다변수 일반예측제어에 의한 증류탑의 제어

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MULTIVARIABLE CONSTRAINED GENERALIZED PREDICTIVE CONTROL OF A BINARY DISTILLATION COLUMN

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Inroduction

Since the GPC had been introduced, a couple of constrained GPC techniques[1][2] were developed for single-input/single-output(SISO) system. In this study, the SISO control procedure is extended for multivariable system to apply to a binary distillation column. The performance of the proposed control is compared with that of the quardratic dynamic matrix control through rigorous simulation. In addition, the property of tuning parameters is investigated to show the tuning guideline of the control.

Process model

The studied process is methanol-water distillation system using a typical binary distillation column and its rigorous model is used in the dynamic simulation. The model consists of material and energy balances and equilibrium relation. The material balances are of total holdup and component holdup in the form of differential equation. For the equilibrium calculation the van Laar equation is employed. While pressure profile is assumed to be of linear distribution throughout stages from the top to the bottom, nonlinear liquid hydraulic with the Francis weir equation is employed in the calculation of liquid flow rate. Vapor holdup is neglected owing to its small amount compared with liquid holdup.

Control algorithm

In the constrained generalized predictive control, the following multivariable input-output model is used.

$$A(q^{-1})\ y(k) = B(q^{-1})\ u(k) + y_0(k) \qquad (1)$$

where $A(q^{-1}) = I - A_1q^{-1} - A_2q^{-2} - \cdots - A_nq^{-n}$,

$$B(q^{-1}) = B_1q^{-1} + B_2q^{-2} + \cdots + B_mq^{-m} \ .$$

Also, y is controlled variable vector, top and bottom product compositions, and u is input vector, reflux, steam and feed flow rates. The y_0 is unmodeled disturbance. Among them, reflux and steam flow rates are manipulated variables and feed flow rate is a measured disturbance. Though the unmodeled disturbance is not customary in adaptive control, it is utilized to eliminate persistent offset as used in the dynamic matrix control[3].

The initial parameters in A and B are found by the one-shot algorithm[4] and the parameters are recursively updated using the recursive least squares method[5] while implemented in control computation.

Multi-step prediction of output is available by successively applying Eq. (1). However, the prediction is readily obtained using the following Diophantine equation without tedious substitution.

$$I = E_j A + q^{-j} F_j \tag{2}$$

where $E_j = \sum_{i=1}^{j} e_{ij} q^{-i+1}$,

$e_{ij} = e_{i,j-1}$, for $i < j$

$e_{ij} = e_{i-2,j-1} A_1 + e_{i-1,j-1} A_2$, for $i=j$

$E_1 = I$,

and $F_j = \sum_{i=1}^{n} f_{ij} q^{-i+1}$,

$f_{ij} = \sum_{i=1}^{n} f_{1,j-k} A_{i+k-1}$,

$f_{10} = I$

Now the multi-step prediction of output using the parameters E_j and F_j is given

$$\hat{y}(k+j) = E_j B u + q^{-j} F_j y \tag{3}$$

and it is put into the control objective function.

When the sum of predicted absolute error is minimized, the control objective of an MIMO system can be formulated as

$$J = \sum_{i=1}^{n} \sum_{j=N_1}^{N_2} Q_{ij} \mid y_{isp} - \hat{y}_i (k+j) \mid + \sum_{i=1}^{m} \sum_{j=1}^{N_3} R_{ij} u_i (k+j-1) \tag{4}$$

where Q_{ij} and R_{ij} are output error and input weights, respectively.

In order to minimize the objective function with the linear programming, an artificial variable is introduced and Eq. (4) is transformed as follows:

$$\text{min.} \quad z \tag{5}$$

$$\text{s. t.} \quad J \leq z ,$$

$$\Delta u_{min} \leq \Delta u \leq \Delta u_{max}$$

and

$$u_{min} \leq u \leq u_{max} .$$

Constraints on input and input variation are largely decided by the physical limitation of the process and they are often determined by the operator's experience. However, they are important tuning parameters in this control scheme and their property is discussed in the next section.

Controller tuning

Many control parameters are involved in this control scheme. Among them the numbers of prediction and control steps are major tuning parameters. In general, the number of prediction steps is set to approximately the rise-time of output[6]. However, it is too large in this study, and therefore the prediction step number is determined as three only considering the total number of model parameters and their recursive estimation time. In the meantime, the number of the first prediction step, N_1, is related to the response delay and is taken as one since no significant delay is observed in the output. The number of control steps is also set to three after deliberating control computation time. It is also related to the movement of manipulated variable, but the constraint on input variation has stronger influence on control performance.

The maximum and minimum values of inputs are physically determined. In other words, they are not adjustable tuning parameter in real application. In this study, however, the limit values are taken as 120% and 80% of steady state values. Maximum and minimum variations of input in single movement are adjustable for the best performance and, in the practical application, field process operator has knowledge on the determination of the limit from his experience. Similarly, trial adjustment is conducted in this study and it is also observed that the constraint has the most significant influence among various tuning parameters in control performance.

Weights, Q_{ij} and R_{ij}, are other tuning parameters. The Q_{ij} is an adjusting weight among output errors and the R_{ij} is weight among inputs. Also, the latter is closely related with limits of input variation as soft constraint. Moreover, weight between output errors and inputs is determined by the ratio of Q_{ij} and R_{ij}. For the analysis of the weights, input constraints in Eq. (5) are relaxed. The property analysis in closed loop system is adopted from the case of the quadratic control of the SISO system[7].

The future input vector is directly computed from the minimization objective, Eq. (4), and it is

$$u = (Q\,G + R)^{-1}\, Q\, (y_{sp} - f) \tag{6}$$

where G and f is found from

$$\hat{y} = G\,u + f\ . \tag{7}$$

Also, $$f\,(k) = \Gamma\, u\,(k\text{-}1) + F\, y\,(k)\ . \tag{8}$$

The first element in input vector of Eq. (6) is

$$\bar{u}\,(k) = \Lambda_1\, [\, y_{sp} - \Gamma\, u\,(k\text{-}1) - F\, y\,] \tag{9}$$

where Λ_1 is the first row of the coefficient matrix of Eq. (6). Two input vectors can be combined and replaced into output vector using Eq. (1). In this case the unmodeled error in the model is assumed to be zero.

$$\Phi\, y = \Lambda_1\, y_{sp}\ , \tag{10}$$

where $\Phi = [\, I, \Lambda_1 \Gamma \,] B^{-1} A + \Lambda_1 [\, 0, F \,]$.
The inverse of matrix B is not directly found in some cases where one or more sets of parameters in Eq. (1) can be used to obtain a square B matrix.

The stability of the output tracking to the set point is determined by the roots of the following characteristic equation of which the roots have to be located inside unit circle for stable closed loop performance.

$$\det [\, \Phi \,] = 0 \tag{11}$$

The property of weights, Q_{ij} and R_{ij}, is not explicitly explained by the stability analysis, but numerical study shows marginal values of the weights. For instance, when $Q_{1j} = 1$ and $Q_{2j} = 3$, the marginal highest value is 0.024 for R_{ij}. In the simulation study, the best performance was obtained with $R_{ij} = 0.005$ for the case. Note that input constraints are relaxed in this computation.

Results and discussion

Performance of the proposed control scheme is examined through the simulation of a rigorous model of a binary distillation column. In order to compare the control performance of this study, the quadatic matrix control with same tuning parameters is used in the control simulation.

Conclusion

The constrained generalized predictive control for single variable system is extended to a multivariable system and its performance is investigated by applying in the control of a binary distillation column. Also, the property of tuning parameters is examined to give tuning guideline.

Simulation study using rigorous model indicates that the proposed control scheme gives satisfactory control performance in both set-point tracking and regulatory control. In the control performance comparison with the quadratic dynamic control, it is observed that the result of the GPC is better than that of the QDMC.

Acknowledgement

Financial support from the Automation Research Center designated by KOSEF is gratefully acknowledged.

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