Tuning of Integrating and Integrating Processes with Dead Time and Inverse Response

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Introduction

The proportional-integral-derivative (PID) controller is widely used in the process industries. The main reason is simple structure, which can be easily understood and implemented in practice. Finding design methods that lead to the optimal operation of PID controllers is therefore of significant interest. The integrating and integrating process with dead time and inverse response process are frequently encountered in the process industries. The common examples of these processes are distillation column, chemical reactor and level control of the boiler steam drum. A recent trends show that the tuning of the controllers for a time-delay integrating process and integrating process with dead time and inverse response has been an active area of research in the literature [Chien and Fruehauf (1990); Luyben (1996, 2003); Morari and Zafiriou (1989)]. Sree & Chidambaram (2005) proposed a PID controller method for an integrator plus time delay process and used to design a PID controller for the isothermal continuous copolymerization process in a CSTR. Luyben (2003) proposed the identification and controller tuning procedures for integrating process with inverse response and dead time system from step response data and claim that due to the process contains an integrator and the proportional-integral controller also contains an integrator, controller tuning is somewhat complex. His proposed method determines the smallest possible value for integral time. Then, using this value, the controller gain that gives a +2 dB maximum closed-loop log modulus is calculated.

The analytically derived IMC-PID tuning [Lee et al. (1998); Morari and Zafiriou (1989); Skogestad (2003)] methods attracted the attention of industrial users recently. This is due to the simplicity and better performance of the internal model control (IMC) based tuning rule. The IMC-PID tuning rule has only one user-defined tuning parameter, which is directly related to the closed-loop time constant. The IMC-PID controller provides good set-point tracking but sluggish disturbance response especially for the process with a small time-delay/time-constant ratio. However, for many process control applications, disturbance rejection is much more important than set-point tracking. Therefore, controller design that emphasizes disturbance rejection rather than set-point tracking is an important design problem that has received renewed interest recently.

However, methods of PID controllers designing for the integrator with long time delay and integrating processes with dead time and inverse response are not discussed extensively and therefore, the present work is directed to design the PID controllers for such systems for the disturbance rejection. The concept of 2DOF control structure is used to cope with setpoint performance. The performance of the proposed tuning rule has been compared with other tuning methods, when the controller is tuned to the same robustness level by evaluating the peak of the maximum sensitivity (*Ms*).

Theory

IMC controller design steps

The IMC controller design involves two steps:

Step 1: A process model $\tilde{G}_P$ is factored into invertible and non invertible parts

$$\tilde{G}_P = P_M P_A \quad (1)$$

where P_M is the portion of the model inverted by the controller; P_A is the portion of the model not inverted by the controller (it is usually a non-minimum phase and contains dead times and/or right half plane zeros); $P_A(0) = 1$.

Step 2: The idealized IMC controller is the inverse of the invertible portion of the process model, i.e., $\tilde{q} = P_M^{-1}$, and to make the IMC controller proper, it is necessary to add the filter. Thus, the IMC controller is designed as

$$q = \tilde{q}f = P_M^{-1} f \quad (2)$$

The filter for the IMC controller can be designed to satisfy two criteria, one is that to make the IMC controller proper and another to cancel the unstable poles or stable poles near zero of G_D.

$$f = \frac{\sum_{i=1}^{m} \beta_i s^i + 1}{\left(\lambda^2 s^2 + 2\lambda\xi s + 1\right)^m} \quad (3)$$

where β_i are determine to cancel the poles of G_D and m is the number which can be adjusted to make the IMC controller proper. Eq. (3) function as a filter with adjustable time constant λ and damping coefficient ξ. Since $1-G_p q\big|_{s=dup_1,\ \ldots,\ dup_m}=0$ where $dup_i \neq 0$.

Thus, the IMC controller is $q=P_M^{-1}\left(\sum_{i=1}^{m}\beta_i s^i+1\right)\Big/\left(\lambda^2 s^2+2\lambda\xi s+1\right)^m$. The resulting IMC controller in Eq. (3) has stable response and the classical feedback controller exactly equivalent to IMC can be obtained from the following relationship

$$G_c=\frac{q}{1-G_p q} \tag{4}$$

The resulting closed-loop output response in Eq. (4) is physically realizable, but it does not have the standard PID controller form. To get the PID controller from the ideal controller G_c, was discussed in detailed Lee et al. (1998).

Proposed Tuning Rule

1. Delay Integrating Process (DIP)

The commonly used delay integrating process model for chemical industries is given below

$$G_p=G_D=\frac{Ke^{-\theta s}}{s} \tag{5}$$

The DIP process can be modeled as the first order plus dead time (FOPDT) by approximating as:

$$G_p=G_D=\frac{Ke^{-\theta s}}{s}=\frac{Ke^{-\theta s}}{s+1/\psi}=\frac{\psi Ke^{-\theta s}}{\psi s+1} \tag{6}$$

where ψ is an arbitrary constant with a sufficiently large value i.e., $1/\psi \ll 1$. The proposed filter structure is $f=(\beta s+1)\big/\left(\lambda^2 s^2+2\lambda\xi s+1\right)$ for the DIP model. Therefore, ideal feedback controller equivalent to the IMC controller is $G_C=\left[(\psi s+1)(\beta s+1)\right]\Big/\left\{K\psi\left[\left(\lambda^2 s^2+2\lambda\xi s+1\right)-e^{-\theta s}(\beta s+1)\right]\right\}$ and expanding G_c in a Maclaurin series in s, and from the Lee et al. (1998) method, the PID parameters can be obtained as:

$$k_C=\frac{\tau_I}{\psi K(\theta-\beta+2\lambda\xi)} \qquad \tau_I=(\psi+\beta)-\frac{\left(\lambda^2-\theta^2/2+\beta\theta\right)}{(\theta-\beta+2\lambda\xi)} \qquad \tau_D=\frac{(\psi\beta)-\dfrac{\left(\theta^3/6-\beta\theta^2/2\right)}{(\theta-\beta+2\lambda\xi)}}{\tau_I}-\frac{\left(\lambda^2-\theta^2/2+\beta\theta\right)}{(\theta-\beta+2\lambda\xi)} \tag{7}$$

The value of β is calculated by solving $\left[1-(\beta s+1)e^{-\theta s}\big/\left(\lambda^2 s^2+2\lambda\xi s+1\right)\right]\Big|_{s=-1/\psi}=0$ to cancel the slow pole of the process and after simplification the β value is given $\beta=\psi\left[1-\left(\lambda^2-2\lambda\xi\psi+\psi^2\right)e^{-\theta/\psi}/\psi^2\right]$.

2. Integrating Process with Dead Time and Inverse

$$G_p=G_D=\frac{K\,(-\tau_a s+1)e^{-\theta s}}{(\tau s+1)s}$$

The above process modeled as the second order plus dead time (SOPDT) by approximating as

$$G_p=G_D=\frac{K\,(-\tau_a s+1)e^{-\theta s}}{(\tau s+1)(s+1/\psi)}=\frac{\psi K\,(-\tau_a s+1)e^{-\theta s}}{(\tau s+1)(\psi s+1)} \tag{8}$$

where ψ is an arbitrary constant with a sufficiently large value. The processes having the inverse response can be easily reduced to dead time. Skogestad (2003) has suggested that an ''inverse response time constant'' T_0^{inv} (negative numerator time constant) may be approximated as a time delay $\left(-T_0^{inv}+1\right)\approx e^{-T_0^{inv}s}$ .This is reasonable since an inverse response has a deteriorating effect on the control, similar to that of a time delay. Therefore the above process model can be reduces as

$$G_p=G_D=\frac{\psi Ke^{-(\theta+\tau_a)s}}{(\tau s+1)(\psi s+1)} \tag{9}$$

The IMC filter is suggested $f=\left(\beta_2 s^2+\beta_1 s+1\right)\Big/\left(\lambda^2 s^2+2\lambda\xi s+1\right)^2$ and the PID controller can be derived similar to Integrating process as discuss above.

Simulation Study

Example 1: **Isothermal continuous copolymerization of styrene-acrylonitrile in CSTR**

Copolymerization is characterized by the presence of two or more distinct monomers. The model equations for CSTR carrying out this copolymerization reaction are given by

$V(dA/dt)=q_1A_f-q_3A-VR_i^{1/2}\left(r_1A^2+AB\right)\big/f(A,B)$ and $V(dB/dt)=q_2B_f-q_3B-VR_i^{1/2}\left(r_2B^2+AB\right)\big/f(A,B)$

with the initial conditions at $t = 0,\ A = A_0\ and\ B = B_0$

$f(A,B) = \left(r_1^2 \xi_1^2 A^2 + 2\phi \xi_1 \xi_2 AB + r_2^2 \xi_2^2 B^2\right)^{0.5}$; $\xi_1 = \left(2k_{t11}/k_{p11}\right)0.5$; $\xi_2 = \left(2k_{t22}/k_{p22}\right)0.5$; $r_1 = k_{p11}/k_{p12}$; $r_2 = k_{p22}/k_{p21}$; $\phi = k_{t12}/\left(k_{t11}k_{t22}\right)^{0.5}$

where R_i=rate of free radical initiation; A=concentration of styrene in the reactor; Af=feed concentration of styrene; B=concentration of acrylonitrile in the reactor; B_f=feed concentration of acrylonitrile; F=mole fraction of styrene monomer; k_{pij}=propagation rate constant of monomer j with radical i; k_{tij}=termination rate constant of radicals i and j; q_1 and q_2=feed rates of A and B; q_3=outlet flow rate of the reactor; r_1=reactivity ratio of styrene; r_2=reactivity ratio of acrylonitrile; V=volume of the reactor.

For solving the above equation, the parameters are listed in Sree and Chidambaram (2005). The simplified linear process model of the copolymerization reactor after relay identification test is given below and details is given in Sree & Chidambaram (2005).

$$G_P = G_D = \frac{0.2082e^{-50.55s}}{s} \tag{10}$$

Luyben (2005), Sree & Chidambaram (2005), and proposed methods were used to design the PID controller for the above process. For the proposed method, a value of *λ=35.7* and *ξ=2.1* was chosen so that *Ms=2.19*. The *λ* has been adjusted to get the similar value of the *Ms* with the Sree & Chidambaram (2005) to obtain the fair comparison. Figure 1 & 2 show the closed-loop output response for the setpoint and disturbance rejection, when a unit-step setpoint & disturbance change occurring in the process. The above figures clearly show the disturbance rejection and set-point response for the proposed controller is better than the other tuning methods. The 2DOF controller is used in the present study for the setpoint response. The proposed method shows the smooth response for both the setpoint and disturbance rejection. Luyben (1996) method has very slow response which also has *Ms= 2.19*. In the disturbance rejection Sree & Chidambaram (2005) has undershoot which is undesirable. The IAE values for disturbance rejection are 1778, 1905 and 6549 (for setpoint IAE 135.9, 170.1 and 171.8) for the proposed, Sree & Chidambaram (2005) and Luyben (1996) methods respectively. The Figs. 1 & 2 and IAE value show that the proposed method has superior performance over other method, keeping same robustness level.

Example 2: **Boiler Steam Drum**

The example of an integrating process that has an inverse response is a boiler steam drum. The level is controlled by manipulating the boiler feed water (BFW) to the drum. The drum is located near the top of the boiler and is connected to it by a large number of tubes. Liquid and vapor water circulate between the drum and the boiler as a result of the density difference between the liquid in the downcomer pipes leading from the bottom of the drum to the base of the boiler and the vapor/liquid mixture in the riser pipes going up through the boiler and back into the steam drum. Luyben (2003) suggested the transfer function after the identification test for the boiler steam drum is integrating process with dead time and inverse response, which is given as

$$G_p = G_D = \frac{0.547\left(-0.418s+1\right)e^{-0.1s}}{\left(1.06s+1\right)s} \tag{11}$$

The PID controller has been designed by the proposed method and Luyben (2003). Figure 3 shows the closed-loop output responses for a unit-step setpoint change occurring at *t=0*, and a unit-step disturbance occurring at *t=20*. For the fair comparison, $\lambda = 0.798$ and $\xi = 1.0$ has been selected to get the similar value of $Ms = 1.89$ with Luyben (2003). Luyben (2003) has proposed the two cases one for the 50% minimum integral time and another is for the 25% minimum integral time. The 25% integral time shows clear advantage in the disturbance rejection followed by more overshoot in the setpoint response. Figure 3 shows that the proposed method has smooth and fast response for the disturbance rejection and setpoint. For the setpoint the 2DOF controller is used where *b=0.3* has been chosen in the proposed study. The IAE value has been calculated for the disturbance rejection, which are 1.56, 3.48 and 6.84 (for setpoint IAE 2.52, 3.63 and 3.82) for the proposed, 25% and 50% integral time by Luyben (2003) method. Therefore, the proposed method shows clear advantage over Luyben (2003) because of less IAE value as well as smooth and fast response.

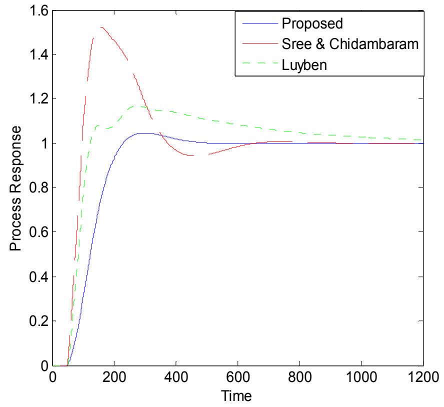

Fig. 1. Setpoint response of the copolymerization rector

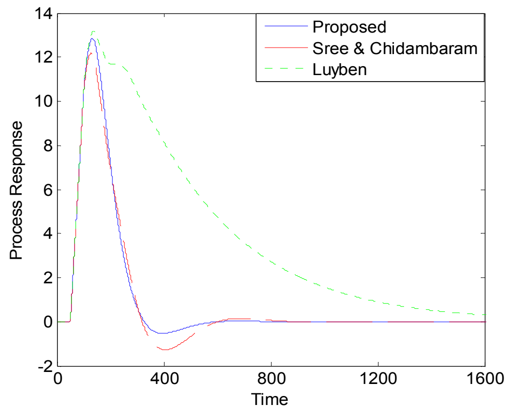

Fig. 2. Disturbance rejection of the copolymerization rector

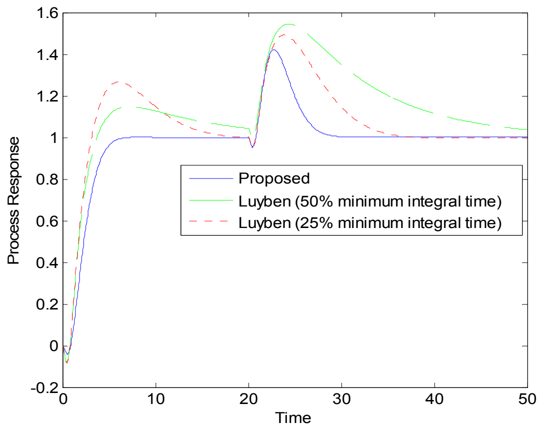

Fig. 3. Process response of the boiler steam drum

Conclusions

The IMC-PID controller has been designed for the integrating and integrating process with dead time and inverse response. The IMC filter has been modified for the long delay integrating process, which clearly shows that the overdamped IMC filter gives the smooth and fast response. The proposed method is used to design the PID controller for the typical copolymerization reactor and boiler steam drum. The proposed method shows the smooth and fast response when the controller is tune with the same robustness level. The processes having inverse response can be treaded by reducing them into FOPDT/SOPDT model. The simulation results demonstrated superiority of the proposed method.

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