# Optimal Design Method Based On Mp Criteria for Multiloop PID Controllers

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## Introduction

Recently, multiloop PID controller is widespread used because of its simple, performance and robustness. The interactions between the control loops are really complicated and correspondingly difficult to make the proper tuning in a MIMO system. For this reason, the proposed method has been used the multiloop diagonal controller's structure for designing multiloop PID controller. In the late 1980s, there were many multiloop PID controller tuning rules which are proposed by the famous authors such as [Luyben, 1986; Grosdidier and Morari, 1987; Skogestad and Morari, 1989; Bassualdo and Marchetti, 1990]. In this article, we propose new tuning rules for multiloop PID controllers to give desired closed-loop responses by an analytical design of multiloop PID controllers. A good combination between diagonal and off-diagonal term in the closed-loop frequency responses that make the interaction effects of the control loop can be adjusted as the desired effect by designer. Considering the process or design goal, it attaches more importance to diagonal or off-diagonal term. In our research, its pair is in equilibrium. The optimum values of closed-loop time constant are found by solving the prososed optimization function. These solutions give the proposed multiloop controllers to minimize integal absolute error (IAE) for all loops.The feasibility of proposed method is verified by several illustrated examples with purpose to show how performance of proposed method, a favorable comparison with some well-known methods.

## Theory

### 1. Proposed Tuning method

### 1.1. Mp Tuning Criterion for the Multiloop Controllers Tuning

In MIMO system, the closed-loop transfer function can be given by:

$$H(s) = G(s)G_c(s)[I + G(s)G_c(s)]^{-1} \tag{1}$$

Where G(s) is process, $G_c(s)$ is the multiloop PID controller.

The closed-loop frequency response for each loop can be found by setting s= j$\omega$.

$$H(j\omega,\lambda) = G(j\omega)G_c(j\omega,\lambda)[I + G(j\omega)G_c(j\omega,\lambda)]^{-1} \tag{2}$$

It can be expressed in terms of its magnitude and phase,

$$H(j\omega,\lambda) = |H(j\omega,\lambda)| \angle H(j\omega,\lambda) \tag{3}$$

The magnitude of $H(i\omega,\lambda)$ is

$$\begin{aligned}|H(j\omega,\lambda)| &= \left|\frac{G(j\omega)G_c(j\omega,\lambda)}{\mathrm{I}+\mathrm{G}(\mathrm{j}\,\omega)\mathrm{G}_{\mathrm{c}}(j\omega,\lambda)}\right| \\ &= \frac{|G(j\omega)G_c(j\omega,\lambda)|}{|\mathrm{I}+\mathrm{G}(\mathrm{j}\,\omega)\mathrm{G}_{\mathrm{c}}(j\omega,\lambda)|}\end{aligned} \tag{4}$$

The phase of $H(j\omega,\lambda)$ is

$$\angle H\ (j\omega,\lambda) = \angle G\ (j\omega) - \angle\left[1 + G\ (j\omega)G_c(j\omega,\lambda)\right] \tag{5}$$

Most MIMO system can be poor performance when their loop gains become high. In this situation, derived from Eq. (4), $H(j\omega,\lambda)$ have to be unity at all frequencies; therefore, the magnitude of $G(j\omega)$ must be infinite, it is not true in practice.

The closed-loop resonant frequency response $\omega_r$ can be calculated by Eq. (5).

The Bandwidth BW is the frequency $|H(j\omega,\lambda)|$ dropping to 70.7 percent at its zero-frequency value.

The constrained optimization problem for set of closed-loop time constant $\lambda$ is described by the function below,

$$\min_{\lambda_i}\left[(1-w)\sum_{i}\sum_{i\neq j} Mp_{ij} + w\sum_{i} Mp_{ii}\right] \tag{6}$$

Subject to $Mp_{ii} \geq Mp_{low}$

Where $Mp_{ij} = \max_{\omega} H_{ij}$, the function of $\lambda$, $Mp_{low}$ is lower bound of diagonal Mp and it should be selected by a value between 1.1 and 1.4[Harris and Mellichamp, 1985]; w is weighting factor for the diagonal Mp. Here, $\lambda_i$ is a design variable and determines the speed of response of output for the paired setpoint change. By solving optimization function (6), we can find optimum value of $\lambda_i$ that minimizes overshoots and disturbance rejection in the loops while all $Mp_{ii}$ must satisfy the constraint. The weighting factor is just given number. Accordingly our studiedness, it is chosen from 0.5 to 0.75 for well-balance in multiloop PID controllers.

### 1.2. Decentralized PI/PID controller design for MIMO systems

The tuning of decentralized PI/PID controllers for MIMO systems is more complicated problem due to loop interaction and we cannot be directly applied the approaches of SISO system to MIMO system. Recently, several tuning schemes for multiloop PI/PID controllers are proposed base on frequency-domain analysis such as BLT method (Luyben, 1986). Ho et al. (1997) proposed an analytical method for tuning multiloop PID controllers by specifying the gain and phase margins, Lee et al. (1997) proposed a multiloop control system design method in which the proportional gains and integral times are designed sequentially. Here, our proposed method is based on frequency-domain analysis also.

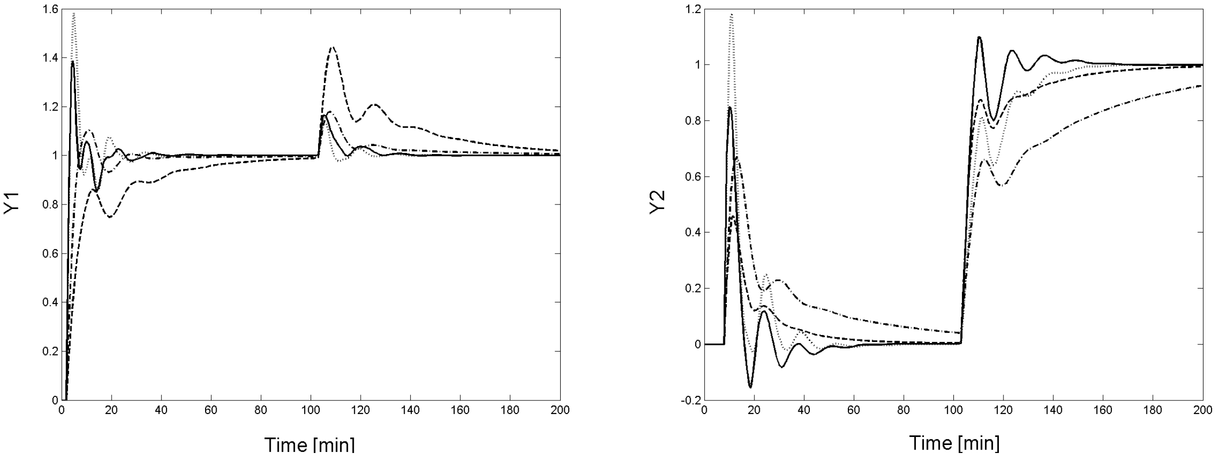

Fig. 1. Set point responses of multiloop PI control systems for Wood and Berry column. (Solid line: proposed, dash dotted line: BLT, dotted line: SAT, dashed line: DLT).

According to Lee at al (1998a), the proportional and derivative gain is given by

$$K_{ci} = f_i^{'}(0) \quad ; \quad \tau_{Di} = \frac{f_i^{''}(0)}{2K_{ci}} \tag{7}$$

Analytical tuning rule for integral gain can be obtained by

$$\tau_{Ii} = -\frac{(G_{ii+}^{'}(0) - n_i \lambda_i) K_{ci}}{(G^{-1}(0))_{ii}} \tag{8}$$

Where $G_{ii+}$ is non –minimum part of $G_{ii}$ and is chosen to be the all pass form, $\lambda_i$ is an adjustable constant for system performance and stability, $n_i$ is chosen as IMC controller to be realizable, $G_{ii+}(0) = 1$ is necessary for the controlled variable to track its setpoint.

## 2. Simulation study

**Example**: Consider an eight trays + reboiler distillation column separating methanol and water (Wood and Berry column model) which was studied by Luyben in 1986.

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \tag{9}$$

In case study, the simulation results show that the proposed tuning method for MIMO systems compares favorably with BLT [7], DLT [1] and SAT [6] methods. The optimum closed-loop time constant $\lambda$ for Wood and Berry column, $\lambda$ , is found by solving the optimization function which was proposed by the Eq.6, it is in turn as 0.013/4.470 for loop 1 and loop 2. The results tuning are listed in table 1. The Fig.1 shows that the proposed method gives the MIMO control systems with well-balanced and robust responses.

Table 1. Tuning results by the proposed PI method and various methods for WB column

| Process | | Proposed | SAT | DLT | BLT |
|---|---|---|---|---|---|
| WB | $\lambda$ | 0.013 | - | 6.0 | - |
| | | 4.470 | - | 6.0 | - |
| | $K_c$ | 1.326 | 0.868 | 0.22 | 0.375 |
| | | -0.108 | -0.087 | -0.14 | -0.075 |
| | $\tau_I$ | 8.557 | 3.246 | 17.2 | 8.29 |
| | | 7.767 | 10.40 | 15.9 | 23.6 |
| | $\tau_D$ | - - | - | - | |
| | IAE1* | 2.720 | 4.00 | 10.94 | 5.11 |
| | | 5.560 | 6.670 | 5.38 | 16.8 |
| | IAE2 | 1.091 | 0.806 | 9.66 | 3.38 |
| | | 7.621 | 11.83 | 10.68 | 32.7 |

IAE1*: IAE for unit step change in loop 1.

## Conclusions

In this paper, the Mp criterion based analytical design of multiloop PID controllers for desired closed-loop responses was proposed based on the analytical method by Lee et al. (2004) in which the proportional gain and derivative time constant are designed by neglecting of off-diagonal elements while the integral term is designed by taking the off-diagonal element fully into account. The design parameter is the closed-loop time constant between set point and the corresponding outputs. Mp tuning criterion was applied to find optimal design parameter which compromises performance and stability in the multiloop system. The resulting multiloop PID controllers have the closed-loop responses to meet desired performance and robustness as closed as possible.

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