Modelling & Design

Oil in a closed funnel

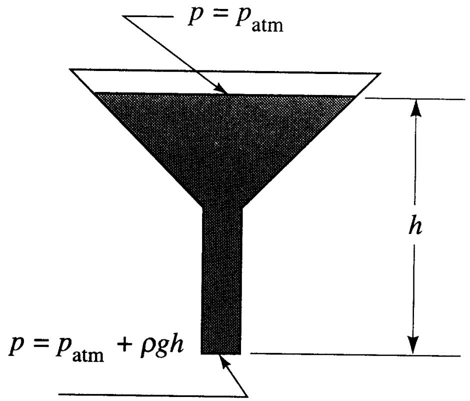

Figure 2.1.5 Closed funnel, no flow.

$$F_{up} = P_2 A = P_1 A + mg = P_1 A + \rho A h g = F_{down}$$

$$P_2 - P_1 = \rho g h$$

if unplug the bottom

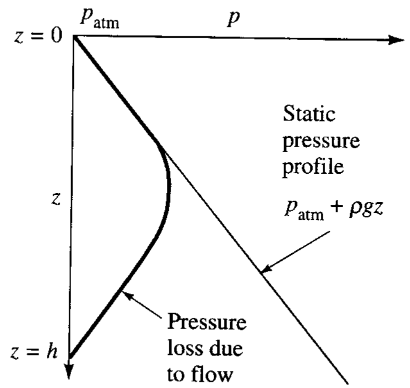

Figure 2.1.6 Pressure distribution with and without flow (static pressure profile).

Figure 2.1.8 Drop hanging on the tip of a capillary.

what if the exit flow is very slow

a drop begins to form slowly on the end of capillary

flow stops if the surface tension is high & capillary tip is small

Surface tension

force per unit length acting
tangentially to the liquid-gas surface

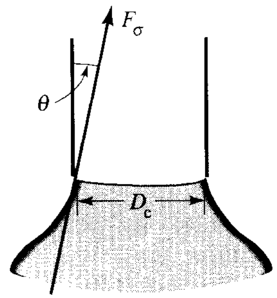

Figure 2.1.9 Forces along the contact line in the neighborhood of *A* in Fig. 2.1.8.

force balance

$$F_\sigma \cos\theta = \sigma\pi D_c \cos\theta = \rho g V$$

hypothesis 1

as drop increases, θ approaches zero

drop will fall when its volume exceeds $V_{\max} = \dfrac{\sigma\pi D_c}{\rho g}$

hypothesis 2

the drop is nearly a sphere of diameter D_d $\left(V = \frac{\pi D_d^3}{6}\right)$

$$D_d = \left(\frac{6\sigma D_c}{\rho g}\right)^{1/3} \quad \text{or} \quad \frac{D_d}{D_c} = \left(\frac{6\sigma}{D_c^2 \rho g}\right)^{1/3}$$

Bond number $Bo = \frac{D_c^2 \rho g}{\sigma}$

$$\frac{D_d}{D_c} = \left(\frac{6}{Bo}\right)^{1/3} = 1.82\, Bo^{-1/3}$$

model for drop size that falls from a slowly dripping capillary

$$N_d = \left(\frac{D_d}{D_c}\right) Bo^{1/3} = 1.82$$

$$N_{d,\exp} = 1.6$$

due to residual liquid

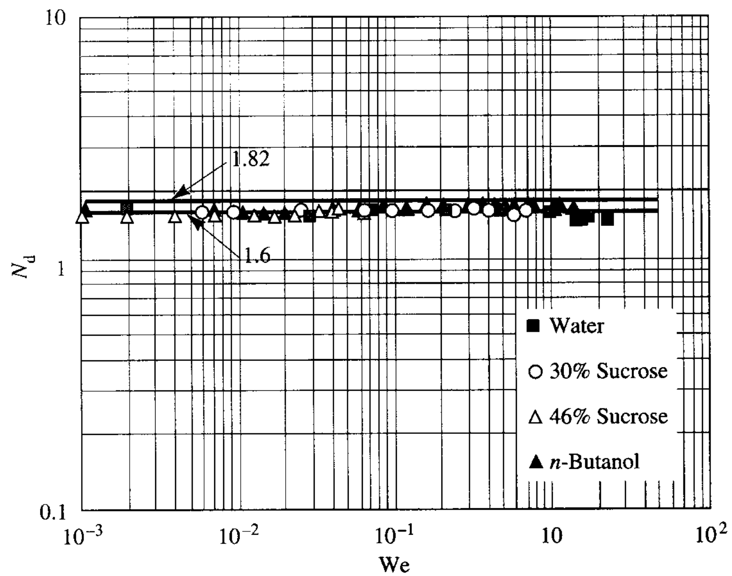

Figure 2.1.10 Dimensionless drop size (see Eq. 2.1.12) for several liquids, as a function of the Weber number.

the theory is not exact, but captures the essential physics of the process at small Bo and small We.

Engineering Design

goal: design a capillary viscometer that will be useful for fluids with viscosities of the order of 1000 poise

interpretation: specify R,L,P

conceptual design:

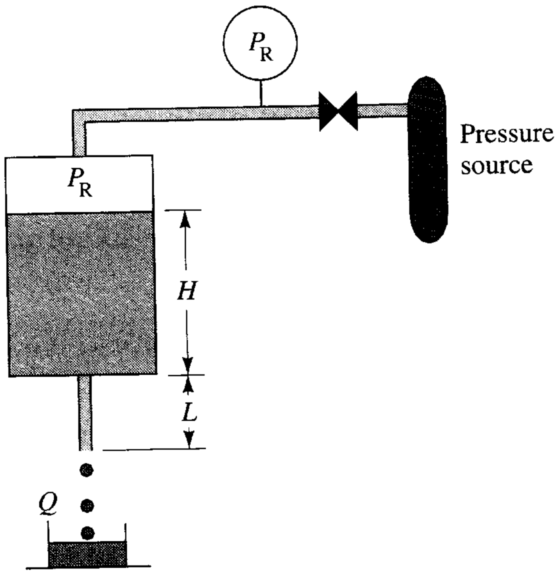

Figure 3.3.1 Conceptual design of a capillary viscometer.

design equation: (model)

$$\mu = \frac{\pi R^4}{8Q}\frac{\Delta P}{L} \quad \text{(H-P)}$$

$$\Delta P = P_R + \rho g H + \rho g L - P_L$$

constraints:

laminar, fully-developed, isothermal, Newtonian

end effect

$$\frac{L_e}{R} = 1.2 + 0.08Re \quad , \quad Re = \frac{\rho u D}{\mu} = \frac{2Q\rho}{\pi R \mu}$$

order of magnitude (rough design proceedure)

assume: we should measure 100cm^3 over 100s (Q=1cm^3/s)

$$\rho = 1g/cm^3 \ , \ R = 0.1cm \ , \ \mu = 100Cpoise$$

$$Re = \frac{2(1)(1)}{\pi(0.1)(1000)} = 0.01 \quad \text{(laminar)}$$

$$\frac{L_e}{R} \cong 1.2 \quad \text{(the flow will be fully developed from the entrance)}$$

to make the entrance length small

$$\frac{L_2}{L} = 0.01 \;,\; L = 100 L_e \cong 100R = 10cm \;\Rightarrow\; \frac{L}{R} = 100$$

design parameter: R=0.1cm, L=10cm

$$\triangle P = \frac{8\mu QL}{\pi R^4} = \frac{8(1000)(1)(10)}{\pi(0.1)^4} = \frac{8}{\pi} \times 10^8 dyn/cm^2 = \frac{800}{\pi} atm = 4000 psi$$

evaluation:

pressure too high (require expensive pressure vessel)

relax some of the specifications

keeping $\frac{L}{R} = 100$, $\triangle P = \frac{8\pi Q}{\pi} \frac{L}{R} \frac{1}{R^3}$

if we increase R by a factor of 2,

$\triangle P = 500psi$, $R = 0.2cm$, $L = 20cm$

hydrostatic effects

$$\rho g H \sim \rho g L = (1)(10^3)(20) = 2 \times 10^4 dyn/cm^2 \cong 0.3 psi$$

negligible when P_R is large

let exit pressure $P_L = 0$ (atmospheric)

$\Rightarrow$ regard all pressures as guage pressure

(relative to atmospheric pressure)

rough design criteria:

$R = 0.2cm$, $L = 20cm$, $\triangle P = 500psi$