and ultimate period (P_{cu})

- Ziegler-Nichols controller setting**

- 1/4 decay ratio (too much oscillatory)

Controller	K_C	t_I	t_D
P	$0.5K_{cu}$	-	-
PI	$0.45K_{cu}$	$P_{cu}/1.2$	-
PID	$0.6K_{cu}$	$P_{cu}/2$	$0.5P_{cu}/8$

- **Modified Ziegler-Nichols setting**

Controller	K_C	t_I	t_D
Original	$0.6K_{cu}$	$P_{cu}/2$	$P_{cu}/8$
Some overshoot	$0.33K_{cu}$	$P_{cu}/2$	$P_{cu}/3$
No overshoot	$0.2K_{cu}$	$P_{cu}/2$	$P_{cu}/3$

- Advantages of continuous cycling method**

- No a priori information on process required
- Applicable to all stable processes

- Disadvantages of continuous cycling method**

- Time consuming
- Loss of product quality and productivity during the tests
- Continuous cycling may cause the violation of process limitation and safety hazards
- Not applicable to open-loop unstable process
- First-order and second-order process without time delay will not oscillate even with very large controller gain

=> Motivates Relay feedback method. (Astrom and Wittenmark)

RELAY FEEDBACK METHOD

- **Relayfeedbackcontroller**

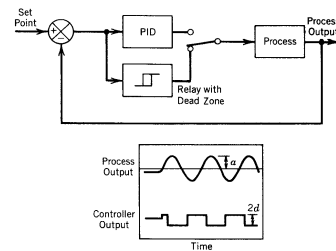
- Forces the system to oscillate by a relay controller
- Require a single closed-loop experiment to find the ultimate frequency information
- No *a priori* information on process is required
- Switch relay feedback controller for tuning
- Find P_{CU} and calculate K_{CU}

$$K_{CU} = \frac{4d}{pa}$$

- User specified parameter: d

Decide d in order not to perturb the system too much.

- Use Ziegler-Nichols Tuning rules for PID tuning parameters



- **Design relations based on integral error criteria**

- 1/4 decay ratio is too oscillatory
- Decay ratio concerns only two peak points of the response
- IAE: Integral of the Absolute Error

$$IAE = \int_0^{\infty} |e(t)| dt$$

- ISE: Integral of the Square Error

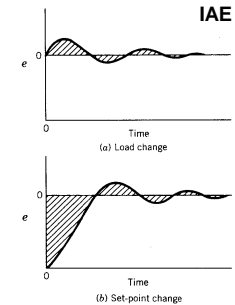
$$ISE = \int_0^{\infty} [e(t)]^2 dt$$

- Large error contributes more
- Small error contributes less
- Large penalty for large overshoot
- Small penalty for small persisting oscillation

- ITAE: Integral of the Time-weighted Absolute Error

$$ITAE = \int_0^{\infty} t |e(t)| dt$$

- Large penalty for persisting oscillation
- Small penalty for initial transient response



DESIGN RELATIONS FOR PID CONTROLLERS

- **Cohen-Coon controller design relations**

- Empirical relation for 1/4 decay ratio for FOPDT model

Table 12.2 Cohen and Coon Controller Design Relations

Controller	Settings	Cohen-Coon
P	K_c	$\frac{1}{K} \frac{\tau}{\theta} [1 + \theta/3\tau]$
PI	K_c	$\frac{1}{K} \frac{\tau}{\theta} [0.9 + \theta/12\tau]$
	τ_I	$\frac{\theta[30 + 3(\theta/\tau)]}{9 + 20(\theta/\tau)}$
PID	K_c	$\frac{1}{K} \frac{\tau}{\theta} \left[\frac{16\tau + 3\theta}{12\tau} \right]$
	τ_I	$\frac{\theta[32 + 6(\theta/\tau)]}{13 + 8(\theta/\tau)}$
	τ_D	$\frac{4\theta}{11 + 2(\theta/\tau)}$

- **Controller design relation based on ITAE for FOPDT model**

Table 12.3 Controller Design Relations Based on the ITAE Performance Index and a First-Order plus Time-Delay Model [6-8]^a

Type of Input	Type of Controller	Mode	A	B
Load	PI	P	0.859	-0.977
		I	0.674	-0.680
		D	0.381	0.995
Load	PID	P	1.357	-0.947
		I	0.842	-0.738
		D	0.381	0.995
Set point	PI	P	0.586	-0.916
		I	1.03 ^b	-0.165 ^b
Set point	PID	P	0.965	-0.85
		I	0.796 ^b	-0.1465 ^b
		D	0.308	0.929

^aDesign relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.

^bFor set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$. [8]

- **Similar design relations based on IAE and ISE for other types of models can be found in literatures.**

• **Example1**

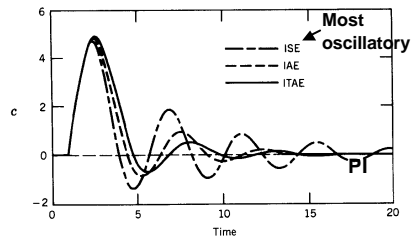
$$G(s) = \frac{10e^{-s}}{2s+1}$$

$$KK_c = (0.859)(1/2)^{-0.977} = 1.69$$

$$\Rightarrow K_c = 0.169$$

$$t/t_I = (0.674)(1/2)^{-0.680} = 1.08$$

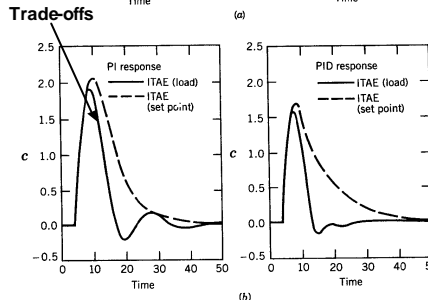
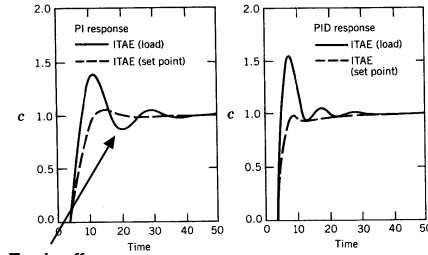
$$\Rightarrow t_I = 1.85$$



Method	K_c	t_I
IAE	0.195	2.02
ISE	0.245	2.44
ITAE	0.169	1.85

Example2

$$G(s) = \frac{4e^{-3.5s}}{7s+1}$$



DIRECT SYNTHESIS METHOD

- **Analysis:** Given $G_c(s)$, what is $y(t)$?
- **Design:** Given $y_d(t)$, what should $G_c(s)$ be?
- **Derivation**

$$\text{Let } G_{OL} = K_m G_c G_p G_v \triangleq G_c G$$

$$\frac{Y(s)}{R(s)} = \frac{G_{OL}}{1+G_{OL}} = \frac{G_c G}{1+G_c G} \Rightarrow G_c = \frac{1}{G} \left(\frac{Y/R}{1-Y/R} \right)$$

$$\text{Specify } (Y/R)_d \Rightarrow G_c = \frac{1}{G} \left(\frac{(Y/R)_d}{1-(Y/R)_d} \right)$$

- If $(Y/R)_d = 1$, then it implies perfect control. (infinite gain)
- The resulting controller may not be physically realizable
- Or, not in PID form and too complicated.
- Design with finite settling time: $(Y/R)_d = \frac{1}{t_c s + 1}$

• **Design relations based on process reaction curve**

- For the processes who have sigmoidal shape step responses (Not for underdamped processes)
- Fit the curve with FOPDT model

$$G(s) = \frac{K e^{-qs}}{(ts+1)} \quad S = K \Delta u / t \quad S^* = S / \Delta u = K / t$$

Table 13.3 Ziegler-Nichols Tuning Relations (Process Reaction Curve Method)

Controller Type	K_c	τ_I	τ_D
P	$\frac{1}{0.5S^*}$	—	—
PI	$\frac{0.9}{0.5S^*}$	3.330	—
PID	$\frac{1.2}{0.5S^*}$	20	0.50

- Very simple
- Inherits all the problems of FOPDT model fitting

• **Examples**

1. Perfect control (K_c becomes infinite)

$$G(s) = \frac{K}{(t_1 s + 1)(t_2 s + 1)} \quad \text{and } (Y/R)_d = 1$$

$$G_c(s) = \frac{1}{G(s)} \left(\frac{1}{1-1} \right) = \frac{\infty}{G(s)} \quad (\text{infinite gain, unrealizable})$$

2. Finite settling time for 1st-order process

$$G(s) = \frac{K}{(ts+1)} \quad \text{and } (Y/R)_d = \frac{1}{t_c s + 1}$$

$$G_c(s) = \frac{1}{G(s)} \left(\frac{1/(t_c s + 1)}{1-1/(t_c s + 1)} \right) = \frac{t_c s + 1}{K t_c s} = \frac{t_c}{K} \left(1 + \frac{1}{t_c s} \right) \quad (\text{PI})$$

3. Finite settling time for 2nd-order process

$$G(s) = \frac{K}{(t_1 s + 1)(t_2 s + 1)} \quad \text{and } (Y/R)_d = \frac{1}{t_c s + 1}$$

$$G_c(s) = \frac{(t_1 + t_2)}{t_c K} \left(1 + \frac{1}{(t_1 + t_2)s} + \frac{t_1 t_2}{(t_1 + t_2)s^2} \right) \quad (\text{PID})$$

• Process with time delay

- If there is a time delay, any physically realizable controller cannot overcome the time delay. (Need time lead)
- Given circumstance, a reasonable choice will be

$$(Y/R)_d = \frac{e^{-qs}}{t_c s + 1}$$

- Examples

$$1. \quad G(s) = \frac{Ke^{-qs}}{(t_s+1)} \quad \text{and} \quad (Y/R)_d = \frac{e^{-qs}}{t_c s + 1} \quad (q_c = q)$$

$$G_c(s) = \frac{1}{G(s)} \left(\frac{e^{-qs}/(t_c s + 1)}{1 - e^{-qs}/(t_c s + 1)} \right) = \frac{t_s + 1}{K} \frac{1}{t_c s + 1 - e^{-qs}} \quad (\text{not a PID})$$

Physically realizable

2. With 1st-order Taylor series approx. ($e^{-qs} \approx 1 - qs$)

$$G_c(s) = \frac{t_s + 1}{K} \frac{1}{(t_c + q)s} = \frac{t}{K(t_c + q)} \left(1 + \frac{1}{t_s} \right) \quad (\text{PI})$$

$$3. \quad G(s) = \frac{Ke^{-qs}}{(t_1 s + 1)(t_2 s + 1)} \quad \text{and} \quad (Y/R)_d = \frac{e^{-qs}}{t_c s + 1} \quad (q_c = q)$$

$$G_c(s) = \frac{(t_1 s + 1)(t_2 s + 1)}{K} \frac{1}{(t_c + q)s} = \frac{(t_1 + t_2)}{K(t_c + q)} \left(1 + \frac{1}{(t_1 + t_2)s} + \frac{t_1 t_2}{(t_1 + t_2)s^2} \right) \quad (\text{PID})$$

INTERNAL MODEL CONTROL (IMC)

• Motivation

- The resulting controller from direct synthesis method may not be physically unrealizable.
- If there is RHP zero in the process, the resulting controller from direct synthesis method will be unstable.
- Unmeasured disturbance and modeling error are not considered in direct synthesis method.

• Source of trouble

- From direct synthesis method

$$G_c = \frac{1}{G} \left(\frac{(Y/R)_d}{1 - (Y/R)_d} \right)$$

Resulting controller may have higher-order numerator than denominator

Direct inversion of process causes many problems

Process is unknown

• Observations on Direct Synthesis Method

- Resulting controllers could be quite complex and may not even be physically realizable.
- PID parameters will be decided by a user-specified parameter: The desired closed-loop time constant (t_c)
- The shorter t_c makes the action more aggressive. (larger K_c)
- The longer t_c makes the action more conservative. (smaller K_c)
- For a limited cases, it results PID form.

- 1st-order model without time delay: PI

- FOPDT with 1st-order Taylor series approx.: PI

- 2nd-order model without time delay: PID

- SOPDT with 1st-order Taylor series approx.: PID

- Delay modifies the K_c .

$$\frac{t}{K t_c} \rightarrow \frac{t}{K(t_c + q)} \quad (\text{1st order}) \quad \frac{(t_1 + t_2)}{K t_c} \rightarrow \frac{(t_1 + t_2)}{K(t_c + q)} \quad (\text{2nd order})$$

- With time delay, the K_c will not become infinite even for the perfect control ($Y/R=1$).

• IMC

- Feedback the error between the process output and model output.

- Equivalent conventional controller: $G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}}$

- Using block diagram algebra

$$C = GP + L \quad P = G_c^* E \quad E = R - (C - \tilde{C}) = R - C + \tilde{G}P$$

$$P = G_c^* (R - C + \tilde{G}P)$$

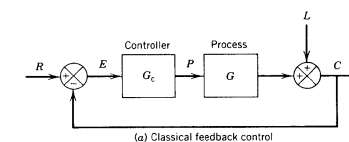
$$\Rightarrow P = G_c^* (R - C) / (1 - G_c^* \tilde{G})$$

$$C = GG_c^* (R - C) / (1 - G_c^* \tilde{G}) + L$$

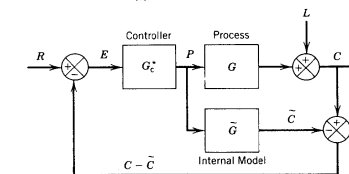
$$(1 + GG_c^* - G_c^* \tilde{G})C = GG_c^* R + (1 - G_c^* \tilde{G})L$$

$$C = \frac{G_c^* G}{1 + G_c^* (G - \tilde{G})} R + \frac{(1 - G_c^* \tilde{G})}{1 + G_c^* (G - \tilde{G})} L$$

$$\text{If } \tilde{G} = G, C = G_c^* GR + (1 - G_c^* G)L$$



(a) Classical feedback control



(b) Internal Model Control

• IMC design strategy

- Factor the process model as

$$\tilde{G} = \underbrace{(\tilde{G}_+ \tilde{G}_-)}_{\text{Uninvertibles}}$$

- \tilde{G}_+ contains any time delays and RHP zeros and is specified so that the steady-state gain is one
 - \tilde{G}_- is the rest of G .
- The controller is specified as

$$G_c^* = \frac{1}{\tilde{G}_-} f$$

- IMC filter f is a low-pass filter with steady-state gain of one
- Typical IMC filter:
$$f = \frac{1}{(t_c s + 1)^r}$$
- The t_c is the desired closed-loop time constant and parameter r is a positive integer that is selected so that the order of numerator of G_c^* is same as the order of denominator or exceeds the order of denominator by one.

IMC based PID controller settings

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ [4]^a

Case	Model	$K_c K$	τ_I	τ_D
A	$\frac{K}{\tau s + 1}$	$\frac{\tau}{\tau_c}$	τ	—
B	$\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\tau_1 + \tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$
C	$\frac{K}{\tau^2 s^2 + 2\zeta \tau s + 1}$	$\frac{2\zeta \tau}{\tau_c}$	$2\zeta \tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta \tau s + 1}, \beta > 0$	$\frac{2\zeta \tau}{\tau_c + \beta}$	$2\zeta \tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{1}{\tau_c}$	—	—
F	$\frac{K}{s(\tau s + 1)}$	$\frac{1}{\tau_c}$	—	τ

^aBased on Eq. 12-30 with $r = 1$.

• Example

- FOPDT model with 1/1 Pade approximation

$$\tilde{G} = \frac{K(1 - qs/2)}{(1 + qs/2)(ts + 1)}$$

$$\tilde{G}_+ = 1 - qs/2 \quad \tilde{G}_- = \frac{K}{(1 + qs/2)(ts + 1)}$$

$$G_c^* = \frac{1}{\tilde{G}_-} f = \frac{(1 + qs/2)(ts + 1)}{K} \frac{1}{(t_c s + 1)}$$

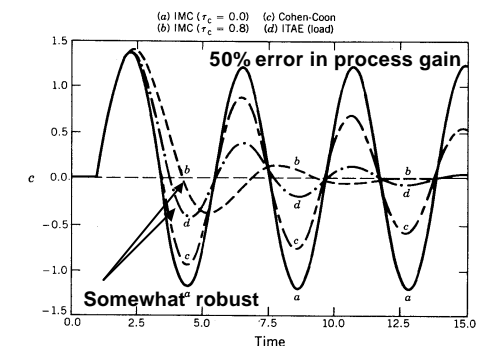
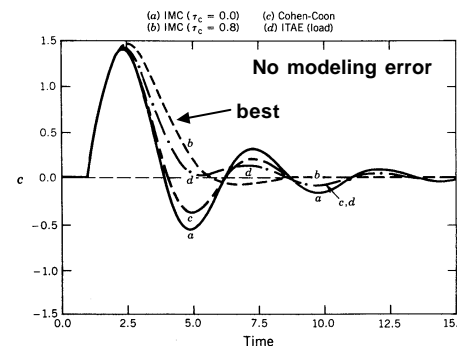
$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} = \frac{(1 + qs/2)(ts + 1)}{K(t_c + q/2)s} \quad (\text{PID})$$

$$K_c = \frac{1}{K} \frac{(t + q/2)}{(t_c + q/2)} \quad t_I = t + q/2 \quad t_D = \frac{tq/2}{t + q/2}$$

COMPARISON OF CONTROLLER DESIGN RELATIONS

- PI controller settings for different methods

$$G(s) = \frac{2e^{-s}}{s + 1}$$



EFFECT OF MODELING ERROR

- **Actual plant**

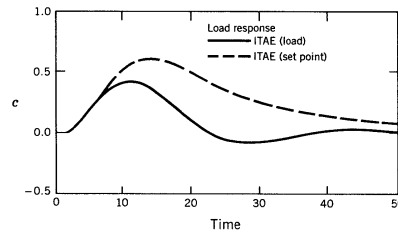
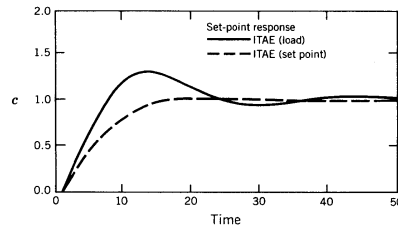
$$G(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}$$

- **Approx. model**

$$\tilde{G}(s) = \frac{2e^{-4.7s}}{12s+1}$$

- Satisfactory for this case
- Use with care

As the estimated time delay gets smaller, the performance degradation will be pronounced.



- **All kinds of tuning method should be used for initial setting and fine tuning should be done!!**

GENERAL CONCLUSION FOR PID TUNING

- The controller gain should be inversely proportional to the products of the other gains in the feedback loop.
- The controller gain should decrease as the ratio of time delay to dominant time constant increases.
- The larger the ratio of time delay to dominant time constant is, the harder the system is to control.
- The reset time and the derivative time should increase as the ratio of time delay to dominant time constant increases.
- The ratio between derivative time and reset time is typically between 0.1 to 0.3.
- The $\frac{1}{4}$ decay ratio is too oscillatory for process control. If less oscillatory response is desired, the controller gain should decrease and reset time should increase.
- Among IAE, ISE and ITAE, ITAE is the most conservative and ISE is the least conservative setting.