Design of Robust PID Controller for the Unstable Dead Time Dominant Processes

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Introduction

In the process control, more than 95% of the control loops are of the proportional integral derivative (PID) type. The main reason is its relatively simple structure, which can be easily understood and implemented in practice. It is well-known that control system design for an open-loop unstable process is more difficult than that for a stable one because of the unstable nature of the dynamics, for which most design tools cannot be applied. The some of the common example for unstable process are the polymerization furnaces, continuous stirred tank reactors (CSTRs), and the batch chemical reactor, which has a strong nonlinearity due to heat generation term in the energy balance. A very recent trend shows that tuning of controllers for a time-delay unstable process has been an active area of research in the literature [1-9]. Due to the effectiveness of internal model control (IMC) in process industry, many efforts have been made to exploit the IMC principle to design the equivalent feedback controllers for unstable processes. Wang *et al.* [7] studied and concluded that that the most of the controller schemes is not working for a large normalized dead time $\theta/\tau > 1$ and the tuning method of Lee *et al.* [2] and Yang *et al.* [9] is only applicable. Recently, Xiang and Nguyen [8] have suggested a control schemes for unstable with dead time by PID controller, the method contain three controllers.

Therefore, the proposed study is concerned for the dead time dominant unstable processes. The IMC filter has been modified from critically damped to underdamped and damping coefficient can be adjusted for the required integral action for improved performance. The dead time dominant first order delay unstable and second order delay unstably process has been studied for the superiority of the proposed method. The robustness of the controller is evaluated using Kharitonov's theorem.

Theory

The IMC has been shown to be a powerful method for control system synthesis [5]. However, for unstable processes the IMC structure cannot be implemented exactly similar to stable process, since any input will make output grow without bound if process is unstable. Nevertheless, as discussed in [5], we could still use IMC approach to design a controller for an unstable process, if only the following conditions are satisfied for the internal stability of the closed-loop system (i) q stable, (ii) $G_p q$ stable, (iii) $(1-G_p q)G_p$ stable.

IMC controller design step

The IMC controller design involves two steps:

Step 1: A process model $\tilde{G}_P$ is factored into invertible and non invertible parts

$\tilde{G}_P = P_M P_A$, where P_M is the portion of the model inverted by the controller; P_A is the portion of the model not inverted by the controller (it is usually a non-minimum phase and contains dead times and/or right half plane zeros); $P_A(0) = 1$.

Step 2: The IMC controller is set as $q = P_M^{-1} f$. Here, q has zeros at $up_1, \ldots, up_k$ because P_M^{-1} is the inverse of the model portion with unstable poles. The filter for the IMC controller can be designed to satisfy two criteria, one is that to make the IMC controller proper and another to cancel the unstable poles or stable poles near zero of G_D.

$f = \left(\sum_{i=1}^{m} \beta_i s^i + 1\right) / \left(\lambda^2 s^2 + 2\lambda\xi s + 1\right)^m$ where β_i are determine to cancel the unstable poles of G_D and m is the number which can be adjusted to make the IMC controller proper. It has function a of adjustable time constant λ and damping coefficient ξ. $1 - G_p q\big|_{s=dup_1,\ \ldots,\ dup_m} = 0$ where $dup_i \neq 0$. Thus, the IMC controller is

$$q = P_M^{-1} \frac{\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)}{\left(\lambda^2 s^2 + 2\lambda\xi s + 1\right)^m} \tag{1}$$

The lead term in IMC filter $\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)$ causes an overshoot in the closed-loop response to a setpoint change. This problem can be resolved if we add a setpoint filter. $f_R = 1/\left(\sum_{i=1}^{m} \beta_i s^i + 1\right)$ The resulting IMC controller in Eq. (1) has stable response and the classical feedback controller exactly equivalent to IMC can be obtained from the following relationship

$$G_c = \frac{q}{1-\tilde{G}_p q} \qquad (2)$$

The resulting closed-loop output response in Eq. (2) is physically realizable, but it does not have the standard PID controller form. For the PID controller from the ideal controller G_c, is discussed in detailed Lee *et al.* [3].

Proposed Tuning Rule

First Order Delayed Unstable Process (FODUP)

Consider a first order delayed unstable process of the form:

$$G_P = G_D = \frac{Ke^{-\theta s}}{\tau s - 1} \qquad (3)$$

where K is the gain, τ is the time constant and θ is the time delay. The proposed IMC filter is found as $f = (\beta s+1)/(\lambda^2 s^2 + 2\lambda\xi s+1)$. Then, the resulting IMC controller becomes $q = (\tau s-1)(\beta s+1)/K(\lambda^2 s^2+2\lambda\xi s+1)$. Therefore, the ideal feedback controller equivalent to the IMC controller is $G_c = (\tau s-1)(\beta s+1)/K\left[(\lambda^2 s^2+2\lambda\xi s+1)-e^{-\theta s}(\beta s+1)\right]$. The dead time term expanded in Maclaurin series and the analytical PID formula can be given as:

$$K_C = \frac{\tau_I}{K(\theta-\beta+2\lambda\xi)} \qquad \tau_I = (-\tau+\beta) - \frac{\left(\lambda^2 - \theta^2/2 + \beta\theta\right)}{(\theta-\beta+2\lambda\xi)} \qquad \tau_D = \frac{(-\tau\beta) - \frac{\left(\theta^3/6 - \beta\theta^2/2\right)}{(\theta-\beta+2\lambda\xi)}}{\tau_I} - \frac{\left(\lambda^2-\theta^2/2+\beta\theta\right)}{(\theta-\beta+2\lambda\xi)}$$

The extra degree of freedom β is calculated by solving $[1-Gq]|_{s=1/\tau}$. That means we want to choose β so that the term [1-Gq] has a zero at the pole of G_D. The value of β after some simplification is given as $\beta = \tau\left[(\lambda^2 + 2\lambda\xi\tau + \tau^2)e^{\theta/\tau}/\tau^2 - 1\right]$.

Robust Stability

Parametric Robustness Analysis

A control system is said to be robust if the closed-loop system is stable even when the model parameters of the actual process are different from that used for controller design. This section is devoted for the robust stability of interval polynomials based on the Kharitonov's Theorem, which is discussed in Bhattacharyya *et al.* [1].

Theorem (Kharitonov's Theorem):

Every polynomial in the family $\chi(s)$ is Hurwitz if and only if the following four polynomials are Hurwitz:

$$k_1(s) = \underline{\chi}_0 + \overline{\chi}_1 s + \overline{\chi}_2 s^2 + \underline{\chi}_3 s^3 + \underline{\chi}_4 s^4 + \overline{\chi}_5 s^5 + \overline{\chi}_6 s^6 \ldots$$

$$k_2(s) = \underline{\chi}_0 + \underline{\chi}_1 s + \overline{\chi}_2 s^2 + \overline{\chi}_3 s^3 + \underline{\chi}_4 s^4 + \underline{\chi}_5 s^5 + \overline{\chi}_6 s^6 \ldots$$

$$k_3(s) = \overline{\chi}_0 + \underline{\chi}_1 s + \underline{\chi}_2 s^2 + \overline{\chi}_3 s^3 + \overline{\chi}_4 s^4 + \underline{\chi}_5 s^5 + \underline{\chi}_6 s^6 \ldots$$

$$k_4(s) = \overline{\chi}_0 + \overline{\chi}_1 s + \underline{\chi}_2 s^2 + \underline{\chi}_3 s^3 + \overline{\chi}_4 s^4 + \overline{\chi}_5 s^5 + \underline{\chi}_6 s^6 \ldots$$

The stability of above four equations formed from Kharitonov polynomials is to be checked. For fixed values of gain K and τ, a perturbation in time delay θ i.e., (θ-Δθ) ≤θ ≤ (θ-Δθ) is substituted in the above coefficient and Kharitonov's four equations are checked for stability using Routh-Hurwitz method. Similarly perturbation in K and τ is also evaluated. The closed-loop characteristic equation ($1+G_{OL}=0$) can be arrange in form of polynomial after the dead time can be approximated by Pade approximation as:

$$\chi(s) = \chi_0 + \chi_1 s + \chi_2 s^2 + \chi_3 s^3 + \chi_4 s^4 + \chi_5 s^5 + \chi_6 s^6 \qquad (9)$$

$\underline{\chi}_i \le \chi_i \le \overline{\chi}_i \;\; (i = 0, 1, 2, 3, 4, 5)$ where $\underline{\chi}_i$ and $\overline{\chi}_i$ are the lower and upper bound for χ_i, respectively. Let's consider the control system design of FODUP process by the PID controller, where $1+G_{OL}=0$ is given as $\left\{\left[k_C K\left(1+\tau_I s+\tau_I\tau_D s^2\right)e^{-\theta s}\right]/\left[\tau_I s(\tau s-1)\right]+1\right\}=0$. The coefficient of the characteristic equation, Eq. (9), for the FODUP is given as:

$\chi_0 = 120 k_C K$, $\chi_1 = -120\tau_I - 60k_C K\theta + 120k_C K\tau_I$, $\chi_2 = 12k_C K\theta^2 - 60k_C K\tau_I\theta + 120k_C K\tau_I\tau_D - 60\tau_I\theta + 120\tau_I\tau$

$\chi_3 = -k_C K\theta^3 + 12k_C K\tau_I\theta^2 - 60k_C K\tau_I\tau_D\theta - 12\tau_I\theta^2 + 60\tau_I\tau\theta$, $\chi_4 = -\tau_I\theta^3 + 12\tau_I\tau\theta^2 - k_C K\tau_I\theta^3 + 12k_C K\tau_I\tau_D\theta^2$,

$\chi_5 = \tau_I\tau\theta^3 - k_C K\tau_I\tau_D\theta^3$

Simulation Study

Example 1. *FODUP*

Consider a large normalized dead time θ/τ=1.5 unstable process Wang *et al.* [7] and Xiang and Nguyen [8].

$$G_P = G_D = \frac{1e^{-1.5s}}{(1s-1)} \qquad (10)$$

The λ and ξ value for the proposed tuning method has been adjusted to give the same *Ms* value to Wang *et al.* [7] and Lee *et al* [2]. The selection of same *Ms* has been done for fair comparison. Xiang and Nguyen [8] method controller setting, which has three controller structure is, $F(s)=1/(5s+1)$ as a setpoint filter, inner loop PD controller $C_1(s)=(1.019+0.59s)$ and outer loop PID controller $C_2(s)=(0.008+0.0054/s+0.0729s)$. To test the performance of the control system, the load disturbance has a step change of magnitude 0.1 and setpoint of magnitude of 1 is added. Fig. 1 and Fig. 2 show the output response of the disturbance rejection and setpoint respectively. The proposed method shows the clear advantage in disturbance rejection and the setpoint response. Lee *et al.* [2] and Wang *et al.* [9] methods shows very slow setpoint and disturbance rejection.

The robustness of the controller is evaluated by using the Kharitonov's Theorem and the parametric uncertainty margin in different process parameters for different controller methods is listed in Table 1. From the Table 1, it is clear that almost equal robustness the proposed method has great advantage in the performance.

Example 2. *SODUP*

Consider the following unstable process studied by Yang *et al.* [9] and Liu *et al.* [4].

$$G_P=G_D=\frac{1e^{-1.2s}}{(1s-1)(0.5s+1)} \tag{11}$$

The above dead time dominated SODUP can be modeled as $G_P=G_D=1e^{-0.6s}/(1s-1)(1.1s+1)$ in the present study. This is due to getting the required integral action. To derive these approximations, consider the following Taylor series approximations of a time delay transfer function, $e^{-\theta s}=1/e^{\theta s}\approx 1/(1+\theta s)$. To test the performance of the control system, the load disturbance has a step change of magnitude 0.05 and setpoint of magnitude 1 is added and the simulation results are provided in Fig. 3 and Fig. 4 respectively. It is seen that the proposed controller leads to obviously improved load disturbance performance. The robustness of the controller is evaluated by using the Kharitonov's Theorem and the uncertainty margin in different process parameters is listed in Table 2. It is clear that proposed methods have good advantage in performance for almost equal robustness level.

Conclusions

The present study deals with the dead time dominant unstable process for the FODUP and SODUP. The control performance has been improved by modifying the filter in IMC design from critically damped to underdamped. In the unstable process, as the process going towards the dead time; it has lacking the integral action, which can be improved by modifying the IMC filter. The detail robustness study has been done for the parametric robustness based on the Kharitonov's Theorem, which clearly show that for almost equal uncertainty margin in process parameters the proposed method have great advantage in the performance.

Tuning Methods	Present Method	Lee *et al.*	Wang *et al.*
K	±0.037	±0.036	±0.036
τ	±0.056	±0.055	±0.055
θ	±0.080	±0.086	±0.086

Table1 Uncertainty in process parameters for example 1

Tuning Methods	Present Method	Liu *et al.*	Yang *et al.*
K	±0.030	±0.030	±0.005
τ_1	±0.045	±0.049	±0.096
τ_2	±0.060	±0.063	±0.128
θ	±0.055	±0.066	±0.131

Table 2 Uncertainty in process parameters for example 2

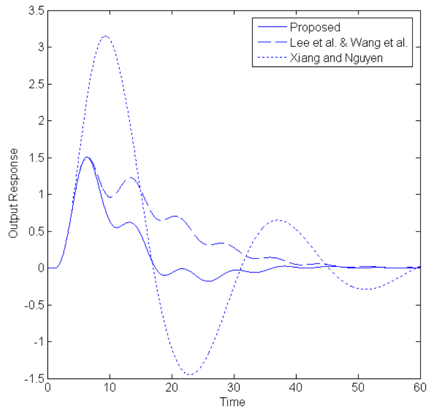

Fig. 1 Disturbance rejection for example 1

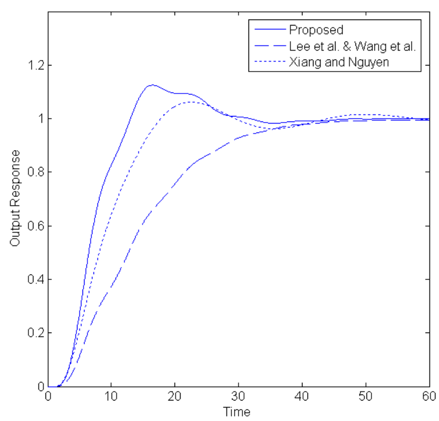

Fig. 2 Setpoint response for example 1

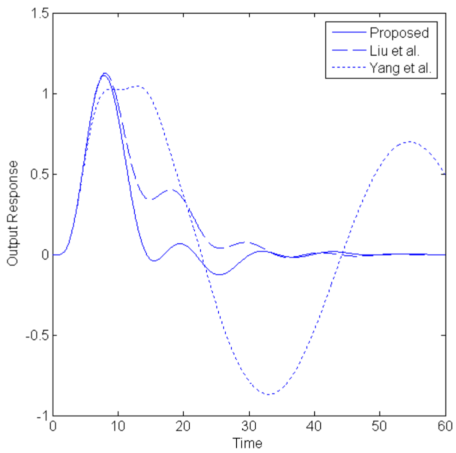

Fig. 3 Disturbance rejection for example 2

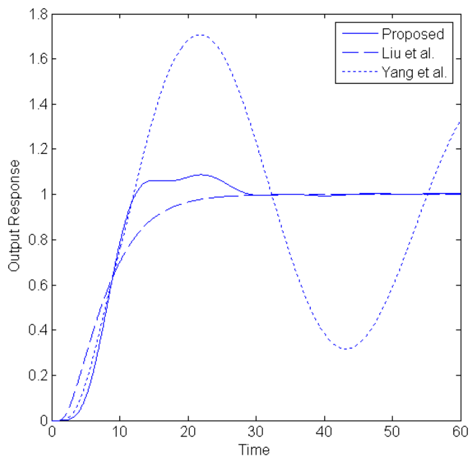

Fig. 4 Setpoint response for example 2

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