Lecture 12. Transport in Membranes (2)

- Module Flow Patterns
	- Perfect mixing
	- Countercurrent flow
	- Cocurrent flow
	- Crossflow
- Membrane Cascades
- External Mass-Transfer Resistances
- Concentration Polarization and Fouling

Module Flow Patterns

- The flow pattern can significantly affect the degree of separation and the membrane area
- For countercurrent and cocurrent flow, permeate fluid at a given location on the downstream side consists of fluid that has just passed through the membrane plus the permeate fluid flowing to that location
- For crossflow, there is no flow of permeate fluid along the membrane

Crossflow Pattern (1)

The pressure ratio, r=P_P/P_F, and the ideal separation factor, $\alpha^\star_{\mathsf{A},\mathsf{B}}$, are assumed constant At the differential element, local mole fractions in the retentate and permeate are x_i and y_i existence of the pressure ratio, $r = p_p / P_F$, and the ideal separation factor, $\alpha^*_{A,B}$, are assumed constant assumed constant At the differential element, local mole fractions in the retentate and permeate $\frac{V_H}{\frac{N_H}{N$ é ù *n* $e^{i\theta}$
bate
 $\left(\frac{n_{\scriptscriptstyle P}}{n_{\scriptscriptstyle F}}\right)$

Penetrant molar flux: dn/dA_M (n_F) $θ$ **: cut, fraction of feed permeated**

F

 n_F)

• Expression for the local permeate composition

$$
\frac{y_A}{1 - y_A} = \alpha_{A,B}^* \left[\frac{x_A - ry_A}{(1 - x_A) - r(1 - y_A)} \right]
$$

• Material balance for A around the differential-volume element

A the differential element, local mole fractions in the retentate and permeate
\n
$$
\frac{P_F \frac{n_1 x_i}{\text{Plug flow}} \text{Differential } \frac{n_2 x_i}{\text{Volumet}}}{\text{Volumet } \frac{d\lambda_M}{\text{Volumet } \frac
$$

Crossflow Pattern (2)

• Integration from the intermediate location of the differential element to the final retentate (from n to ${\sf n}_{\sf R}$; from ${\sf x}_{\sf A}$ to ${\sf x}_{\sf R_{\sf A}}$) **Crossflow Pattern (2)**

from the intermediate location of the differential elemententate (from n to $n_{\overline{n}}$; from x_A to $x_{\overline{n}_A}$)
 $\frac{x_A}{x_{R_A}}\sqrt{\frac{1-x_{R_A}}{1-x_A}}\left(\frac{1-x_{R_A}}{1-x_A}\right)^{\left(\frac{1}{\alpha-1}\right)}$

on of A in the final **Crossflow Pattern (2)**
the intermediate location of the differentic
te (from n to n_R; from x_A to x_{RA})
 $\left(\frac{1}{\alpha-1}\right)\left(\frac{1-x_{R_A}}{1-x_A}\right)^{\left(\frac{1}{\alpha-1}\right)}$ **Crossflow Pattern (2)**

on from the intermediate location of the differential element to

retentate (from n to n_R; from x_A to x_{R_A})
 $\left[\left(\frac{x_A}{x_{R_A}}\right)^{\left(\frac{1}{\alpha-1}\right)}\left(\frac{1-x_{R_A}}{1-x_A}\right)^{\left(\frac{1}{\alpha-1}\right)}\right]$

ction of A **Crossflow Pattern (2)**
gration from the intermediate location of the differential element to
final retentate (from n to n_B; from x_A to x_{BA})
 $= n_R \left[\left(\frac{x_A}{x_{R_A}} \right)^{\left(\frac{1}{\alpha - 1} \right)} \left(\frac{1 - x_{R_A}}{1 - x_A} \right)^{\left(\frac{1}{\alpha -$

Crossflow Pattern
tegration from the intermediate location of the dif
ne final retentate (from n to
$$
n_B
$$
; from x_A to x_{B_A})

$$
n = n_R \left[\left(\frac{x_A}{x_{R_A}} \right)^{\left(\frac{1}{\alpha - 1} \right)} \left(\frac{1 - x_{R_A}}{1 - x_A} \right)^{\left(\frac{1}{\alpha - 1} \right)} \right]
$$

ole fraction of A in the final permeate

• Mole fraction of A in the final permeate

Crossflow Pattern (2)
\nintegration from the intermediate location of the differential element to
\nne final retentate (from n to n_R; from x_A to x_{R_A})
\n
$$
n = n_R \left[\left(\frac{x_4}{x_{R_A}} \right)^{\left(\frac{1}{\alpha - 1} \right)} \left(\frac{1 - x_{R_A}}{1 - x_A} \right)^{\left(\frac{1}{\alpha - 1} \right)} \right]
$$
\nAole fraction of A in the final permeate
\n
$$
y_{P_A} = \int_{x_{P_A}}^{x_{R_A}} y_A \, dn / \theta n_F = x_{R_A}^{\left(\frac{1}{1 - \alpha} \right)} \left(\frac{1 - \theta}{\theta} \right) \left[(1 - x_{R_A})^{\left(\frac{\alpha}{\alpha - 1} \right)} \left(\frac{x_{P_A}}{1 - x_{P_A}} \right)^{\left(\frac{\alpha}{\alpha - 1} \right)} - x_{R_A}^{\left(\frac{\alpha}{\alpha - 1} \right)} \right]
$$
\nDifferential rate of mass transfer of
\nacross the membrane
\n
$$
y_A \, dn = \frac{P_{M_A} dA_M}{I_M} (x_A P_F - y_A P_P) \qquad A_M = \int_{x_{R_A}}^{x_{P_A}} \frac{I_M y_A \, dn}{P_{M_A} (x_A P_F - y_A P_P)}
$$

• Differential rate of mass transfer of A across the membrane

$$
y_A d\boldsymbol{n} = \frac{P_{M_A} dA_M}{l_M} (x_A P_F - y_A P_P)
$$

• Total membrane surface area

$$
(1 - x_{F_A})
$$

\n
$$
[1 - x_{F_A}]
$$

\n
$$
A_M = \int_{x_{F_A}}^{x_{F_A}} \frac{l_M y_A dn}{P_{M_A}(x_A P_F - y_A P_P)}
$$

Cascades (1)

- Cascades: aggregates of stages
	- Accomplish separations that cannot be achieved in a single stage
	- $-$ Reduce the amounts of separating $\qquad \text{Feed}$ agent required
	- Make efficient use of raw materials
- Two or more streams are intimately contacted
	- \rightarrow Promote rapid mass and heat transfer
	- \rightarrow The separated phases leaving the stage approach equilibrium

Cascades (2)

- Single section of stages
	- Streams entering and leaving are only from the ends
	- Used to recover components from a feed stream

Linear countercurrent

Linear crosscurrent

- Two sections of stages
	- Consist of one section above the feed and one below
	- Used to make a sharp separation between two selected feed components, key components

Membrane Cascades (1)

- Membrane-separation systems often consist of multiplemembrane modules because a single module may not be large enough to handle the required feed rate
	- A number of modules of identical size in parallel with retentates and permeates from each module combined
	- The parallel units function as a single stage
	- In multiple stages, the combined retentate from each stage becomes the feed for the next stage

Membrane Cascades (2)

- Single-stage membrane-separation process read
- : a single membrane module or a number of such modules arranged in parallel or in series without recycle

- The extent to which a feed mixture can be separated is limited and determined by the separation factor, α
- The separation factor depends on module flow patterns, the permeability ratio (ideal separation factor), the cut (θ) , and the driving force for membrane mass transfer
- To improve purity and recovery, membrane stages are cascaded with recycle

Membrane Cascades (3)

• Multiple-stage countercurrent recycle cascades

- Permeate is enriched in components of high permeability in an enriching section
- The retentate is enriched in components of low permeability in a stripping section
- $-$ For a cascade, additional factors that affect the degree of separation are the number of stages and the recycle ratio (permeate recycle rate/permeate product rate)

Membrane Cascades (4)

- It is best to manipulate the cut and reflux at each stage so as to force compositions of the two streams entering each stage to be identical (ideal)
	- : This corresponds to the least amount of entropy production for the cascade and, thus, the highest second-law efficiency
- In the case of gas permeation, compression costs are high \rightarrow often limited to just two or three stages

Membrane Cascades (5)

Two-stage enriching cascade with additional premembrane stage

- Addition of a premembrane stage may be attractive when
	- (1) feed concentration is low in the component to be passed preferentially through the membrane
	- (2) desired permeate purity is high
	- (3) separation factor is low
	- (4) a high recovery of the more permeable component is desired

External Mass-Transfer Resistances (1)

- When mass-transfer resistances external to read side membrane the membrane are not negligible,
	- Gradients exist in the boundary layers (or films) adjacent to the membrane surfaces
	- Reduces the driving force for mass transfer across the membrane and, therefore, the flux of penetrant

- Gas permeation by solution-diffusion is slow compared to diffusion in gas boundary layers or films
	- \rightarrow external mass-transfer resistances are negligible
- Membrane processes involving liquid (dialysis, reverse osmosis, pervaporation): diffusion in liquid boundary layers and films is slow
	- \rightarrow concentration polarization (accumulation of non-permeable species on the upstream surface of the membrane) cannot be neglected

External Mass-Transfer Resistances (2)

• Mass transfer of liquids with a porous membrane (at steady state)

External Mass — Transfer Results of Results	
Mass transfer of liquids with a porous membrane (at steady state)	
$N_i = k_{i_r}(c_{i_r} - c_{i_0}) = \frac{D_{e_i}}{I_M}(c_{i_0} - c_{i_1}) = k_{i_r}(c_{i_t} - c_{i_r})$	Feed side membrane
$N_i = \frac{c_{i_r} - c_{i_p}}{1 + \frac{I_M}{D_{e_i}} + \frac{I_M}{R_{i_p}}}$	Ans. Answer coefficients
Mass transfer of liquids with a nonporous membrane (at steady state)	
$N_i = k_{i_r}(c_{i_r} - c_{i_0}') = \frac{K_i D_i}{I_M}(c_{i_0}' - c_{i_1}') = k_{i_r}(c_{i_1}' - c_{i_p})$	Feed side membrane
$N_i = \frac{c_{i_r} - c_{i_p}}{1 + \frac{I_M}{I_M} + \$	

External Mass-Transfer Resistances (3)
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 $\frac{M}{D_{e_i}}\left(\text{or } \frac{I_M}{K_i D_i}\right), \frac{1}{K_{i_p}}$
and on fluid properties. flow**ansfer Resistances (3)**
 $\frac{1}{\sum_{i_r}, \frac{I_M}{D_{e_i}}\left(\text{or } \frac{I_M}{K_i D_i}\right), \frac{1}{k_{i_r}}}$
epend on fluid properties, flow-

• Resistances to mass flux:
$$
\frac{1}{k_{i_r}}
$$
, $\frac{l_M}{D_{e_i}} \left(\text{or } \frac{l_M}{K_i D_i} \right)$, $\frac{1}{k_{i_p}}$

- **External Mass-Transfer Resistances (3)**
• Resistances to mass flux: $\frac{1}{k_{i_r}} \cdot \frac{l_{w}}{R_{i_r}} \left(\text{or } \frac{l_{w}}{R_{i,D_i}} \right) \cdot \frac{1}{k_{i_r}}$
• Mass transfer coefficients depend on fluid properties, flow-
channel geometry, and flow **nsfer Resistances (3)**
 l<u>u</u> (or $\frac{l_{M}}{R_{\rho_{e}}}\left(\text{or } \frac{l_{M}}{K_{i}D_{i}}\right)$, $\frac{1}{k_{i_{e}}}$
and on fluid properties, flow-
gime **ransfer Resistances (3)**
 $\frac{1}{k_{i_r}}, \frac{l_{M}}{D_{e_i}} \left(\text{or } \frac{l_{M}}{K_i D_i} \right), \frac{1}{k_{i_r}}$
depend on fluid properties, flow-
regime **sfer Resistances (3)**
 $\frac{\left(\text{or } \frac{I_M}{K_i D_i}\right) \frac{1}{k_{i_p}}}{\frac{1}{K_i D_i}}$

d on fluid properties, flow-

d
 $\frac{I_M}{K_i D_i}$ and he replaced by $\frac{I_M}{K_i}$ and $\frac{1}{K_i}$ • Mass transfer coefficients depend on fluid properties, flowchannel geometry, and flow regime $\left(\text{or } \frac{l_{M}}{K_{i}D_{i}}\right), \frac{1}{k_{i_{p}}}$
d on fluid properties, flc
me
<u>ind</u> can be replaced by
tion of mass-transfer co
- Membrane resistances $\frac{l_M}{R}$ and $\frac{l_M}{r_R}$ can be replaced by $\frac{l_M}{R}$ or $\frac{1}{R}$ *i* M and M can hor $\frac{\bm l_M}{\bm D_{e_i}}$ and $\frac{\bm l_M}{\bm K_i\bm D_i}$ can be replace *M* can ha ranlacad l_M con ho *i* M or $\frac{1}{\sqrt{2}}$ M_i M_i l_M or 1 P_{M_i} P_{M_i} **1**
- **ISTER Resistances (3)**
 $\frac{L}{k_i} \left(\text{or } \frac{I_M}{K_i D_i} \right)$, $\frac{1}{k_i}$

Ind on fluid properties, flow-

Ime
 $\frac{I_M}{K_i D_i}$ can be replaced by $\frac{I_M}{P_{M_i}}$ or $\frac{1}{\overline{P}_{M_i}}$

ation of mass-transfer coefficients • The empirical film-model correlation of mass-transfer coefficients for channel flow vistances to mass flux: $\frac{1}{k_{i_r}} \cdot \frac{l_{st}}{D_{e_i}} \left(\text{or } \frac{l_{st}}{K_i D_i} \right)$, $\frac{1}{k_{i_r}}$

ss transfer coefficients depend on fluid properties

innel geometry, and flow regime

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transfer coefficients depend on fluid properties,
nel geometry, and flow regime
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inel geometry, and flow regime

prane resistances $\frac{I_M}{D_e}$ and $\frac{I_M}{K_i D_i}$ can be replaced

mpirical film-model correlation of mass-transf

iannel flow
 $N_{\rm th} = k_i d_H/D_i$ $\frac{1}{k_{i_r}}$, $\frac{l_{M}}{D_{e_i}}\left(\text{or } \frac{l_{M}}{K_i D_i}\right)$, $\frac{1}{k_{i_p}}$
depend on fluid properties, flow-
w regime
and $\frac{l_{M}}{K_i D_i}$ can be replaced by $\frac{l_{M}}{P_{M_i}}$ or $\frac{1}{\overline{P}_{M_i}}$
correlation of mass-transfer coefficients $\frac{l_M}{D_{e_i}}$ and $\frac{l_M}{K_i D_i}$ can be replanded correlation of mass-transformation of mass-transformation of mass-transformation of mass $\frac{b}{Re} N_{\text{Sc}}^{0.33} (d_H/L)^d$ Fraction of mass-transfer cooks $\frac{l_M}{l_{R_e}}$ and $\frac{l_M}{K_i D_i}$ can be replaced by $\frac{l}{P}$
del correlation of mass-transfer cooks $\frac{b}{Re} N_{Sc}^{0.33} (d_H/L)^d$ ass flux: $\frac{1}{k_{i_r}}$, $\frac{l_M}{D_{e_i}}$ (or $\frac{l_M}{K_i D_i}$), $\frac{1}{k_{i_r}}$
 Hicients depend on fluid properties, flow-
 nces $\frac{l_M}{D_{e_i}}$ and $\frac{l_M}{K_i D_i}$ can be replaced by $\frac{l_M}{P_{M_i}}$ or $\frac{1}{P_{M_i}}$
 -model correlation

$$
N_{\rm Sh} = k_i d_H / D_i = a N_{\rm Re}^b N_{\rm Sc}^{0.33} (d_H / L)^d
$$

d_H: hydraulic diameter *v* : velocity

Concentration Polarization and Fouling (1)

- Concentration polarization occurs in membrane separators when the membrane is permeable to A, but relatively impermeable to B
	- Molecules of B are carried by bulk flow to the upstream surface of the membrane, where they accumulate, causing their concentration at the surface of the membrane to increase in a polarization layer
	- $-$ The equilibrium concentration of B in this layer is reached when its back-diffusion to the bulk fluid on the feed-retentate side equals its bulk flow toward the membrane

Concentration Polarization and Fouling (2)

- Concentration polarization is most common in pressure-driven membrane separations involving liquids, such as reverse osmosis and ultrafiltration, where it reduces the flux of A
	- The polarization effect can be serious if the concentration of B reaches its solubility limit on the membrane surface
	- \rightarrow A precipitate of gel may form, the result being fouling on the membrane surface or within membrane pores, with a further reduction in the flux of A

• The most straightforward way of minimizing concentration polarization is to reduce the film thickness by increasing turbulent mixing at the membrane surface