3 - 9 DO balance

가) Reaeration

Do deficit : D = C_y - C

$$\frac{dD}{dt} = -\frac{dC}{dt}$$

i) '산소소비속도' ↑하면 '산소결핍속도 $\frac{dD}{dt}$ ↑

ii) D↑ 할수록 reaeration 속도 ↑

나) Algae photosynthesis

i) at light

$$CO_2 + 2H_2O \xrightarrow{hv} CH_2O + O_2 + H_2O$$

(new cell)

ii) no lihgt (endogenous catabolism)

$$CH_2O + O_2 \rightarrow CO_2 + H_2O$$

iii) Algae에 의한 DO 기여도는 time dependent

iv) 낮 pH ↑

밤 pH ↓

$$2HCO_3^- \rightleftharpoons CO_3^{2-} + H_2O + CO_2$$

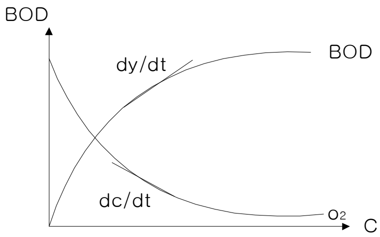
$$CO_3^{2-} + H_2O \rightleftharpoons 2OH^- + CO_2$$

$$CO_2 + H_2O \rightleftharpoons H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$$

3 - 10 DO model

가) O_2 소비속도

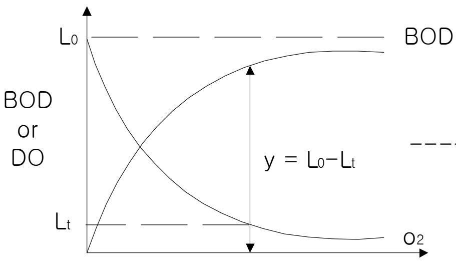
$$\frac{dy}{dt} = -\frac{dc}{dt} \quad ------ ①$$



from 산소결핍속도 ($D = C^* - C$)

$$\frac{dD}{dt} = -\frac{dc}{dt} \quad ------ ②$$

from ①②
$$\frac{dy}{dt} = \frac{dD}{dt} \quad ------ ③$$



------ ④

$$\frac{dy}{dt} = -\frac{dL_t}{dt} = kL_t \quad ------ ⑤$$

산소소비속도 (1st order)

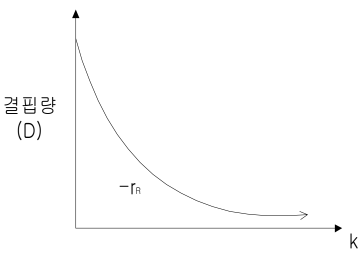
from ③⑤ 결핍속도 = 산소소비속동

$$\frac{dD}{dt} = kL_t \qquad \rightarrow \quad Y_0 = k_1 L_t \quad ------ ⑥$$

나) O_2 addition rate

$Y_R = -k_2 D$ ------ ⑦ (1st order)

k_2 : reaeration rate constant

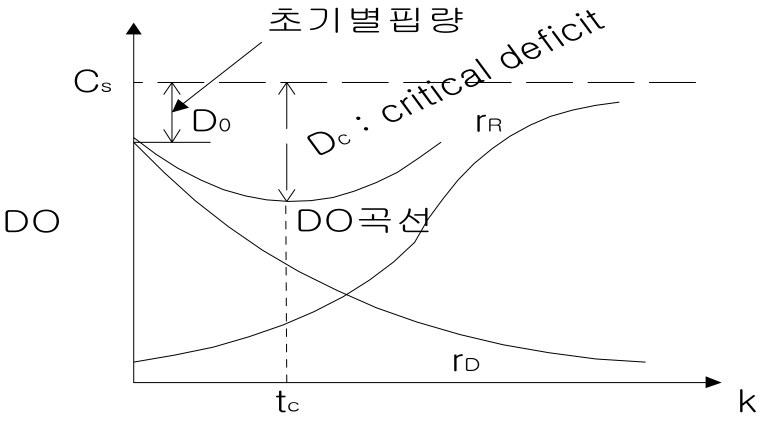


from ⑥⑦

$$\frac{dD}{dt} = Y_0 + Y_R = k_1 L_t - k_2 D \quad ------⑧$$

산소소비속도 reaeration속도

* Oxygen Sag curve



L_t = $L_0 e^{-k1t}$ $L_t = L_0 e^{-k_1 t}$ ------ ⑨

from ⑧⑨ $\frac{dD}{dt} + k_2 D = k_1 L_0 e^{-k_1 t}$ ------ ⑩

firot differential eq $\frac{dy}{dx} + py = \alpha$

ⅰ) p=상수, α=f(x)

ⅱ) exp [∫p dx]를 양변에 곱한다.

$$D = \frac{k_1 L_0}{k_2 - k_1}(e^{-k_1 t} - e^{-k_2 t}) + D_0 e^{-k_2 t}$$ ------ ⑪

* 최대 산소 결핍량 (D_C)

at t = t_c, D = D_C, $\frac{dD}{dt} = 0$

from ⑩ $0 + k_2 D_c = k_1 L_0 e^{-k_1 t_c}$

$D_c = \frac{k_1}{k_2} L_0 e^{-k_1 t_c}$ (D_C : critical deficit)

* D = D_C 일때의 time (t_c)

from ⑪ $\frac{dD}{dt} = 0$

$$t_c = \frac{1}{k_2 - k_1} \ln[\frac{k_2}{k_1}(1 - D_0 \frac{k_2 - k_1}{k_1 L_0})]$$

↓

critical time

example 3-3 Applying the BOD sag curve

Θ_1 = 15,000 ㎥/d
BOD_5 (y_1) = 40 mg/ℓ
DO_1 = 2 mg/ℓ
T_1 = 25 ℃

2차 effluent

Θ_2 = 0.5 ㎥/s
BOD_5 (y_2) = 3 mg/ℓ
DO_2 = 8 mg/ℓ
T_2 = 22 ℃

* complete mixing
mixture velocity u = 0.2 m/sec
at 20 ℃ *BOD 반응상수 k_1 = 0.23/day
*reaeration 상수 k_2 = 0.4/day

100㎞ 까지 DO profile을 구하시오

<sol>

ⅰ) at mixing point

$Q = Q_1 + Q_2 = 0.67$ ㎥/s

$$BOD_5 = \frac{y_1 Q_1 + y_2 Q_2}{Q_1 + Q_2} = 12.4 \text{ mg/ℓ}$$

$$BOD_u(y_u) = L_0 = \frac{y}{1-\exp(-k_1 t)} = \frac{12.4}{1-\exp\{(-0.23)(5)\}} = 18.2 \text{ mg/ℓ}$$

$$DO = \frac{DO_1 Q_1 + DO_2 Q_2}{Q_1 + Q_2} = 6.3 \text{mg/ℓ}$$

$$T = \frac{T_1 Q_1 + T_2 Q_2}{Q_1 + Q_2} = 22.8 ℃$$

ⅱ) temperature에 의한 k_1, k_2의 보정

from eq (2-23) $k_T = k_{20}\,\Theta^{(T-20)}$

㉠ k_1의 경우 Θ = 1.047

㉡ k_2의 경우 Θ = 1.016

$k_1 = 0.23 \times 1.047^{(22.8-20)} = 0.26$/day

$k_2 = 0.4 \times 1.016^{(22.8-20)} = 0.42$/day

iii) 22.8 ℃에서 saturated DO (C_s)

from Appendix C-3

T(℃)	O2의 Henry's constant (104 atm.mol fraction)
20℃	4.01
30℃	4.75
22.8℃	4.217

$$X_{O_2} = \frac{P_{O_2}}{H_{O_2}} = \frac{0.21}{4.217\times10^4} = 4.9\times10^{-6} \text{ ------ ①}$$

$$X_{O_2} = \frac{n_{O_2}}{n_{O_2} + n_{liq}} \text{ ------ ②}$$

from ①② 1ℓ 기준 $4.9\times10^{-6} = \frac{n_{O_2}}{n_{O_2} + 55.6 gmol/\ell}$

$$n_{O_2} = 2.77\times10^{-4} gmol/\ell \left[\frac{32\times10^3 mg}{gmol}\right]$$

$$= 8.7 mg/\ell$$

iv) initial O_2 deficit D_0

D_0 = 8.7 - 6.5 = 2.2 mg/ℓ

v) $t_c = \frac{1}{k_2 - k_1} \ln[\frac{k_2}{k_1}(1 - D_0 \frac{k_2 - k_1}{k_1 L_0}] = 2.5 day$

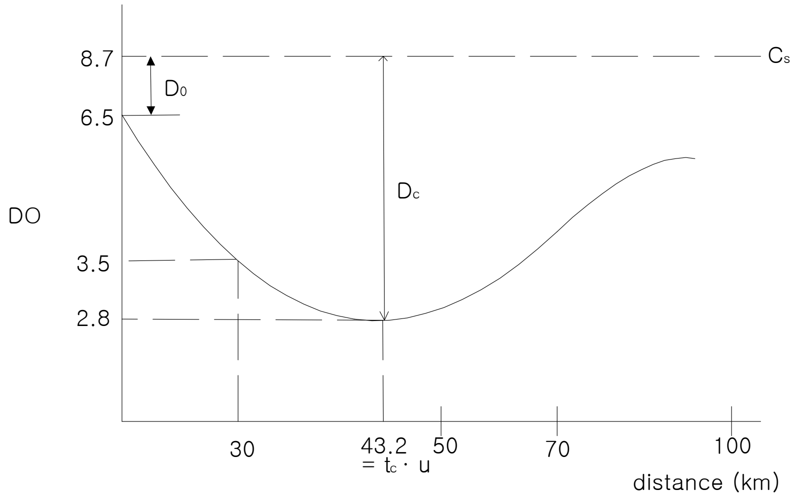
$$D_C = \frac{k_1}{k_2} L_0 e^{-k_1 t_c} = 5.9 \text{mg}/\ell$$

t_c 일때의 distance (χ) = u · t_c = 0.2 (m/s)(2.5 day) = 43.2 ㎞

vi) $D = \frac{k_1 L_0}{k_2 - k_1}(e^{-k_1 t} - e^{-k_2 t}) + D_0 e^{-k_2 t}$

distance (χ)	t=(u/χ) (day)	D (결핍량) (mg/ℓ)	DO (C_s - D)
0	0	2.2	6.5
20	1.16	5.1	3.4
43.2	2.5	5.9	2.8
70	4.3	5.2	3.5
100	5.8	4.1	4.6

vii) DO profile



* oxygen sag curve의 해석

i) critical deficit 의 position & magnitude

$= f\ (k_1,\ k_2,\ L_0,\ D_0,\ u)$

ii) t_c는 k_1, k_2 에 강하게 의존

iii) D_c는 L_0에 강하게 의존

* Linitations of the oxygen sag curve

→ detailed water - quality survey에 의한 다음 stream condition을 예측 할 수 있다.

1) BOD variables

i) BOD source 가 하나라고 가정 하였기에 부정확

ii) k_1 에 포함되지 않은 oxygen demand

· Algae respiration in the absence of sun light

· nitrification

· presence of sludge deposit

→ BOD ↑시킨다

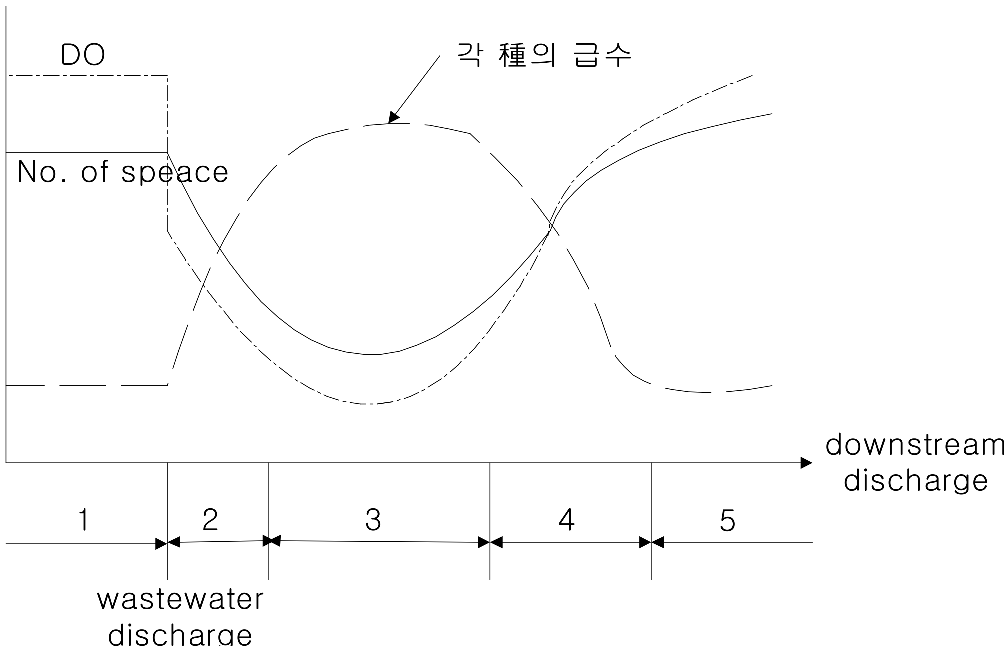
2) reaeration variables

i) algae photosynthesis

ii) river chanel이 steady-state로 가정

→ k_2는 flow characteristics에 의해 세분화 되어야 한다

3 – 11 Organic dischange & steram ecology



		physical 특성	chemical	biologyical
2	Degradation	· turbidity ↑	· DO ↓ (C_s의 40%까지)	· bacteria, protozoa mold 우세
3	active Degradation	· 탁도 더욱 ↑ · scum 형성 · septic condition	· DO ↓후↑ (C_s의 0~40%) · 혐기성 소화 gas 발생	· 혐기성 미생물수 peak
4	Recovery	· turbidity ↓	· DO 더욱 ↑ · nitrite, nitrate 생성	· protozoa appear
5	clean water	· 자연조선으로 회복	· DO is close to saturation	