Evaluation of rule of thumb to predict gas flow directions in gas assisted injection molding

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INTRODUCTION

In this paper the authors shall combine the resistance of cavity of two square plates with that of pipes to determine the gas direction under the foresaid geometry. The flow model of Newtonian fluid was previously suggested under the fan-shaped geometry including relatively thin cavity of two square plates when $\frac{\rho \bar{v}_r H}{\mu}(\frac{H}{R_0}) << 1$, $(\frac{H}{R_0})^2 \frac{1}{\hat{\theta}^2} << 1$ and $(\frac{H}{R_0})^2 << 1$. [Lim,1999, 2004] However one may frequently encounter the problem of relatively thick cavity between two square-plates where $(\frac{H}{R_0})^2$ is replaced by ε (that is the order of 10^{-1}) and $\hat{\theta}^2$ is the order of one. For these conditions an approximated flow model and the rule of thumb shall be introduced to show whether the resistence of the relatively thick cavity of two square plates may affect the gas direction in GAIM under the fore-said geometry, and the result of simulation shall be compared with the result of rule of thumb for both conditions.

METHODS

1. Theory

For incompressible fluids, continuity equation in cylindrical coordinates becomes:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0 \tag{1}$$

when v_θ is assumed to be zero velocity.

Momentum equation for Newtonian fluid, neglecting gravity, becomes:

$$\rho(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z}) = -\frac{\partial P}{\partial r} + \mu[\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \Theta^2} + \frac{\partial^2 v_r}{\partial z^2}] \tag{2}$$

$$\rho(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z}) = -\frac{\partial P}{\partial z} + \mu[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}] \tag{3}$$

Then continuity and momentum equations are rendered into dimensionless form as below.

$$\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r}\tilde{v}_r) + \frac{\partial \tilde{v}_z}{\partial \tilde{z}} = 0 \tag{4}$$

$$\frac{\rho \bar{v}_r H}{\mu}(\frac{H}{R_0})[\frac{\partial \tilde{v}_r}{\partial \tilde{t}} + \tilde{v}_r \frac{\partial \tilde{v}_r}{\partial \tilde{r}} + \tilde{v}_z \frac{\partial \tilde{v}_r}{\partial \tilde{z}}]$$
$$= -\frac{\partial \tilde{P}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{v}_r}{\partial \tilde{z}^2} + (\frac{H}{R_0})^2(\frac{\partial}{\partial \tilde{r}}(\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r}\tilde{v}_r))) + (\frac{H}{R_0})^2 \frac{1}{\hat{\Theta}^2}(\frac{1}{\tilde{r}^2}\frac{\partial^2 \tilde{v}_r}{\partial \tilde{\Theta}^2}) \tag{5}$$

$$\frac{\rho\bar{v}_r H}{\mu}\left(\frac{H}{R_0}\right)^3\left[\frac{\partial\tilde{v}_z}{\partial\tilde{t}}+\tilde{v}_r\frac{\partial\tilde{v}_z}{\partial\tilde{r}}+\tilde{v}_z\frac{\partial\tilde{v}_z}{\partial\tilde{z}}\right]$$
$$=-\frac{\partial\tilde{P}}{\partial\tilde{z}}+\left[\left(\frac{H}{R_0}\right)^4\left\{\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left(\tilde{r}\frac{\partial\tilde{v}_z}{\partial\tilde{r}}\right)+\frac{1}{\hat{\Theta}^2}\frac{1}{\tilde{r}^2}\frac{\partial^2\tilde{v}_z}{\partial\tilde{\Theta}^2}\right\}+\left(\frac{H}{R_0}\right)^2\frac{\partial^2\tilde{v}_z}{\partial\tilde{z}^2}\right] \tag{6}$$

When $\frac{\rho\,\bar{v}_r H}{\mu}\left(\frac{H}{R_0}\right) << 1$, $\left(\frac{H}{R_0}\right)^2\frac{1}{\hat{\theta}^2} << 1$ and $\left(\frac{H}{R_0}\right)^2$ << 1, the flow model of Newtonian fluid was previously suggested under the fan-shaped geometry.[Lim,1999, 2004] However one may frequently encounter the problem of fan-shaped cavity between two square plates where $\left(\frac{H}{R_0}\right)^2$ is the order of 10^{-1} instead of the limiting condition of $\left(\frac{H}{R_0}\right)^2 << 1$, and $\hat{\theta}^2$ is the order of one. Then $\left(\frac{H}{R_0}\right)^2$ and $\left(\frac{H}{R_0}\right)^2\frac{1}{\hat{\theta}^2}$ may be replaced by ε that is the order of 10^{-1}.

In addition, for $\frac{\rho\,\bar{v}_r H}{\mu}\left(\frac{H}{R_0}\right) << 1$, Eqs. (5) and (6) may be reduced into quasi-steady state equations as:

$$0=-\frac{\partial\tilde{P}}{\partial\tilde{r}}+\frac{\partial^2\tilde{v}_r}{\partial\tilde{z}^2}+\varepsilon\left(\frac{\partial}{\partial\tilde{r}}\left(\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}(\tilde{r}\tilde{v}_r)\right)+\frac{1}{\tilde{r}^2}\frac{\partial^2\tilde{v}_r}{\partial\theta^2}\right)+0(\varepsilon^2) \tag{7}$$

$$0=-\frac{\partial\tilde{P}}{\partial\tilde{z}}+\varepsilon\frac{\partial^2\tilde{v}_z}{\partial\tilde{z}^2}+0(\varepsilon^2) \tag{8}$$

On the other hand $\tilde{P}$, $\tilde{v}_r$, and $\tilde{v}_z$ may be perturbed around $(\tilde{P})_0$, $\tilde{v}_{r0}$, and $\tilde{v}_{z0}$, using perturbation technique, in terms of ε as:

$$\tilde{P}=(\tilde{P})_0+(\tilde{P})_1\varepsilon+0(\varepsilon^2) \tag{9}$$

$$\tilde{v}_r=\tilde{v}_{r0}+\tilde{v}_{r1}\varepsilon+0(\varepsilon^2) \tag{10}$$

$$\tilde{v}_z=\tilde{v}_{z0}+\tilde{v}_{z1}\varepsilon+0(\varepsilon^2) \tag{11}$$

Then the pressure distribution of 0(1) becomes

$$\tilde{P}_0=\frac{(\tilde{P}_1)_0-(\tilde{P}_0)_0}{\ln\frac{R_1}{R_0}}\ln\tilde{r}+(\tilde{P}_0)_0 \tag{12}$$

And the pressure distribution of 0(ε) becomes

$$(\tilde{P})_1=\frac{(\tilde{P}_1)_1-(\tilde{P}_0)_1}{\ln\frac{R_1}{R_0}}\ln\tilde{r}+(\tilde{P}_0)_1 \tag{13}$$

Thus:

$$\Delta P_{\text{fan-plates}}=\frac{12\mu Q}{H^3\hat{\theta}}\ln\frac{R_o}{R_1}=\frac{12\mu r<V_r>}{H^2}\ln\frac{R_o}{R_1} \tag{14}$$

when $\frac{\rho\,\bar{v}_r H}{\mu}\left(\frac{H}{R_0}\right) << 1$, and $\left(\frac{H}{R_0}\right)^2\frac{1}{\hat{\theta}^2}$ and $\left(\frac{H}{R_0}\right)^2$ are the orders of 10^{-1}.

Eq. (14) is eligible to the problem of fan-shaped geometry in case that not only $(\frac{H}{R_0})^2 << 1$ but also $(\frac{H}{R_0})^2$ is even the order of 10^{-1}.

2. Resistance of heterogeneous geometries

Figures 1 and 2 show cavities composed of two pipes, pipe 1 and pipe 2, respectively, connected in parallel. Relatively thick cavities of two square plates are attached to each side of these pipes in Fig. 1. The left- relatively thick cavity of two square plates is replaced by two runners as in Fig. 2. Pipe 1 is composed of pipe 11 and pipe 12 connected in series, and pipe 2 is composed of pipe 21 and pipe 22. These four pipes have the same length and may or may not have the same diameter. The polymer and gas injection points are located at the center of the front side of a relatively thick cavity between two square plates in the left hand side. Pipe 1 is located at the upper side and pipe 2 is at the lower side.

2.1. Definition of proposed resistance

The definition of resistance may be developed and proposed to be $r*$ as a resistance to $V*$ of the initial velocity of melt polymer at the nearest geometry to a gas injection point while the resistance to flow rate was previously defined as r.[Lim and Lee, 2003] Consequently the proposed resistance of steady state flow of a Newtonian liquid under the following geometry may be rearranged as below.

2.2. Proposed resistance for four conduits

$$\frac{r_2 *}{r_1 *} = \frac{r_2 D_{21}^{\ 2}}{r_1 D_{11}^{\ 2}} \tag{15}$$

2.3. Proposed rule of thumb under the geometry composed of a cavity between two SFP and four conduits

One may consider that the vertex angle of fan-shaped path (i.e., $\hat{\theta}$) for gas penetration may be divided into two sections for both the upper and the lower sides. For each side the vertex angle may become $\frac{\hat{\theta}}{2}$.

$$\frac{V_1 *}{V_2 *} = \frac{r_2 *}{r_1 *} \tag{16}$$

where

$$r_1 * = (\frac{R_0}{2})(\frac{\hat{\theta}}{2})H[\frac{12\mu}{H^3 \frac{\hat{\theta}}{2}} \ln \frac{R_0}{R_1} + \frac{128\mu}{\pi}(\frac{L_{11}}{D_{11}^{\ 4}} + \frac{L_{12}}{D_{12}^{\ 4}})] \tag{17}$$

$$r_2 * = (\frac{R_0}{2})(\frac{\hat{\theta}}{2})H[\frac{12\mu}{H^3 \frac{\hat{\theta}}{2}} \ln \frac{R_0}{R_1} + \frac{128\mu}{\pi}(\frac{L_{21}}{D_{21}^{\ 4}} + \frac{L_{22}}{D_{22}^{\ 4}})] \tag{18}$$

3. Simulations and Model-predictions

The commercial software of MOLDFLOW (version of MPI 4.0) was used to perform the simulations of the cases. The cavities of pipes (center) as well as two cavities between two SFP (left and right) and the cavities of pipes (center), a runner as well as a cavity between two SFP (right) were involved in the configuration as in Fig. 1 and Fig. 2, respectively.

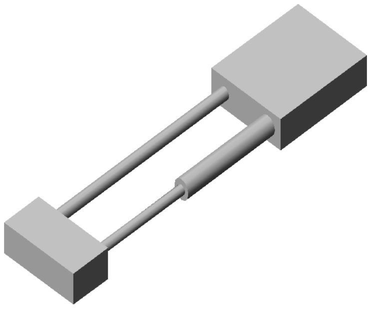

Fig. 1

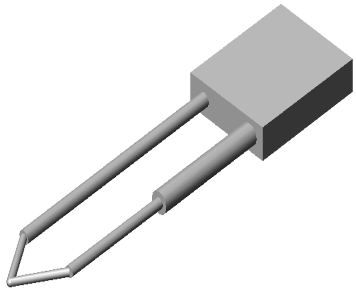

Fig. 2

RESULTS AND DISCUSSION

The results of simulation (e.g., Figs. 3 and 4) were generally consistent with those of the suggested rule of thumb in qualitative way to determine gas directions in gas assisted injection molding even though a relatively large value of 0.36 was applied as the value of ε to describe a relatively thick cavity of two square plates.

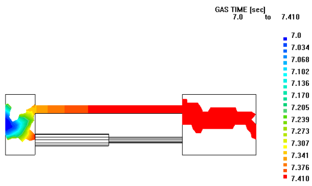

Fig. 3

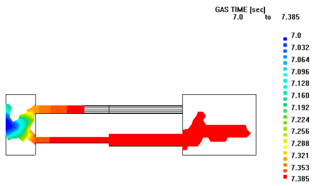

Fig. 4

CONCLUSION

The results of simulation were compared with the results of rule of thumb (RT1) containing the approximated flow model as well as those of another rule of thumb (RT2) without the resistence of the relatively thick cavity of two square plates. RT1 may be used as a criterion to determine where the gas flows earlier between to upper pipes and to lower pipes while RT2 may be used to determine where the gas flows faster between inside upper pipes and inside lower pipes. The results of simulations were generally consistent with the former in qualitative way to determine gas directions in gas assisted injection molding even though a relatively large value of 0.36 was applied as the value of ε to describe a relatively thick cavity of two square plates.

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