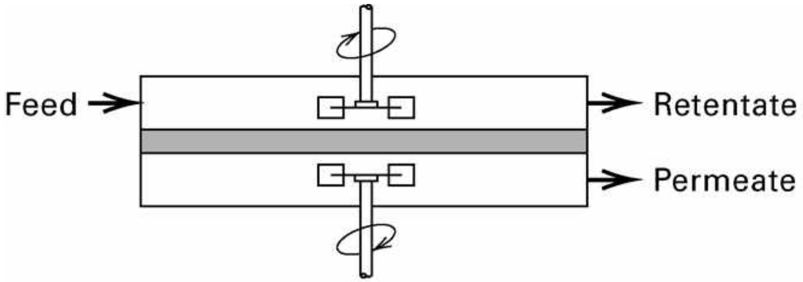
Lecture 12.
Transport in Membranes (2)

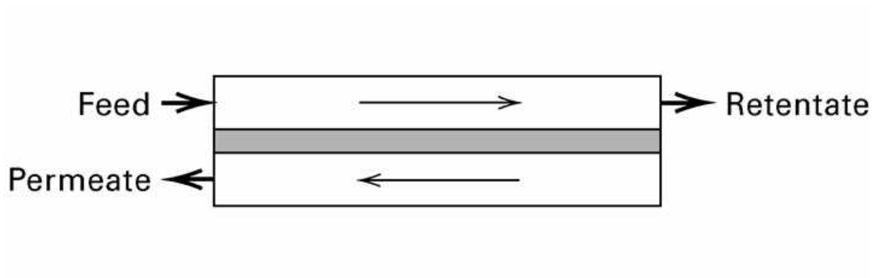
- Module Flow Patterns
  - Perfect mixing
  - Countercurrent flow
  - Cocurrent flow
  - Crossflow
- Membrane Cascades
- External Mass-Transfer Resistances
- Concentration Polarization and Fouling

Module Flow Patterns

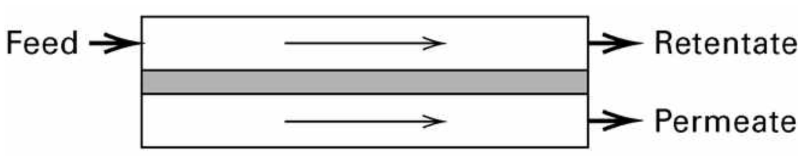
Perfect mixing



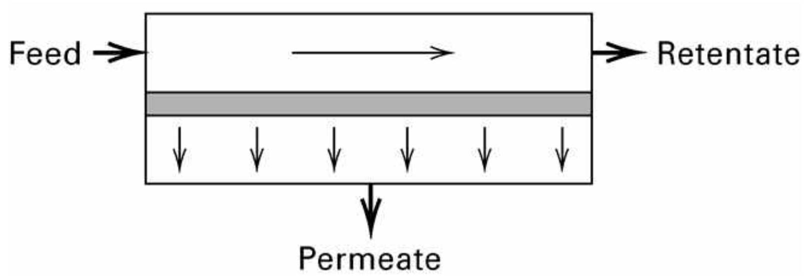
Countercurrent flow



Cocurrent flow

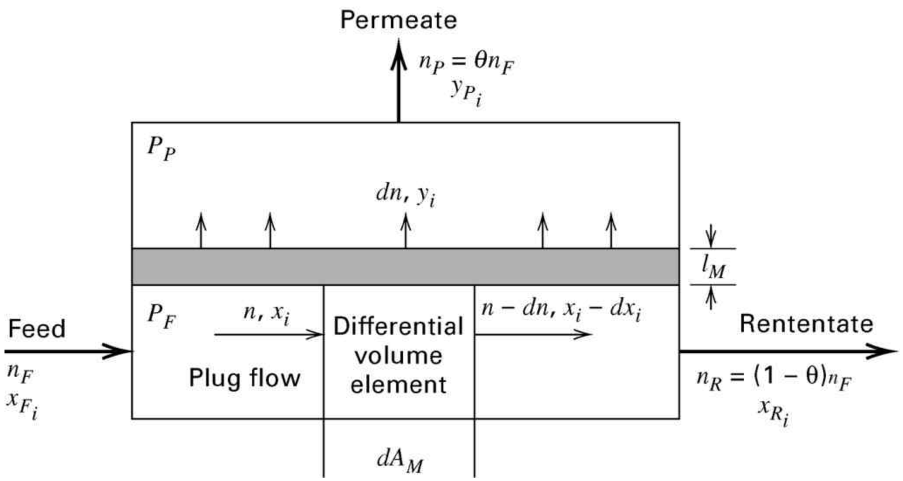


Crossflow



- The flow pattern can significantly affect the degree of separation and the membrane area
- For countercurrent and cocurrent flow, permeate fluid at a given location on the downstream side consists of fluid that has just passed through the membrane plus the permeate fluid flowing to that location
- For crossflow, there is no flow of permeate fluid along the membrane

Crossflow Pattern (1)



The pressure ratio, $r=P_P/P_F$, and the ideal separation factor, $\alpha^*_{A,B}$, are assumed constant

At the differential element, local mole fractions in the retentate and permeate are x_i and y_i

θ: cut, fraction of feed permeated $\left(\dfrac{n_P}{n_F}\right)$

Penetrant molar flux: dn/dA_M

- Expression for the local permeate composition

$$\frac{y_A}{1-y_A} = \alpha^*_{A,B}\left[\frac{x_A - ry_A}{(1-x_A)-r(1-y_A)}\right]$$

- Material balance for A around the differential-volume element

$$y_A dn = d(nx_A) = x_A dn + ndx_A$$

$$\frac{dn}{n} = \frac{dx_A}{y_A - x_A} = \left[\frac{1+(\alpha-1)x_A}{x_A(\alpha-1)(1-x_A)}\right]dx_A$$

$$\alpha = \alpha^*_{A,B}$$

Crossflow Pattern (2)

- Integration from the intermediate location of the differential element to the final retentate (from n to n_R; from x_A to x_{RA})

$$n = n_R\left[\left(\frac{x_A}{x_{R_A}}\right)^{\left(\frac{1}{\alpha-1}\right)}\left(\frac{1-x_{R_A}}{1-x_A}\right)^{\left(\frac{1}{\alpha-1}\right)}\right]$$

- Mole fraction of A in the final permeate

$$y_{P_A} = \int_{x_{F_A}}^{x_{R_A}} y_A\, dn/\theta n_F \;= x_{R_A}^{\left(\frac{1}{1-\alpha}\right)}\left(\frac{1-\theta}{\theta}\right)\left[(1-x_{R_A})^{\left(\frac{\alpha}{\alpha-1}\right)}\left(\frac{x_{F_A}}{1-x_{F_A}}\right)^{\left(\frac{\alpha}{\alpha-1}\right)} - x_{R_A}^{\left(\frac{\alpha}{1-\alpha}\right)}\right]$$

- Differential rate of mass transfer of A across the membrane

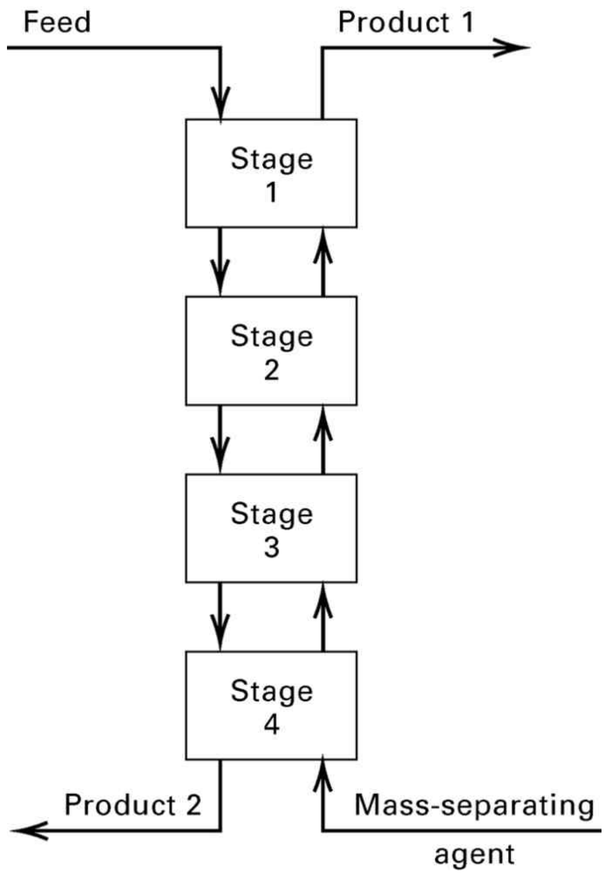
$$y_A dn = \frac{P_{M_A} dA_M}{l_M}(x_A P_F - y_A P_P)$$

- Total membrane surface area

$$A_M = \int_{x_{R_A}}^{x_{F_A}} \frac{l_M y_A dn}{P_{M_A}(x_A P_F - y_A P_P)}$$

Cascades (1)

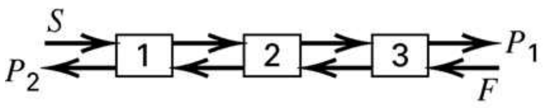
- Cascades: aggregates of stages
  - Accomplish separations that cannot be achieved in a single stage
  - Reduce the amounts of separating agent required
  - Make efficient use of raw materials
- Two or more streams are intimately contacted
  - → Promote rapid mass and heat transfer
  - → The separated phases leaving the stage approach equilibrium



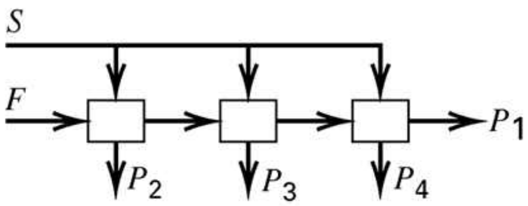
Cascades (2)

- Single section of stages
  - Streams entering and leaving are only from the ends
  - Used to recover components from a feed stream

Linear countercurrent

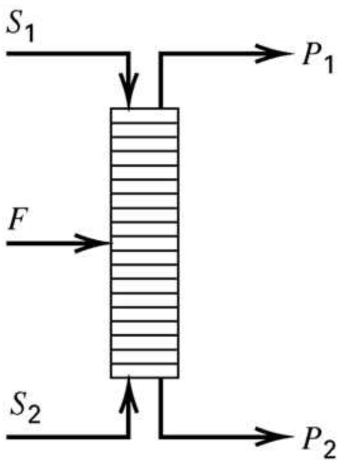


Linear crosscurrent



- Two sections of stages
  - Consist of one section above the feed and one below
  - Used to make a sharp separation between two selected feed components, key components

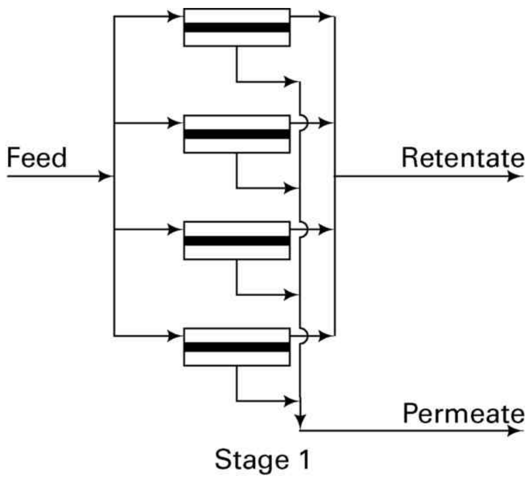
Two-section, countercurrent



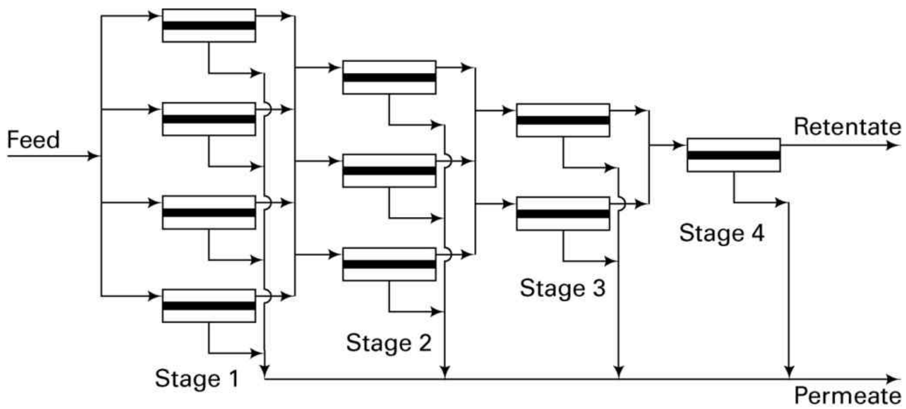
Membrane Cascades (1)

- Membrane-separation systems often consist of multiple-membrane modules because a single module may not be large enough to handle the required feed rate
  - A number of modules of identical size in parallel with retentates and permeates from each module combined
  - The parallel units function as a single stage
  - In multiple stages, the combined retentate from each stage becomes the feed for the next stage

One stage

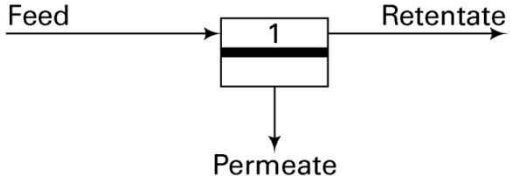


Multiple stage



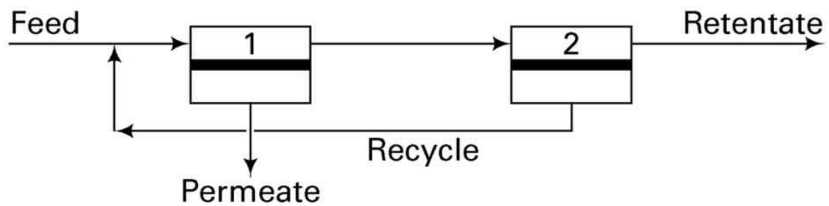
Membrane Cascades (2)

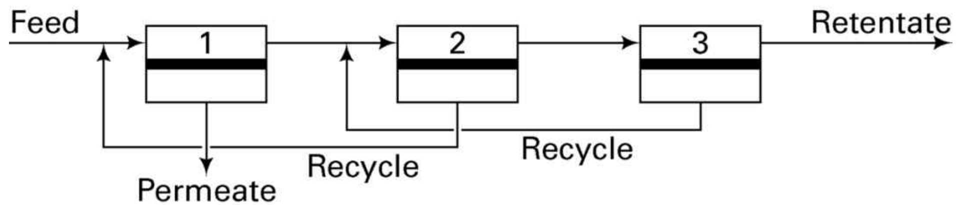
- Single-stage membrane-separation process
  : a single membrane module or a number of such modules arranged in parallel or in series without recycle



  - The extent to which a feed mixture can be separated is limited and determined by the separation factor, α
  - The separation factor depends on module flow patterns, the permeability ratio (ideal separation factor), the cut (θ), and the driving force for membrane mass transfer

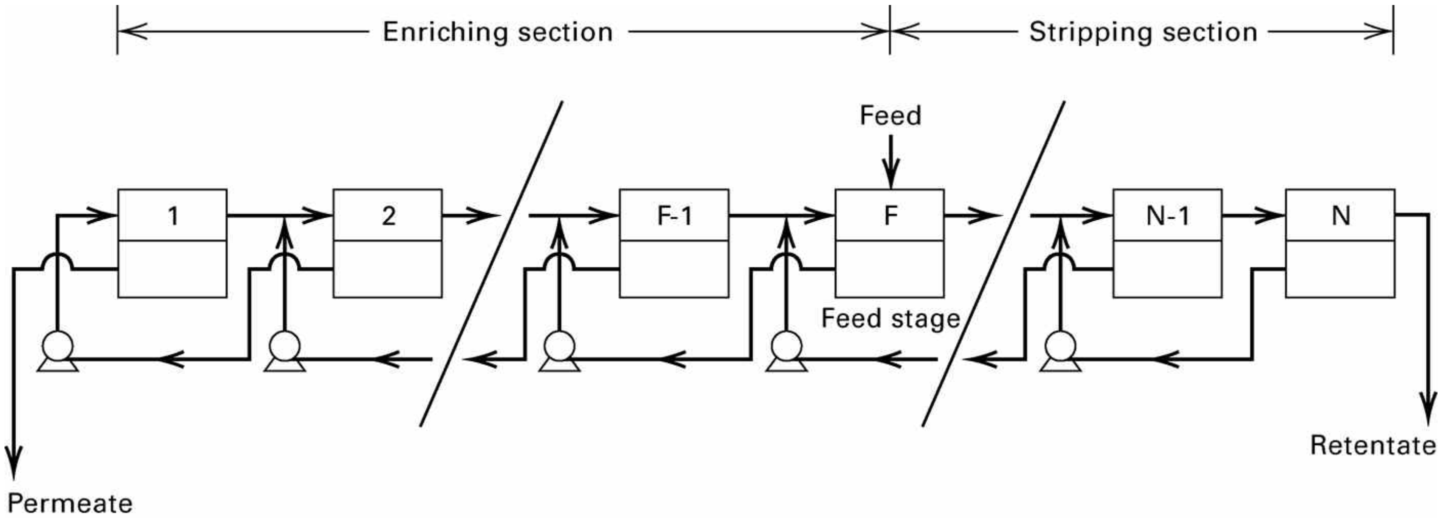
- To improve purity and recovery, membrane stages are cascaded with recycle





Membrane Cascades (3)

- Multiple-stage countercurrent recycle cascades



- Permeate is enriched in components of high permeability in an enriching section
- The retentate is enriched in components of low permeability in a stripping section
- For a cascade, additional factors that affect the degree of separation are the number of stages and the recycle ratio (permeate recycle rate/permeate product rate)

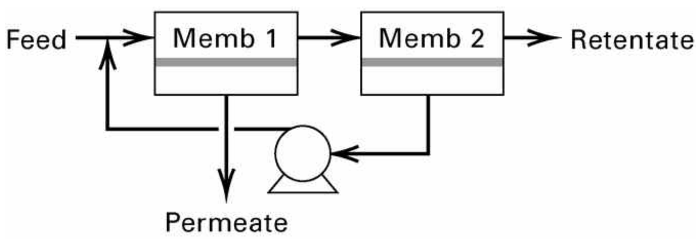
Membrane Cascades (4)

- It is best to manipulate the cut and reflux at each stage so as to force compositions of the two streams entering each stage to be identical (ideal)

  : This corresponds to the least amount of entropy production for the cascade and, thus, the highest second-law efficiency

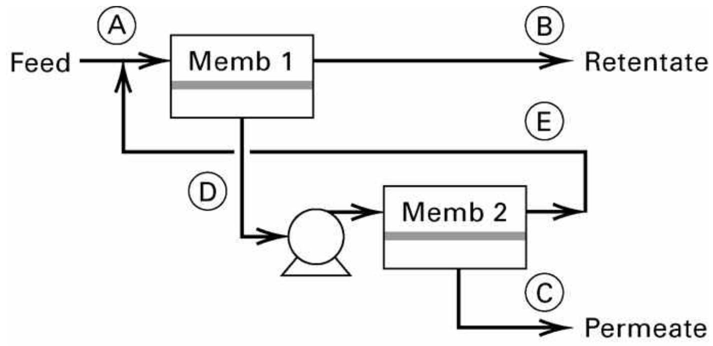
- In the case of gas permeation, compression costs are high → often limited to just two or three stages

Two-stage stripping cascade



– Designed to obtain a purer retentate

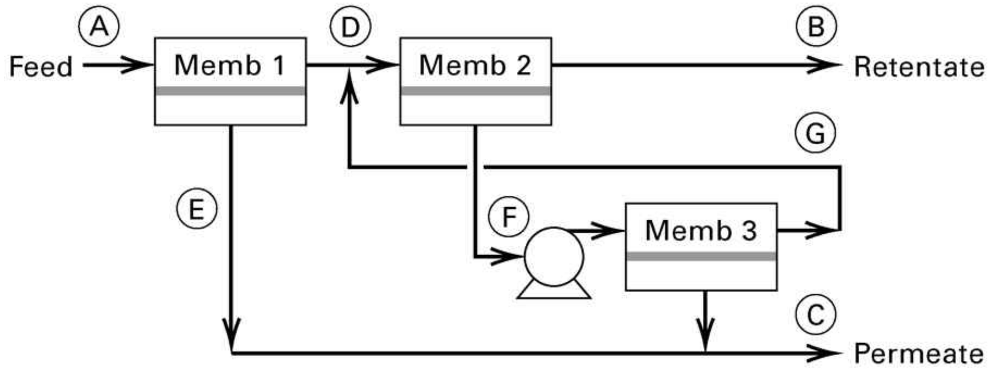
Two-stage enriching cascade



– Designed to obtain a purer permeate

Membrane Cascades (5)

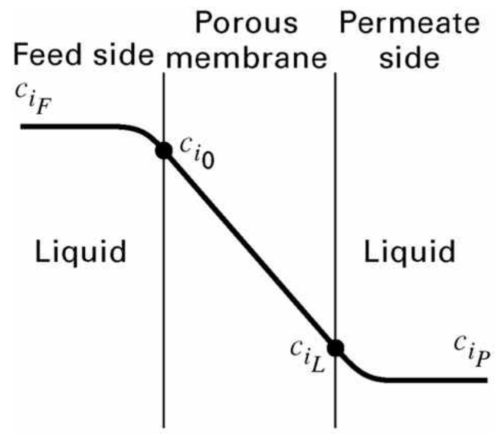
Two-stage enriching cascade with additional premembrane stage



- Addition of a premembrane stage may be attractive when
  - (1) feed concentration is low in the component to be passed preferentially through the membrane
  - (2) desired permeate purity is high
  - (3) separation factor is low
  - (4) a high recovery of the more permeable component is desired

External Mass-Transfer Resistances (1)

- When mass-transfer resistances external to the membrane are not negligible,
  - Gradients exist in the boundary layers (or films) adjacent to the membrane surfaces
  - Reduces the driving force for mass transfer across the membrane and, therefore, the flux of penetrant



- Gas permeation by solution-diffusion is slow compared to diffusion in gas boundary layers or films

  → external mass-transfer resistances are negligible

- Membrane processes involving liquid (dialysis, reverse osmosis, pervaporation): diffusion in liquid boundary layers and films is slow

  → concentration polarization (accumulation of non-permeable species on the upstream surface of the membrane) cannot be neglected

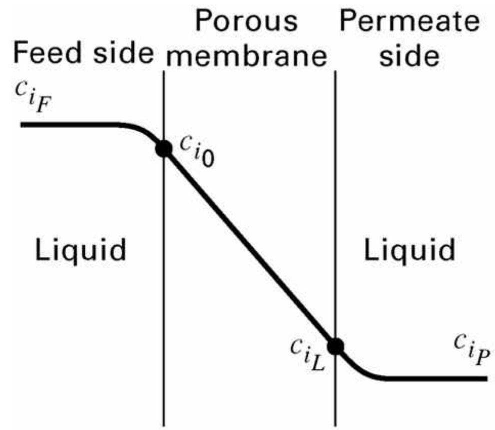
External Mass-Transfer Resistances (2)

- Mass transfer of liquids with a porous membrane (at steady state)

$$N_i = k_{i_F}(c_{i_F} - c_{i_0}) = \frac{D_{e_i}}{l_M}(c_{i_0} - c_{i_L}) = k_{i_P}(c_{i_L} - c_{i_P})$$

$$N_i = \frac{c_{i_F} - c_{i_P}}{\dfrac{1}{k_{i_F}} + \dfrac{l_M}{D_{e_i}} + \dfrac{1}{k_{i_P}}}$$

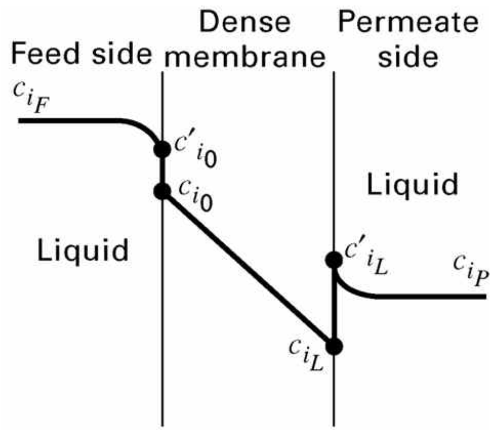
k_{iF} & k_{iP}
: mass-transfer coefficients



- Mass transfer of liquids with a nonporous membrane (at steady state)

$$N_i = k_{i_F}(c_{i_F} - c'_{i_0}) = \frac{K_i D_i}{l_M}(c'_{i_0} - c'_{i_L}) = k_{i_P}(c'_{i_L} - c_{i_P})$$

$$N_i = \frac{c_{i_F} - c_{i_P}}{\dfrac{1}{k_{i_F}} + \dfrac{l_M}{K_i D_i} + \dfrac{1}{k_{i_P}}}$$



External Mass-Transfer Resistances (3)

- Resistances to mass flux: $\frac{1}{k_{i_F}}, \frac{l_M}{D_{e_i}}\left(\text{or } \frac{l_M}{K_i D_i}\right), \frac{1}{k_{i_P}}$
- Mass transfer coefficients depend on fluid properties, flow-channel geometry, and flow regime
- Membrane resistances $\frac{l_M}{D_{e_i}}$ and $\frac{l_M}{K_i D_i}$ can be replaced by $\frac{l_M}{P_{M_i}}$ or $\frac{1}{\bar{P}_{M_i}}$
- The empirical film-model correlation of mass-transfer coefficients for channel flow

$$N_{\mathrm{Sh}} = k_i d_H / D_i = a N_{\mathrm{Re}}^b N_{\mathrm{Sc}}^{0.33} (d_H / L)^d$$

$$N_{\mathrm{Re}} = d_H v \rho / \mu$$

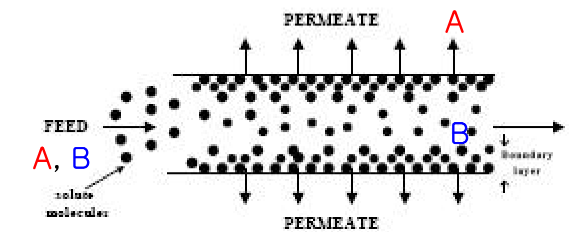
$$N_{\mathrm{Sc}} = \mu / \rho D_i$$

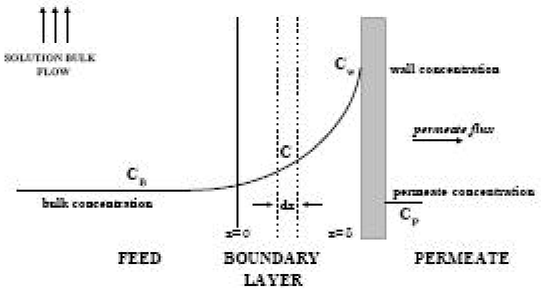
d_H : hydraulic diameter

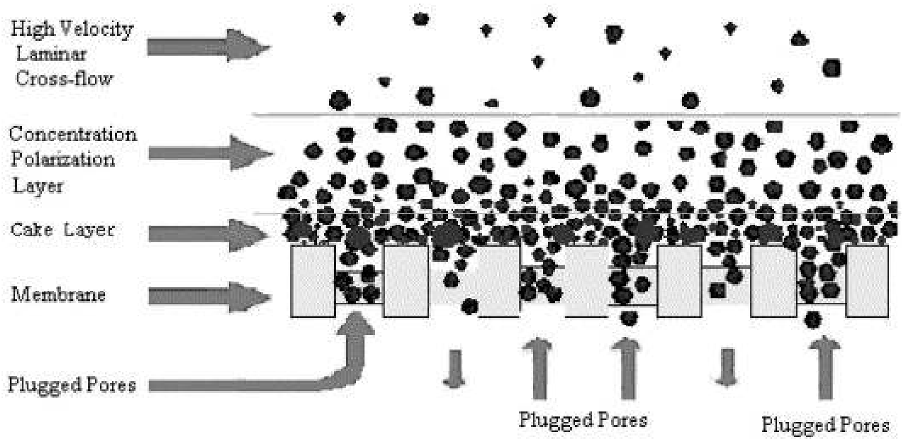
v : velocity

Concentration Polarization and Fouling (1)

- Concentration polarization occurs in membrane separators when the membrane is permeable to A, but relatively impermeable to B
  - Molecules of B are carried by bulk flow to the upstream surface of the membrane, where they accumulate, causing their concentration at the surface of the membrane to increase in a polarization layer
  - The equilibrium concentration of B in this layer is reached when its back-diffusion to the bulk fluid on the feed-retentate side equals its bulk flow toward the membrane



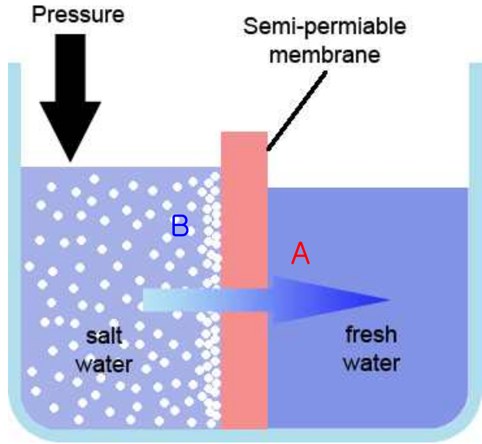


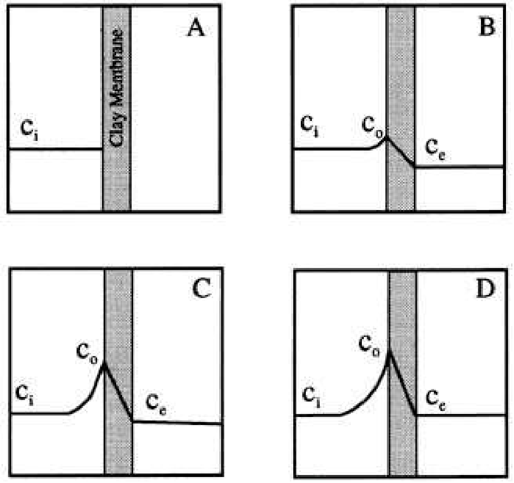


Concentration Polarization and Fouling (2)

- Concentration polarization is most common in pressure-driven membrane separations involving liquids, such as reverse osmosis and ultrafiltration, where it reduces the flux of A
  - The polarization effect can be serious if the concentration of B reaches its solubility limit on the membrane surface

  → A precipitate of gel may form, the result being fouling on the membrane surface or within membrane pores, with a further reduction in the flux of A





- The most straightforward way of minimizing concentration polarization is to reduce the film thickness by increasing turbulent mixing at the membrane surface