PID Controller Design Strategy for First Order Time Delay Processes

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Introduction

Proportional integral derivative (PID) controllers have been the most popular [1-6] and widely used controllers in process industries because of their simplicity, robustness and wide range of applicability with near-optimal performance. However, it has been noticed that many PID controllers are often poorly tuned and need to resolve this matter. The effectiveness of the internal model control (IMC) design principle has made it attractive in process industries, where many attempts have been made to exploit the IMC principle to design PID controllers for both stable and unstable processes Morari and Zafiriou [1]. The IMC-PID tuning rules have the advantage of only using a single tuning parameter to achieve a clear trade-off between the closed-loop performance and robustness. The PID tuning methods proposed by Rivera *et al.* [2], Morari and Zafiriou [1], Horn *et al.* [3], Lee *et al.* [4] and Skogestad [6] are typical examples of the IMC-PID tuning method. The direct synthesis for disturbance (DS-d) method proposed by Chen and Seborg [5] computing the ideal feedback controller which gives a predefined desired closed-loop response. The control performance can be significantly enhanced by cascading the PID controller with a lead/lag filter, as given by Eq. (1).

$$G_c = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1+as}{1+bs} \qquad (1)$$

The structure of the PID controller cascaded with a filter was also suggested by many researchers [1-4]. The PID·filter controller in Eq. (1) can easily be implemented in modern control hardware. The PID controller based on the IMC principle provides excellent set-point tracking, but has a sluggish disturbance response, especially for processes with a small time-delay/time-constant ratio. Since disturbance rejection is much more important than set-point tracking for many process control applications, a controller design that emphasizes the former rather than the latter is an important design goal that has been the focus of renewed research recently.

In the present study, a simple and efficient method is proposed for the design of a PID·filter controller with enhanced performance. A closed-loop time constant λ guideline is recommended for a wide range of time-delay/time-constant ratio.

IMC Controller Design Procedure

In the IMC block diagram (Morari and Zafiriou [1]) the output responses is related as (nominal case *i.e.*, $G_P = \tilde{G}_P$), $y = G_P q r + (1-\tilde{G}_P q)G_D d$, where G_P is the process, $\tilde{G}_P$ the process model, q the IMC controller, f_R the set-point filter. According to the IMC parameterization (Morari and Zafiriou [1]), the process model $\tilde{G}_P$ is factored into two parts:

$$\tilde{G}_P = p_m p_A \qquad (2)$$

where p_m is the portion of the model inverted by the controller, p_A is the portion of the model not inverted by the controller and $p_A(0)=1$. The noninvertible part usually includes the dead time and/or right half plane zeros and is chosen to be all-pass.

To obtain a good response for processes with poles near zero, the IMC controller q should be designed to satisfy the following conditions. (i) If the process G_p has poles near zero at $z_1, z_2, \ldots, z_m$, then q should have zeros at $z_1, z_2, \ldots, z_m$. (ii) If the process G_D has poles near zero, $z_{d1}, z_{d2}, \ldots, z_{dm}$, then $1-G_P q$ should have zeros at $z_{d1}, z_{d2}, \ldots, z_{dm}$.

Since the IMC controller q is designed as $q = p_m^{-1} f$, the first condition is satisfied automatically. The second condition can be fulfilled by designing the IMC filter f as $f = \left(\sum_{i=1}^{m}\beta_i s^i + 1\right)/(\lambda s+1)^r$, where λ is an adjustable parameter which controls the tradeoff between the performance and robustness; r is selected to be large enough to make the IMC controller (semi-)proper; β_i are determined by Eq. (3) to cancel the poles near zero in G_D.

$$1-G_P q\big|_{s=z_{d1},\ldots,z_{dm}} = \left|1-\frac{p_A(\sum_{i=1}^{m}\beta_i s^i+1)}{(\lambda s+1)^r}\right|_{s=z_{d1},\ldots,z_{dm}} = 0 \quad (3)$$

Then, the IMC controller comes to be $q = p_m^{-1}(\sum_{i=1}^{m}\beta_i s^i+1)/(\lambda s+1)^r$. Thus, the closed-loop response is

$$y=\frac{p_A\left(\sum_{i=1}^{m}\beta_i s^i+1\right)}{(\lambda s+1)^r}r+\left(1-\frac{p_A\left(\sum_{i=1}^{m}\beta_i s^i+1\right)}{(\lambda s+1)^r}\right)G_D d \quad (4)$$

From the above design procedure, one can achieve a stable closed-loop response by using the IMC controller.

PID·filter Design for FOPDT Process

The ideal feedback controller that is equivalent to the IMC controller can be expressed in terms of the internal model $\tilde{G}_P$ and the IMC controller q $G_c = q/\left(1-\tilde{G}_P q\right)$. Substituting Eqs. (3) and (6) into (8) gives the ideal feedback controller:

$$G_c = p_m^{-1}\frac{(\sum_{i=1}^{m}\beta_i s^i+1)}{(\lambda s+1)^r}\Bigg/ 1-\frac{p_A\left(\sum_{i=1}^{m}\beta_i s^i+1\right)}{(\lambda s+1)^r} \quad (5)$$

Let us consider the first order plus dead time (FOPDT) process, which is most widely utilized in the chemical process industries, as a representative model.

$$G_P = G_D = \frac{Ke^{-\theta s}}{\tau s+1} \quad (6)$$

The IMC filter structure is $f=(\beta s+1)/(\lambda s+1)^2$. It is noticed that the IMC filter in this form was also utilized by Lee *et al.* [4] and Horn *et al.* [3]. The resulting IMC controller becomes $q=(\tau s+1)(\beta s+1)/K(\lambda s+1)^2$. Therefore, the ideal feedback controller is obtained as

$$G_c=\frac{(\tau s+1)(\beta s+1)}{K\left[(\lambda s+1)^2-e^{-\theta s}(\beta s+1)\right]} \quad (7)$$

Since the ideal feedback controller in Eq. (7) does not have the PID·filter controller form, the remaining issue is how to design the PID·filter controller that approximates the ideal feedback controller most closely. Approximating the dead time $e^{-\theta s}$ with a 2/2 Pade expansion $e^{-\theta s}=\left(1-\theta s/2+\theta^2 s^2/12\right)/\left(1+\theta s/2+\theta^2 s^2/12\right)$, results in G_c as

$$G_c=\frac{(\tau s+1)\left(1+\frac{\theta s}{2}+\frac{\theta^2 s^2}{12}\right)(\beta s+1)}{K\left[(\lambda s+1)^2\left(1+\frac{\theta s}{2}+\frac{\theta^2 s^2}{12}\right)-(\beta s+1)\left(1-\frac{\theta s}{2}+\frac{\theta^2 s^2}{12}\right)\right]} \quad (8)$$

The 2/2 Pade approximation is precise enough to convert the ideal feedback controller into a finite dimensional feedback controller with barely any loss of accuracy and rearranging Eq. (8) gives

$$G_c=\frac{\left(1+\theta s/2+\theta^2 s^2/12\right)}{K(2\lambda-\beta+\theta)s}\frac{(\tau s+1)(\beta s+1)}{\left[1+\left(\beta\theta/2+\lambda\theta+\lambda^2\right)s/(2\lambda-\beta+\theta)+\left(-\beta\theta^2/12+\lambda\theta^2/6+\lambda^2\theta/2\right)s^2/(2\lambda-\beta+\theta)+\left(\lambda^2\theta^2/12\right)s^3/(2\lambda-\beta+\theta)\right]} \quad (9)$$

As seen in Eq. (16), the resulting controller has the form of the PID controller cascaded with a high order filter. The analytical PID formula can be obtained as

$$K_C=\frac{\theta}{2K(2\lambda-\beta+\theta)};\quad \tau_I=\frac{\theta}{2};\quad \tau_D=\frac{\theta}{6} \quad (10)$$

The value of the extra degree of freedom β is selected so that it cancels out the open-loop pole at $s=-1/\tau$ that causes a sluggish response to load disturbances. From Eq. (3), this requires $\left[1-(\beta s+1)e^{-\theta s}/(\lambda s+1)^2\right]_{s=-1/\tau}=0$. Thus, the value of β is obtained as

$$\beta=\tau\left[1-\left(1-\frac{\lambda}{\tau}\right)^2 e^{-\theta/\tau}\right] \quad (11)$$

Furthermore, it is obvious from Eq. (3) that the remaining part of the denominator in Eq. (9) contains the factor $(\tau s+1)$. Therefore, the filter parameter b in Eq. (1) can be obtained below

$$\left(cs^2+bs+1\right)=\frac{1+\dfrac{\left(\dfrac{\beta\theta}{2}+\lambda\theta+\lambda^2\right)}{(2\lambda-\beta+\theta)}s+\dfrac{\left(-\dfrac{\beta\theta^2}{12}+\dfrac{\lambda\theta^2}{6}+\dfrac{\lambda^2\theta}{2}\right)}{(2\lambda-\beta+\theta)}s^2+\dfrac{\left(\dfrac{\lambda^2\theta^2}{12}\right)}{(2\lambda-\beta+\theta)}s^3}{(\tau s+1)} \tag{12}$$

and substituting $s=0$ as

$$b=\frac{\left(\beta\theta/2+\lambda\theta+\lambda^2\right)}{(2\lambda-\beta+\theta)}-\tau \tag{13}$$

The filter parameter a in Eq. (1) can be easily obtained from Eq. (11) as $a=\beta$. Since the high order cs^2 term has little impact on the overall control performance in the control relevant frequency range, the remaining part of the fraction in Eq. (9) can be successfully approximated to a simple first order lead/lag filter as $(1+as)/(1+bs)$. Our simulation result (although not shown in this paper) also confirms the validity of this model reduction.

Simulation Study

Example 1. Lag time dominant process $(\theta/\tau=0.01)$

Consider the following FOPDT process (Chen and Seborg, [5]):

$$G_p=G_D=\frac{100e^{-1s}}{100s+1} \tag{14}$$

The proposed PID·filter controller is compared with other controllers (Proposed method: λ =1.131, K_c=0.124, τ_I= 0.50, τ_D=0.167, a=3.222, b=0.139, $f_R=(1.45s+1)/(3.22s+1)$, ITAE disturbance= 14.55, ITAE setpoint= 1.96; Lee *et al.*: λ =1.330, K_c=0.806, τ_I= 3.947, τ_D=0.3068, $f_R=1/(3.66s+1)$, ITAE disturbance= 19.77, ITAE setpoint= 8.46; DS-d: λ =1.202, K_c=0.826, τ_I= 0.826, τ_D=4.059, $f_R=(2.03s+1)/(1.43s^2+4.06s+1)$, ITAE disturbance= 20.43, ITAE setpoint= 3.13; Horn *et al.:* λ =1.689, K_c=15.038, τ_I= 15.038, τ_D=0.497, a=4.311, b=100.2, c=21.34, $f_R=1/(4.31s+1)$, ITAE disturbance= 31.18, ITAE setpoint= 12.45; Rivera *et al.*: λ =0.408, K_c=0.714, τ_I= 100.50, τ_D=0.4975, b=0.145, ITAE disturbance= 3785.0, ITAE setpoint= 3.86; Lee *et al.*(CF): λ =0.248, K_c=0.805, τ_I=100.407, τ_D=0.399, ITAE disturbance= 3354.0, ITAE setpoint= 3.15). In order to ensure a fair comparison, all of the controllers compared are tuned to have $Ms=1.94$ by adjusting λ. Figure 1 compares the set-point and load responses obtained using the proposed method, the DS-d method, and the methods proposed by Lee *et al.* and Horn *et al.* and the set-point filter has been used enhanced servo response. It is important to note that the set-point filter used for the set-point response has a clear benefit when the process is lag time dominant. From the above performance indices, ITAE value and Fig.1 show that the proposed method has great advantage over other method. The simulation results (although not shown in this paper) also confirms that the robustness of the proposed controller has better compare to other methods.

Closed-loop time constant λ guideline

In the proposed tuning rule, the closed-loop time constant λ controls the tradeoff between the robustness and performance of the control system. As λ decreases, the closed-loop response becomes faster and can become unstable. On the other hand, as λ increases, the closed-loop response becomes stable but sluggish. A good tradeoff is obtained by choosing λ to give an Ms value in the range of $1.2\sim2.0$. The λ guideline for several robustness levels is plotted in Fig. 2.

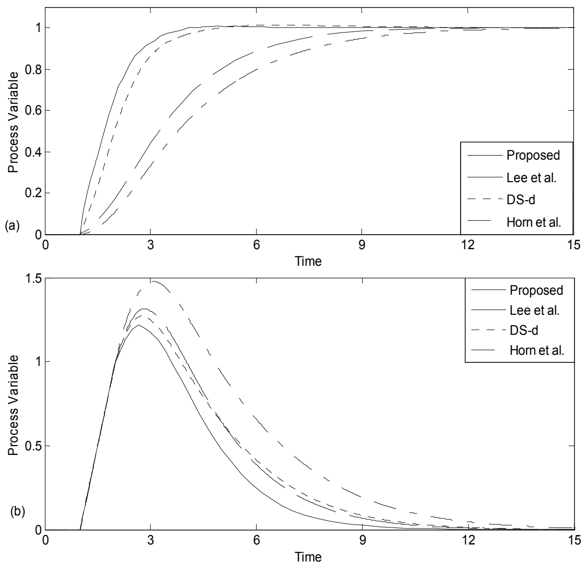

Fig. 1 Simulation results for example 1

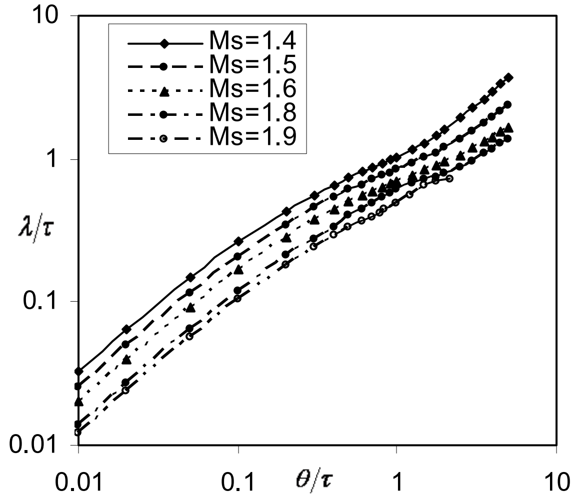

Fig. 2 λ guideline for the proposed tuning method

Conclusions

A simple analytical design method for a PID controller cascaded with a lead/lag filter was proposed based on the IMC principle in order to improve its disturbance rejection performance. The proposed method also includes a set-point filter to enhance the set-point response like as the 2DOF controller suggested by Lee *et al.,* [4], Horn *et al.,* [3] and Chen and Seborg, [5]. The proposed PID·filter controller consistently provides superior performance over the whole range of the θ/τ ratio, while the other controllers based on the IMC-PID design methods take their advantage only in a limited range of the θ/τ ratio. In particular, the proposed controller shows excellent performance when the lag time dominates. The closed-loop time constant λ guideline was also proposed for a wide range of θ/τ ratio.

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