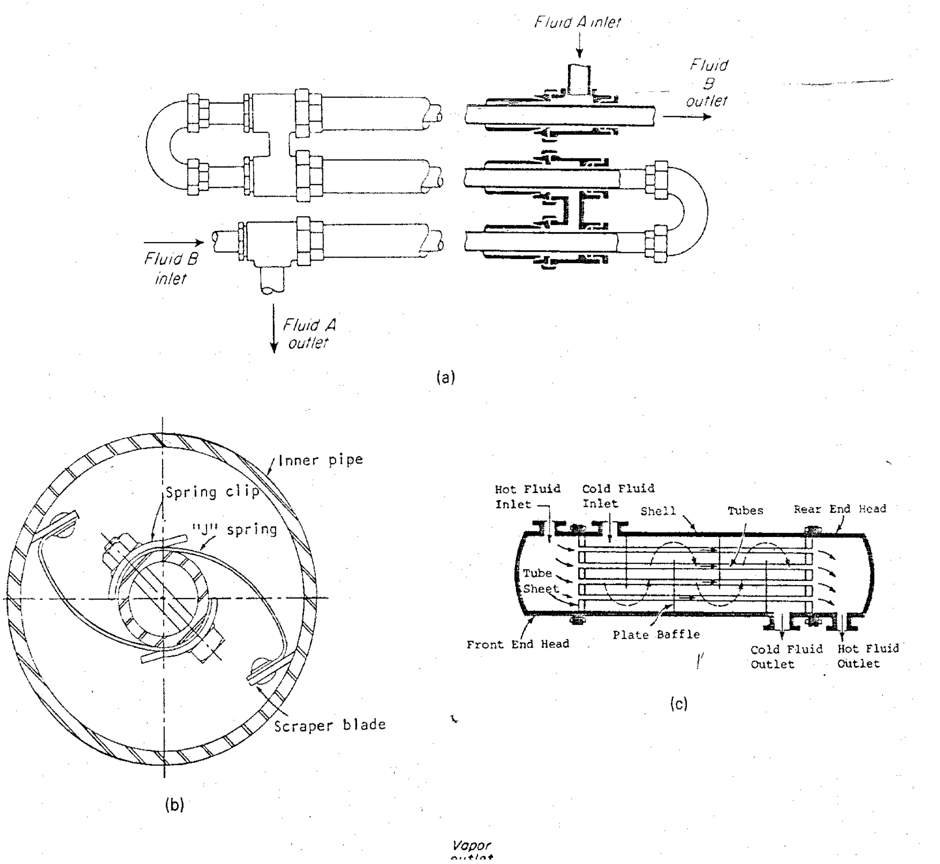
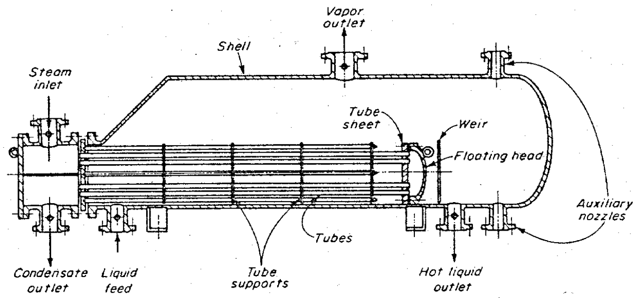
# Chapter 2

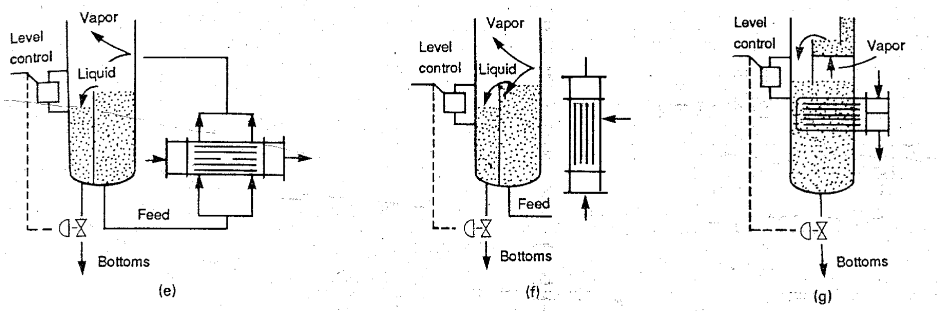
# Heat Transfer Equipments

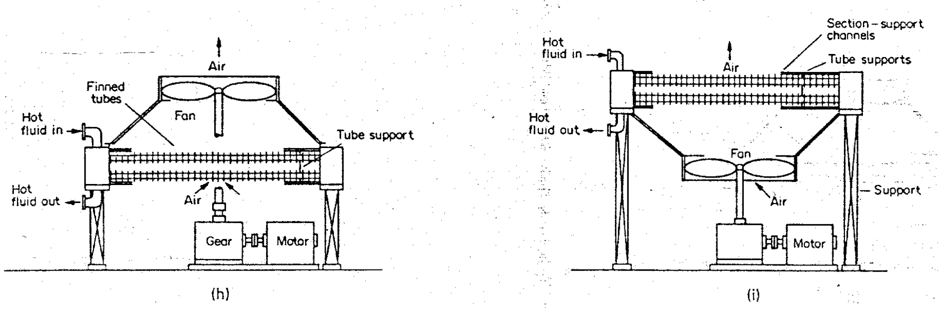
## 2.1 Convectional Heat Transfer, Heat Exchangers

### 2.1.1 Examples of Various Heat Exchangers



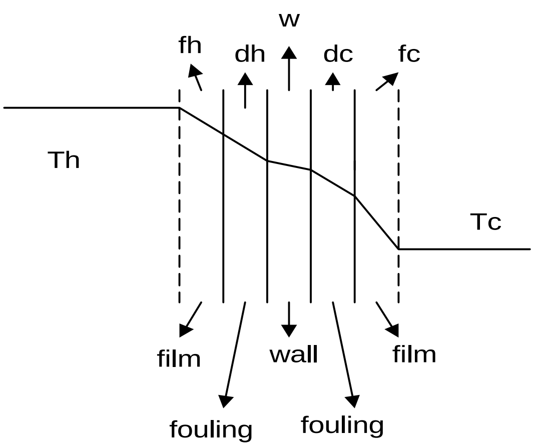


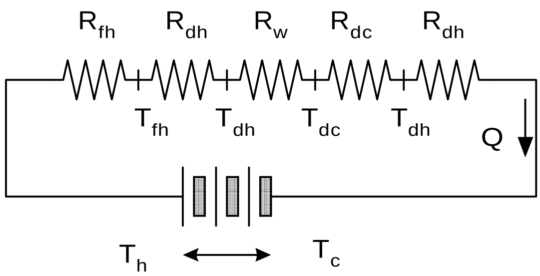




## 2.1.2 Basic Heat Balances

### Local and Overall Heat Transfer Coefficients



$$\begin{aligned} Q &= UA(T_h - T_c) = h_{fh}A(T_h - T_{fh}) = h_{dh}A(T_{fh} - T_{dh}) \\ &= h_wA(T_{dh} - T_{dc}) = h_{dc}(T_{dc} - T_{fc}) = h_{fc}(T_{fc} - T_c) \end{aligned} \tag{2.1}$$

$1/UA$, $1/h_iA$'s represent the thermal resistances, and $T_i - T_j$'s represent the driving forces (emfs).

Using the Ohm's law, total thermal resistance is represented by

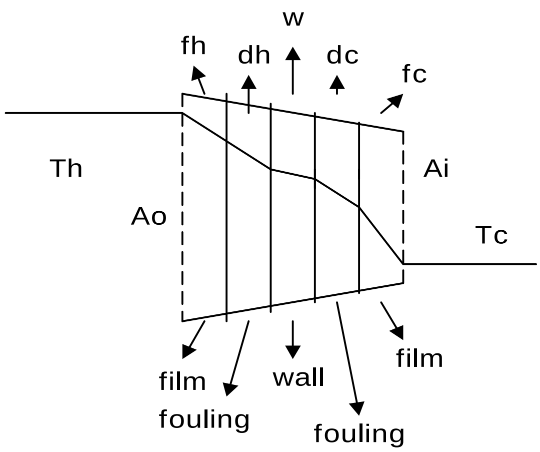
$$\frac{1}{UA} = \frac{1}{h_{fh}A} + \frac{1}{h_{dh}A} + \frac{1}{h_wA} + \frac{1}{h_{dc}A} + \frac{1}{h_{fc}A}$$

Here, $h_w$ is replaced by the following formula:

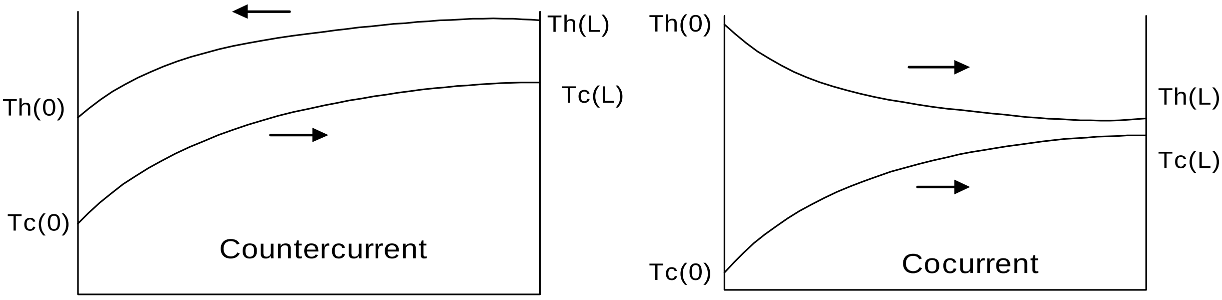
$$Q = k_wA\frac{\Delta T_w}{\Delta z_w} = \left(\frac{k_w}{\Delta z_w}A\right)\Delta T_w = (h_wA)\Delta T_w \quad \rightarrow \quad h_w = \frac{k_w}{\Delta z_w}$$

Eq. (2.1) holds for the case when heat transfer areas are different for respective thermal resistances. In this case, $A$'s are replaced by the associated heat transfer areas. Accordingly, two different overall heat transfer coefficients can be defined on basis of two different reference areas.

$$\frac{1}{U_iA_i} = \frac{1}{U_OA_O} = \frac{1}{h_{fh}A_{fh}} + \frac{1}{h_{dh}A_{dh}} + \frac{\Delta z_w}{k_wA_w} + \frac{1}{h_{dc}A_{dc}} + \frac{1}{h_{fc}A_{fc}}$$

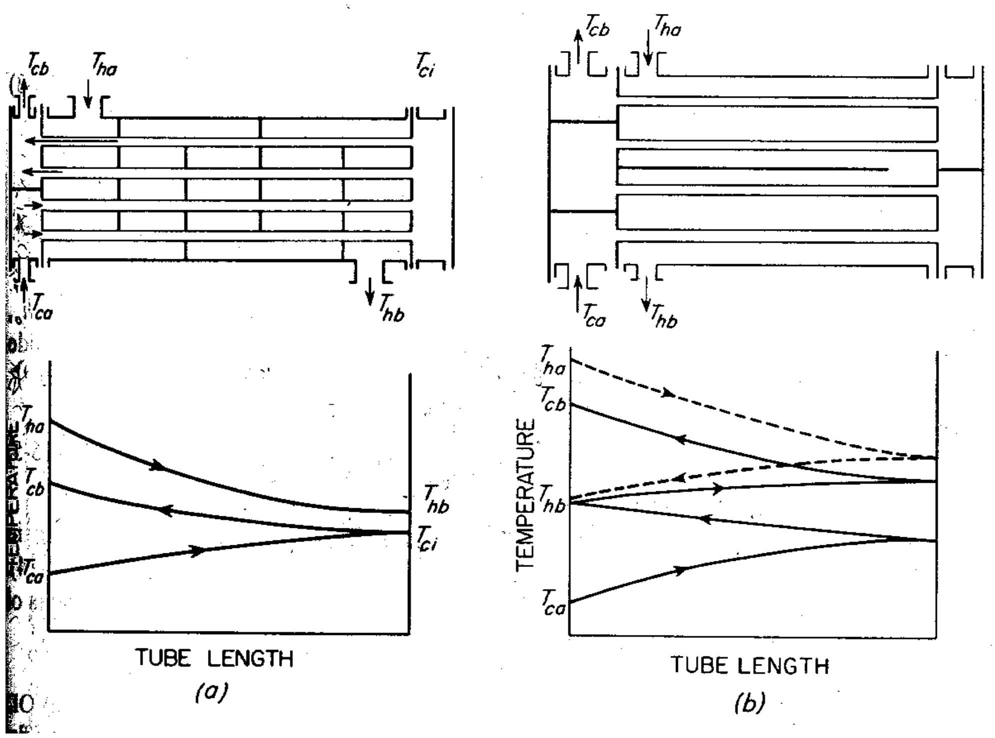



**Single-Pass Heat Exchanger**




$$
\begin{aligned}
Q &= UA\Delta T_{oa-lm} \quad \text{where} \quad \Delta T_{oa-lm} = \frac{(T_h(L) - T_c(L)) - (T_h(0) - T_c(0))}{\ln \frac{T_h(L)-T_c(L)}{T_h(0)-T_c(0)}} && (2.2) \\
&= m_c c_{pc}(T_c(L) - T_c(0)) && (2.3) \\
&= m_h c_{ph}(T_h(L) - T_h(0)) \quad \text{for cocurrent} && (2.4) \\
&= m_h c_{ph}(T_h(0) - T_h(L) \quad \text{for countercurrent} && (2.5)
\end{aligned}
$$

**Multi-Pass Heat Exchangers**



$$Q_T = UAF\Delta T_{oa-lm}$$

where

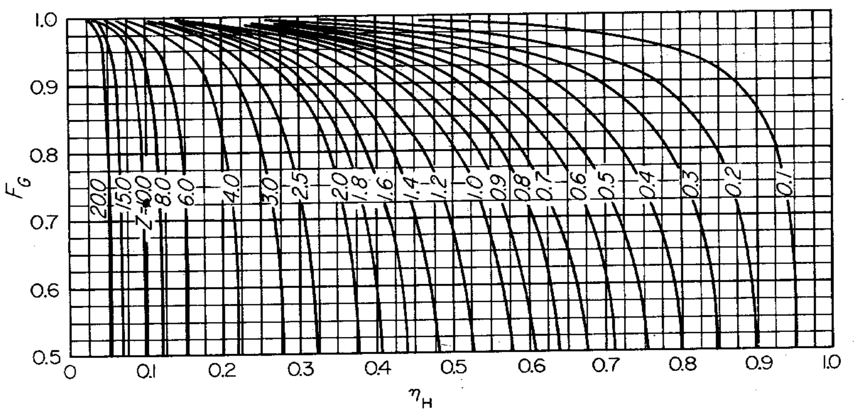
$$\Delta T_{oa-lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}, \quad \Delta T_1 = T_{hb} - T_{ca}, \quad \Delta T_2 = T_{ha} - T_{cb}$$

and $F$ is a correction factor given as a function of $Z$ and $\eta_H$(heating efficiency).

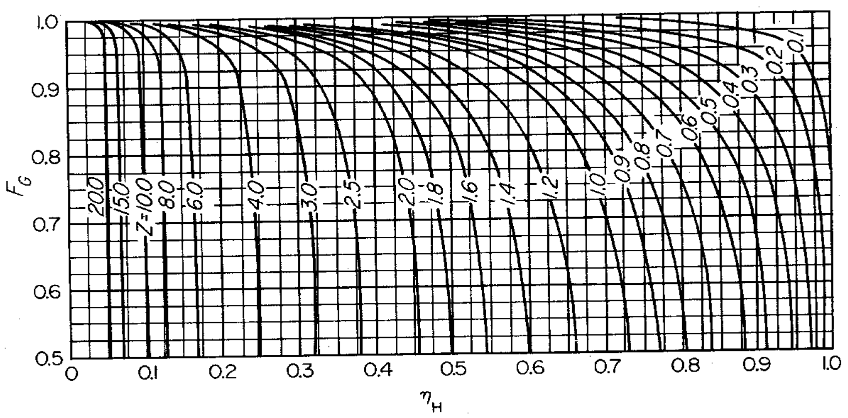
$$Z = \frac{T_{ha} - T_{hb}}{T_{cb} - T_{ca}} = \frac{m_c c_{pc}}{m_h c_{ph}}$$

$$\eta_H = \frac{T_{cb} - T_{ca}}{T_{ha} - T_{ca}} = \frac{\text{actual heat transfer}}{\text{maximum heat transfer}}$$

When $F < 0.7$, the heat exchanger is considered inefficient. Change the design.




(a)




(b)

**Heat Exchanger Design Problem**

Assumptions

- Hot stream is the process stream. $T_h(0),\ T_h(L),\ m_h$ are given.
- Cold stream is the utility stream. $T_c(0)$ is given.

The problem is to determine $A,\ m_c,\ T_c(L)$.

1. Total heat demand, $Q$, is computed using (2.4) or (2.5)
2. From (2.3), one can see that $m_c$ and $T_c(L)$ are not uniquely determined. They are dependent. If $m_c$ is increased, $T_c(L)$ is decreased, and vice versa. Usually, $T_c(L)$ is specified first and then $m_c$ is calculated accordingly.

   For cocurrent Hes, we call $T_h(L) - T_c(L)$ the *minimum temperature approach.*

   For countercurrent HEs, min temp approach $= \min(T_h(L) - T_c(L),\ T_h(0) - T_c(0))$.

   The minimum temperature approach is give to be larger than $10^oF$.
3. Finally,

$$A = \frac{Q}{U\Delta T_{oa-lm}}$$

### 2.1.3 Heat Transfer Coefficient

Discretion is needed in choosing the correct formula of the heat transfer coefficients.

Formulas are different according to

1. With phase change (condensation or vaporization) vs. without phase change
2. Laminar flow vs. turbulent flow
3. Parallel flow vs. cross (normal) flow
4. Natural convection vs. forced convection
5. Drop-wise condensation vs. film-type condensation
6. Nucleate boiling vs. film boiling

7. Packed bed, agitated vessel, $\cdots$

| Condition | $h$ $(Btu/hrft^2F)$ |
|---|---|
| Drop-wise condensation of steam | 10,000-20,000 |
| Film-type condensation of steam | 1,000-3,000 |
| Boiling water | 200-9,000 |
| Film-type condensation of organic vapors | 200-400 |
| Heating or cooling of water | 50-3,000 |
| Heating or cooling of organic solvents | 30-500 |
| Heating or cooling of oils | 10-120 |
| Superheated steam | 5-30 |
| Heating or cooling of air | 0.2-20 |

### 2.1.4 Pressure Drop

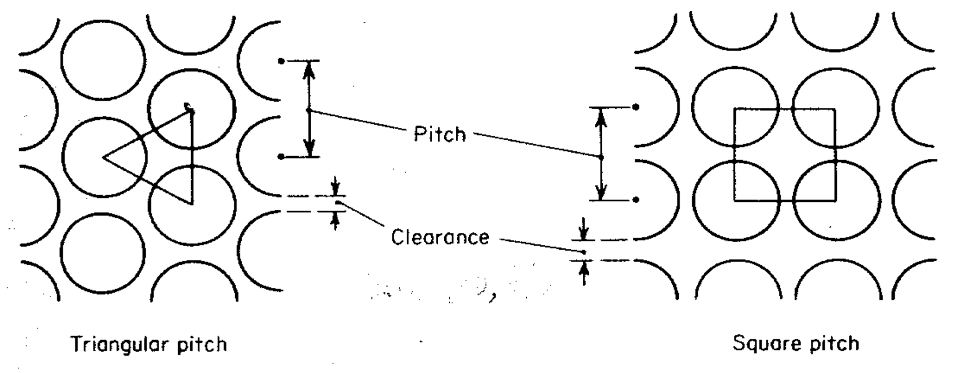
◊ Tube-side pressure drop

◊ Shell-side pressure drop

### 2.1.5 Considerations in Selection of Heat Transfer Equipment

#### Tube size and Pitch

Normally, $L$ = 8 or 12 or 16 ft; OD = 5/8" - 1.5" (mostly 3/4" or 1").



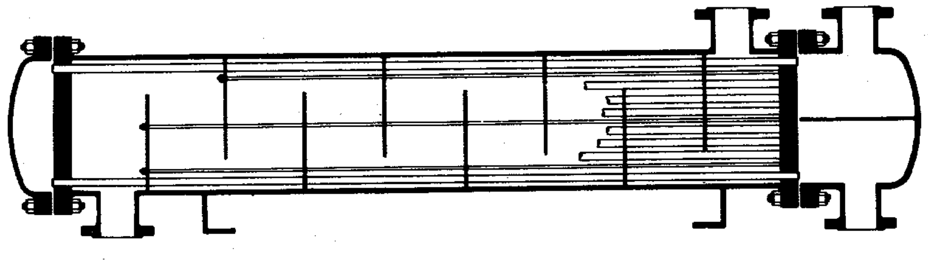
pitch = 1.25 - 1.5 OD, clearance $\geq$ max(1/4 OD, 3/16")

**Shell Size**

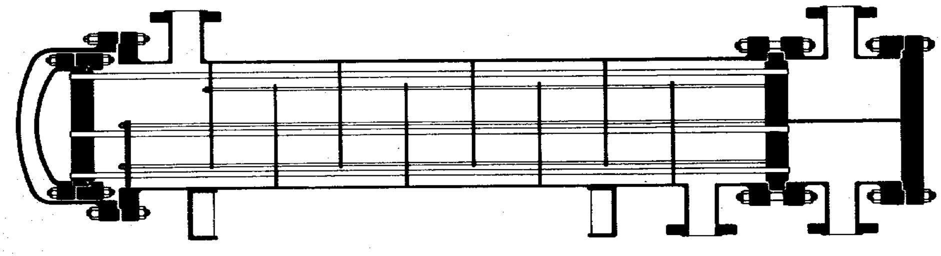
Usually nominal pipes with size up to 24" are used, but a larger size can be used as needed.

**Thermal Strain**

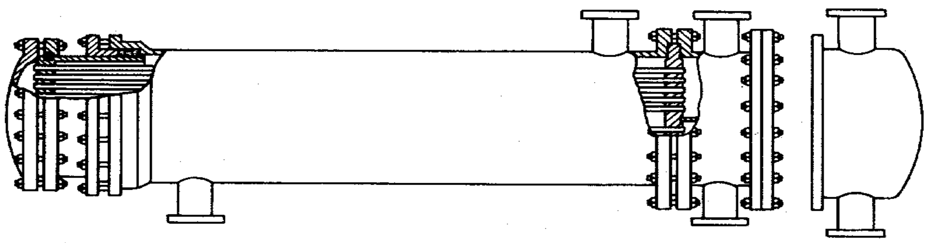
Fixed head type: used for max $\Delta T \leq 50^o F$



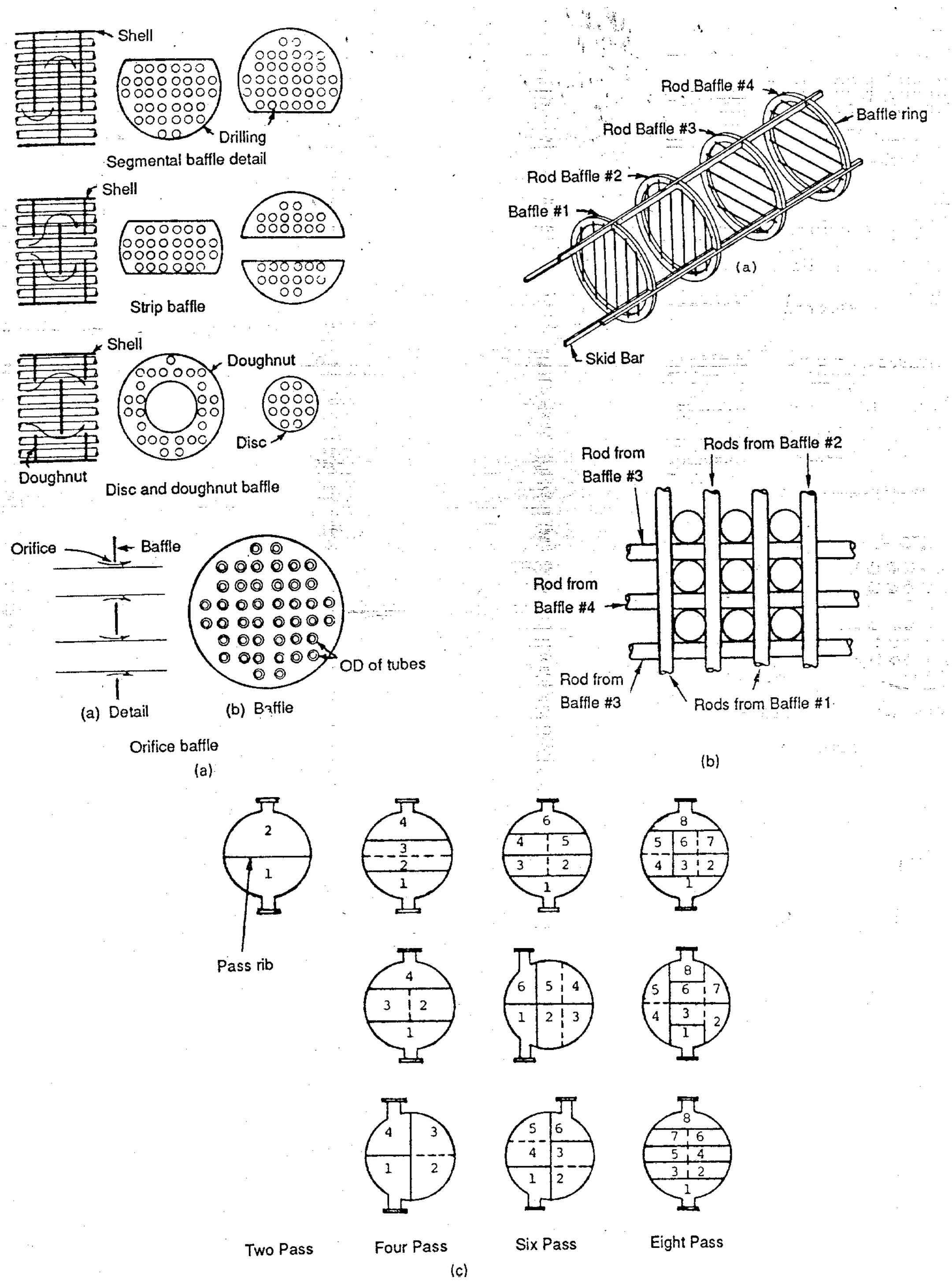
Internal floating head type



External floating head type with slip joint



## Arrangement of Cross Baffles and Tube-Side Passes




- The baffle spacing is determined between 1/4 - 1 shell diameter.
- The segmented baffle is most popular. The baffle height is normally designed as 75 % of the inside diameter of the shell (25% cut segmental baffle).

**Cleaning and Maintenance**

- Tube internal is easy to clean than external.
- Usually tubes are designed as removable bundles.

**Fluid Velocities and Location of Fluids**

- High velocity increases heat transfer ( $\rightarrow$ lowers heat transfer area). Also decreases fouling by sedimentation. But increases pumping cost.
- Location is determined under the consideration of fouling, corrosion, pressure drop, material cost, $\cdots$. As a rule, more corrosive and/or dirtier fluid is located in the tube side.

**Use of Water in Heat Exchangers**

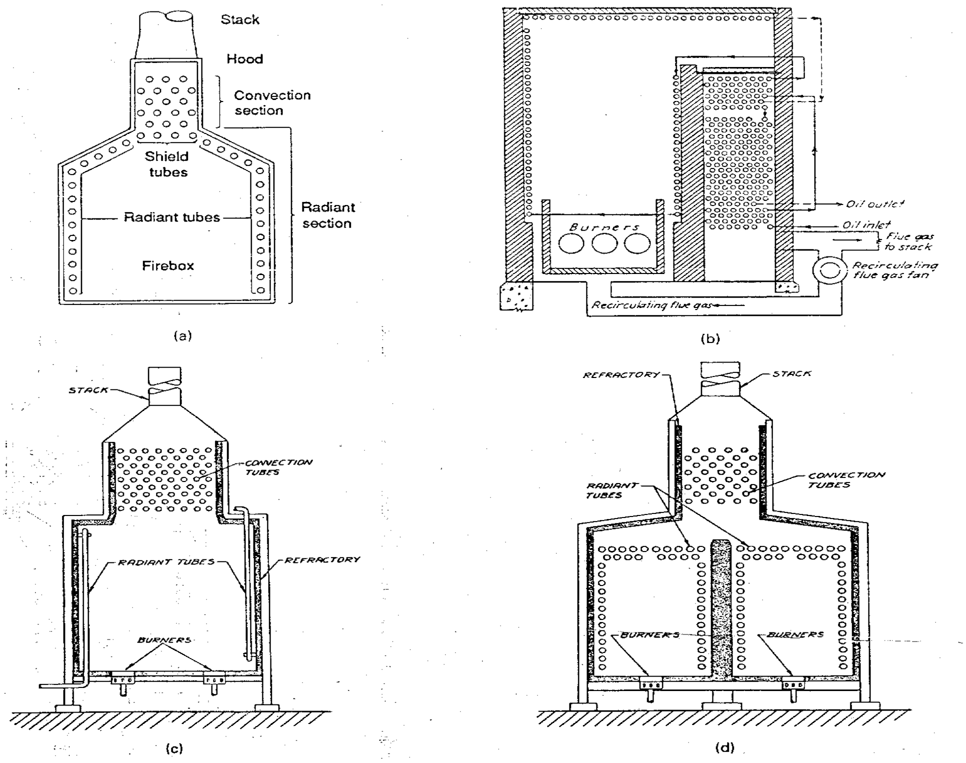
- At high temp., considerable corrosion action by dissolved oxygen.
- Fouling by dissolved minerals becomes excessive when $T \geq 50^o C$.
- Usually located in the tube side.
- velocity $\geq$ 3 ft/sec.

**Heating Media**

- Up to 150$^o$C - steam (** vapor pressure = 200 psig at 195$^o$C)
- Up to 330$^o$C - heat transfer oil such as Dow Therm or Mineral Oil
- Up to 550$^o$C - molten salt : 40% NaNO2 + 7% NaNO3+ 53% KNO3, melting point =146$^o$C

## 2.2 Radiational Heat Transfer

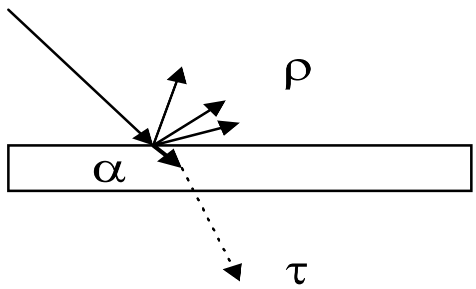
### 2.2.1 Examples of Process Furnaces




Crude oil heaters, boilers, various reactors, where thermal decomposition takes place, such as naphtha cracker, EDC $\rightarrow$ VCM, etc.

### 2.2.2 Fundamental Relationships

**Radiation falling on a body**




absorptivity($\alpha$) + transmissivity($\tau$) + reflectivity($\rho$) = 1
Black body : $\alpha = 1, \quad \tau = \rho = 0$

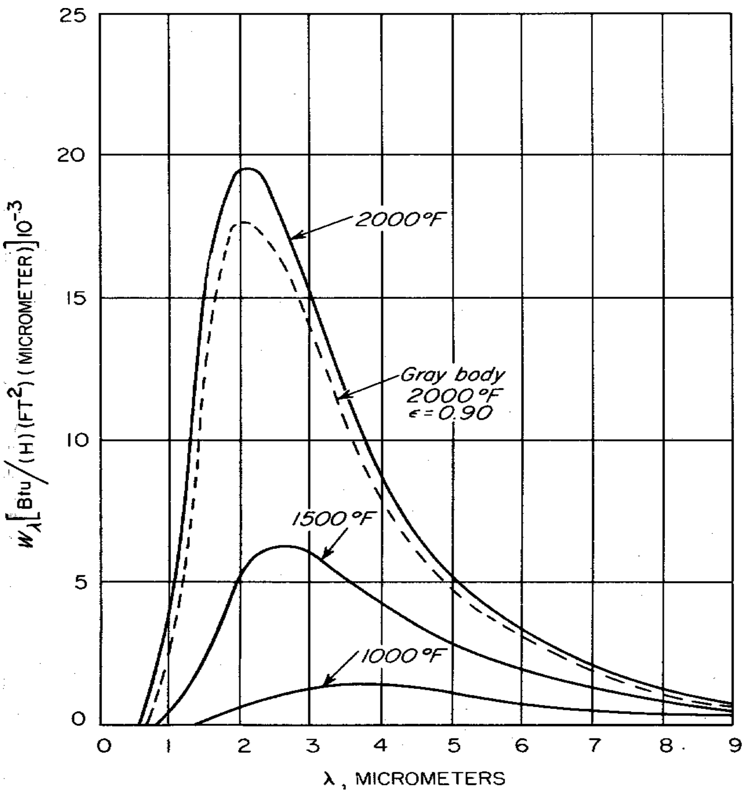
**Monochromatic radiation** from a *black body*:

$$E_b(\lambda)[\text{Btu/hr ft}^2\ \mu] = \frac{2\pi hc^2\lambda^{-5}}{e^{hc/k\lambda T} - 1}$$

where $h$ = Plank's constant, $k$ = Boltzmann's constant.

**Wien's displacement law**

$$\lambda_{max}T = \text{constant} = 2897.6\ \mu\ K$$




**Total radiation**

$$E_b[\text{Btu/hr ft}^2] = \int_0^\infty E_b(\lambda)d\lambda = \sigma T^4$$

where $\sigma = 0.1713 \times 10^{-8}[\text{Btu/hr ft}^2\ \text{R}^4]$ – Stephan-Boltzman constant.

**Emissivity**

$$\varepsilon(\lambda) = \frac{E(\lambda)}{E_b(\lambda)} = 0 \sim 1$$

**Gray body**: Emissivity is constant independent of $\lambda$.

$$E_g = \varepsilon \sigma T^4$$

**Under thermal equilibrium**,

$$\alpha(\text{absorptivity}) = \varepsilon(\text{emissivity})$$

holds for any non-grey or non-black body. Consider a body of a surface area $A$ and absorptivity $\alpha$ in a black enclosure. Let $E$ and $q_i$ be the radiant emission and radiant influx from and on the body per unit area. At equilibrium,

$$EA = q_i A \alpha$$

Assume that we replace the body with a black body of the same shape and size. In this case, it holds that

$$E_b A = q_i A$$

Taking the ratio, we have

$$\frac{E}{E_b} = \alpha$$

### 2.2.3 Radiation Shape Factor

**View factor**

Consider two black surfaces $A_1$ and $A_2$ shown in the figure. To find a general expression for the energy exchange between these surfaces, we first define the *radiation shape factor* (or *view factor*) as

$$F_{mn} = \text{fraction of energy leaving surface m which reaches surface n}$$

Since Net radiation between body 1 and body 2 = radiation from 1 to 2 - radiation from 2 to 1, we have

$$Q_{1-2} = E_{b1} A_1 F_{12} - E_{b2} A_2 F_{21}$$

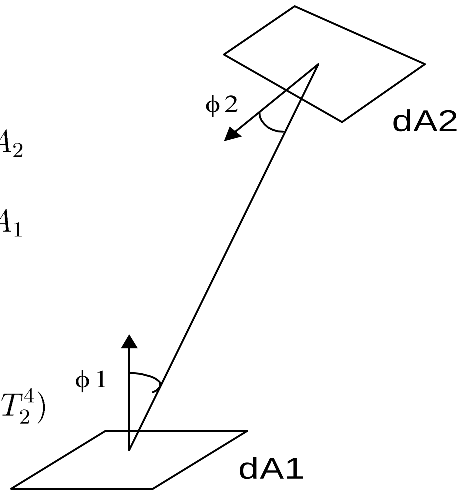
In the case that both surfaces are at the same temperature, $Q_{12} = 0$ and $E_{b1} = E_{b2}$. Hence,

$$A_1F_{12} = A_2F_{21} \quad \cdots \quad \text{reciprocity theorem}$$

Therefore,

$$Q_{12} = A_1F_{12}(E_{b1} - E_{b2}) = A_2F_{21}(E_{b1} - E_{b2})$$

Now,




$$\begin{aligned} dQ_{dA_1 \to dA_2} &= \frac{\sigma T_1^4}{\pi r^2} \cos\phi_1 dA_1 \cos\phi_2 dA_2 \\ dQ_{dA_2 \to dA_1} &= \frac{\sigma T_2^4}{\pi r^2} \cos\phi_2 dA_2 \cos\phi_1 dA_1 \end{aligned}$$

$$\begin{aligned} dQ_{12} &= dQ_{dA_1 \to dA_2} - dQ_{dA_2 \to dA_1} \\ &= \frac{\sigma}{\pi r^2} \cos\phi_1 dA_1 \cos\phi_2 dA_2 (T_1^4 - T_2^4) \end{aligned}$$

Therefore,

$$A_1F_{12} = A_2F_{21} = \int_{A_1}\int_{A_2} \frac{1}{\pi r^2} \cos\phi_1 \cos\phi_2 dA_1 dA_2$$

When $A_i, i = 2, 3, \cdots n$ completely surround the hemi-sphere over $A_1$,

$$F_{11} + F_{12} + \cdots + F_{1n} = 1$$

### 2.2.4 Heat Exchange between Non-black Bodies, Radiation Network Analysis

More complicated since incident radiation energy can be reflected back and forth several times.

Let

$$\begin{aligned} G &= \text{irradiation} = \text{total radiation incident upon a surface per unit time and unit area} \\ J &= \text{radiosity} = \text{total radiation which leaves a surface per unit time and unit area} \end{aligned}$$

$$J = \varepsilon E_b + \rho G$$

Assuming that $\tau = 0$, $\rho = 1 - \alpha = 1 - \varepsilon$. Therfore

$$J = \varepsilon E_b + (1 - \varepsilon) G$$

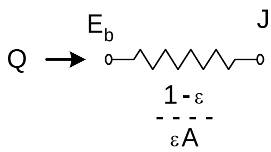
The net energy leaving the surface is

$$\frac{Q}{A} = J - G = \varepsilon E_b + (1 - \varepsilon) G - G$$

or

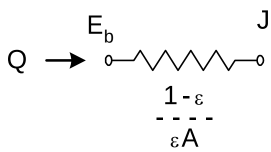
$$Q = \frac{E_b - J}{(1 - \varepsilon)/\varepsilon A} \quad \Leftrightarrow \quad \text{current} = \frac{\text{driving force}}{\text{resistance}}$$

We call $(1 - \varepsilon)/\varepsilon A$ the surface resistance.



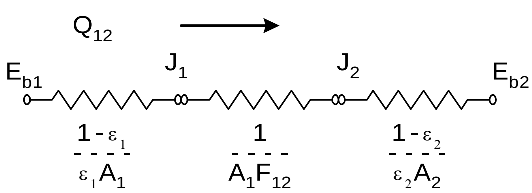

Now, we consider the exchange of radiation energy by two surface $A_1$ and $A_2$.

$$Q_{12} = A_1 F_{12} J_1 - A_2 F_{21} J_2 = A_1 F_{12} (J_1 - J_2) \quad \Rightarrow \quad Q_{12} = \frac{J_1 - J_2}{1/A_1 F_{12}}$$




We can now establish the radiation heat transfer network using the above basic formulas.

(Ex.1) Two surfaces which see each other and nothing else.



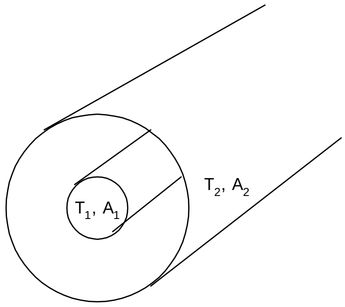
$$Q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + 1/A_1 F_{12} + (1-\varepsilon_2)/\varepsilon_2 A_2}$$

For infinite parallel plates, $A_1 = A_2 = A$ and $F_{12} = 1$. Hence,

$$\frac{Q_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

For tow long concentric cylinders,

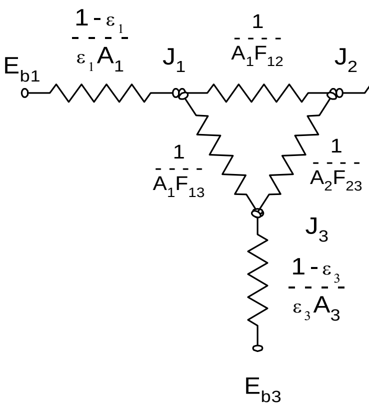
$$\frac{Q_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{1/\varepsilon_1 + (A_1/A_2)(1/\varepsilon_2 - 1)}$$



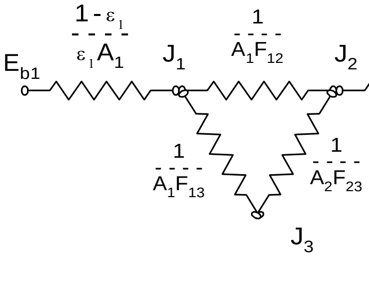

When $A_1/A_2 \to 0$,

$$\frac{Q_{12}}{A} = \varepsilon_1 \sigma(T_1^4 - T_2^4)$$

(Ex.2) Three-body problems




Three surfaces which see each other and nothing else




Two surfaces enclosed by a third surface which is nonconducting but re-radiating

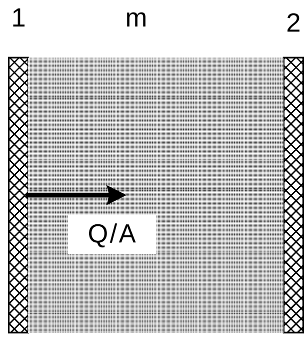
In the above,

$$F_{13} = 1 - F_{12}$$
$$F_{23} = 1 - F_{21}$$

(EX. 3) Radiation through an Absorbing and Transmitting Medium

Assume a situation where gas (absorbing and transmitting but non-reflecting) is filled between two parallel plates.

$$1 = \alpha_m + \tau_m = \varepsilon_m + \tau_m$$




Energy exchange between plate 1 and 2 is

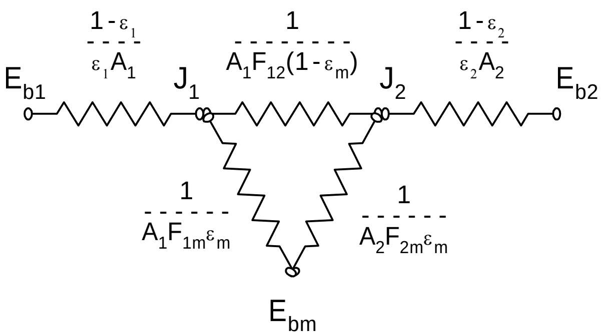
$$\begin{aligned} Q_{12} &= A_1F_{12}\tau_m J_1 - A_2F_{21}\tau_m J_2 = A_1F_{12}(1-\varepsilon_m)(J_1 - J_2) \\ &= \frac{J_1 - J_2}{1/A_1F_{12}(1-\varepsilon_m)} \end{aligned}$$

Energy exchange between plate 1 and the gas is

$$Q_{1m} = A_1F_{1m}\alpha_m J_1 - A_mF_{m1}\varepsilon_m E_{bm}$$

Using the fact that $A_mF_{m1} = A_1F_{1m}$ and $\alpha_m = \varepsilon_m$,

$$Q_{1m} = A_1F_{1m}\varepsilon_m(J_1 - E_{bm}) = \frac{J_1 - E_{bm}}{1/A_1F_{1m}\varepsilon_m}$$




Here, $F_{12} = F_{1m} = F_{2m} = 1$ and $E_{bm} = \sigma T_m^4$.

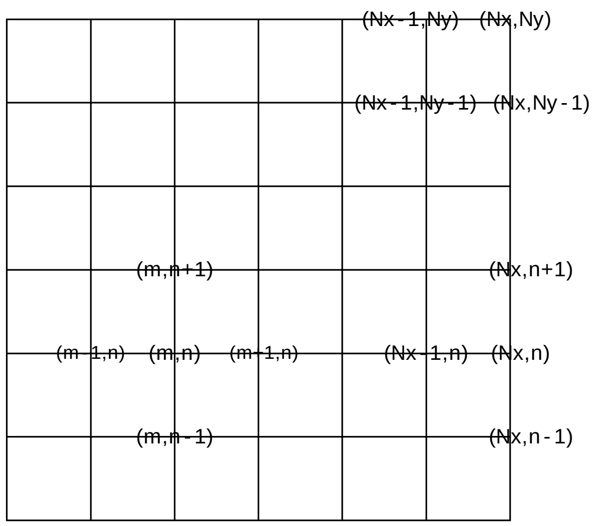
## 2.3 Conduction Heat Transfer

In this section, we introduce a numerical technique to solve the two-dimensional heat conduction problem given in project 2.

Problem

$$
\begin{aligned}
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} &= -\left(\frac{1}{kw}\right) q(x,y) \\
\text{with} \quad -k\frac{\partial T}{\partial x} &= h(T_a - T(0,y)) \;\; \text{at} \;\; x=0 \\
k\frac{\partial T}{\partial x} &= h(T_a - T(L,y)) \;\; \text{at} \;\; x=L \\
-k\frac{\partial T}{\partial y} &= h(T_a - T(x,0)) \;\; \text{at} \;\; y=0 \\
k\frac{\partial T}{\partial y} &= h(T_a - T(x,H)) \;\; \text{at} \;\; y=H
\end{aligned}
$$

**Finite Difference Approximation**




At the internal points $(m,n)$,

$$
\begin{aligned}
\left.\frac{\partial T}{\partial x}\right|_{(m,n)} &\approx \frac{1}{\Delta x}(T_{m+1/2,n} - T_{m-1/2,n}) \\
\left.\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)\right|_{(m,n)} &\approx \frac{1}{\Delta x}\left(\frac{1}{\Delta x}(T_{m+1,n} - T_{m,n}) - \frac{1}{\Delta x}(T_{m,n} - T_{m-1,n})\right) \\
&= \frac{1}{\Delta x^2}(T_{m+1,n} - 2T_{m,n} + T_{m-1,n})
\end{aligned}
$$

If we choose $\Delta x = \Delta y$ and define

$$\bar{q}(x,y) = \frac{\Delta x^2 \Delta y^2}{kw} q(x,y)$$

Then we have

$$T_{m,n} = \frac{1}{4}\left(T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1} + \bar{q}_{m,n}\right) \tag{2.6}$$

At the boundary points $(Nx, n)$,

$$k\Delta y \frac{T_{Nx-1,n} - T_{Nx,n}}{\Delta x} + k\frac{\Delta x}{2}\left(\frac{T_{Nx,n+1} - T_{Nx,n}}{\Delta y} + \frac{T_{Nx,n-1} - T_{Nx,n}}{\Delta y}\right) = \Delta y h (T_{Nx,n} - T_a)$$

Since $\Delta x = \Delta y$, rearranging gives

$$T_{Nx,n} = \frac{1}{h\Delta h/k + 2}\left(T_{Nx-1,n} + \frac{T_{Nx,n+1}}{2} + \frac{T_{Nx,n-1}}{2} + \frac{h\Delta y}{k}T_a\right) \tag{2.7}$$

At the boundary points $(Nx, Ny)$,

$$k\frac{\Delta y}{2}\frac{T_{Nx-1,Ny} - T_{Nx,Ny}}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{Nx,Ny-1} - T_{Nx,Ny}}{\Delta y} = \Delta y h (T_{Nx,Ny} - T_a)$$

Hence,

$$T_{Nx,Ny} = \frac{1}{2(h\Delta h/k + 1)}\left(T_{Nx-1,Ny} + T_{Nx,Ny-1} + 2\frac{h\Delta y}{k}T_a\right) \tag{2.8}$$

**Gauss-Siedel Iteration**

$N_x \times N_y$ gird temperatures are unknowns. The FDM can be rearranged to a linear simultaneous equation

$$Ax = b \quad \text{with} \quad x^T = [T_{1,1}\ T_{1,2}\ \cdots\ T_{Nx,NY}]$$

When $N_x = N_y = 100$, $\dim(x) = 10,000$ and $A$ becomes $10,000 \times 10,000$. Therefore, solution by matrix inversion is intractable. $\Rightarrow$ Gauss-Siedel Iteration

Using the identity,

$$Ax = b \quad \Rightarrow \quad x = (I - A)x + b$$

we consider an iteration scheme

$$x_{k+1} = (I - A)x_k + b \quad \rightarrow \quad \text{Gauss-Siedel Iteration}$$

$\{x_k\}$ converges to a limit only when $\rho(I - A) = |\lambda_{max}(I - A)| < 1$.

In fact, (2.6)-(2.8) are written for GS iteration.