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# Competition Between Intermolecular and Intramolecular Association in Polyatomic Molecules : Theory and Simulation

A. Garcia-Cruellar, D. Ghonasgi, and W.G. Chapman  
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Korea University, Chem. & Bio. Engineering  
Kim, Yong-soo

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# Introduction

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- Unusual phase behavior of associating polymers
  - ◆ Liquid-Liquid equilibria of Nylon-6 / tetrafluoroethanol / carbon dioxide at 373.15 K
  - ◆ Solutions of Telechelic polymers
  - ◆ The competition between **inter-** and **intramolecular** association
  - ◆ Wertheim's theory
    - Model for intermolecular associations
    - Unable to predict the intramolecular association
- This work
  - ◆ Development of the model for the intramolecular association
  - ◆ Comparison of Monte Carlo simulation results and the theory

# Metropolis Monte Carlo Simulation

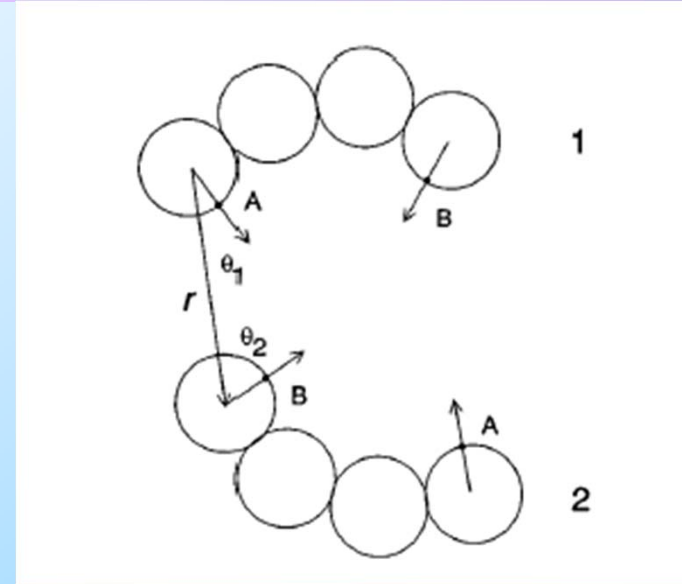
- Potential energy

$$\phi = \phi_{HS} + \phi_{inter}^{assoc} + \phi_{intra}^{assoc}$$

$$\phi_{HS}(\mathbf{r}, \Omega_1, \Omega_2) = \begin{cases} \infty & \text{if } r_{ij} < \sigma \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_{inter}^{assoc}(\mathbf{r}, \Omega_{t1}, \Omega_{t2}) = \begin{cases} -\epsilon_{inter} & \text{if } r < r_c, \theta_1 < \theta_c, \text{ and } \theta_2 < \theta_c \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_{intra}^{assoc}(\mathbf{r}, \Omega_{t1}, \Omega_{t2}) = \begin{cases} -\epsilon_{intra} & \text{if } r < r_c, \theta_1 < \theta_c, \text{ and } \theta_2 < \theta_c \\ 0 & \text{otherwise} \end{cases}$$



# Metropolis Monte Carlo Simulation

- NVT ensemble

- ◆ Starting from an

- Ordinary flexi

- ◆ Association sites

- ◆ Displacement and Reorientation

algorithm of Dickman and Hall [1981]

- Rearrangement of atoms in chain molecules
- Accepted ratio of 40 % of the configurations generated

(1) Random uniform displacement :  $\mathbf{x}_i^{(j)} \rightarrow \mathbf{x}_i^{(j)} + \mathbf{a}$

(2) Bond vectors  $\mathbf{e}_i^{(j)} = \mathbf{x}_i^{(j+1)} - \mathbf{x}_i^{(j)}$  ( $j = 1, \dots, n-1$ ) along chain to independent random displacements, where  $|\mathbf{e}_i^{(j)}| = 1$  by adding a random vector  $\mathbf{b}_j$  to  $\mathbf{e}_i^{(j)}$  and normalizing the resultant to unit length

“translational-jiggling”

- NPT ensemble

- ◆ The length of the simulation cell are changed to keep the pressure constant.

- ◆ About 40 % of the volume changes are accepted

# Theory for Intra- and Intermolecular Association

- The change in configurational Helmholtz free energy

$$\frac{A^{assoc}}{NkT} = \ln X_0 + X_0^{intra} - X_A$$

- ◆ Solving nonlinear equations as follows

$$X_A = \frac{X_0^{int ra}}{1 + \rho X_A \Delta_{AB}^{int er}} \quad \Delta_{AB}^{int er} = 4\pi g_{HS}(\sigma) K_{AB} [\exp(\epsilon_{AB}^{int er} / kT) - 1]$$

$$X_0 = \frac{X_0^{int er}}{1 + \Delta^{int ra}} \quad \Delta^{int ra} = D [\exp(\epsilon^{int ra} / kT) - 1]$$

$$X_0 = X_A^2 / X_0^{int ra}$$

$$X_0^{int er} + X_0^{int ra} - X_0 = 1$$

# Theory for Intra- and Intermolecular Association

- Thermodynamic variables
  - ◆ Configurational internal energy

$$\frac{\partial(A^{assoc} / NkT)}{\partial\beta} = \frac{1}{X_0} \frac{\partial X_0}{\partial\beta} - \frac{\partial X_A}{\partial\beta} + \frac{\partial X_0^{int ra}}{\partial\beta} = \frac{U}{N}$$

- ◆ Compressibility factor (Equation of state)

$$Z^{assoc} = \eta \frac{\partial(A^{assoc} / NkT)}{\partial\eta} = \eta \left( \frac{1}{X_0} \frac{\partial X_0}{\partial\eta} - \frac{\partial X_A}{\partial\eta} + \frac{\partial X_0^{int ra}}{\partial\eta} \right)$$

where,  $\eta = \pi m\rho\sigma^3 / 6$

- Equation of state for the reference state [Chapman et al. 1988]

$$Z = m \left( \frac{1 + \eta + \eta^2 - \eta^3}{(1 - \eta^3)} \right) - (m - 1) \eta \frac{5 - 2\eta}{(1 - \eta)(2 - \eta)} - (m - 1)$$

# Results and Discussions

- At low density, there is significant difference between Wertheim's theory and the present theory.

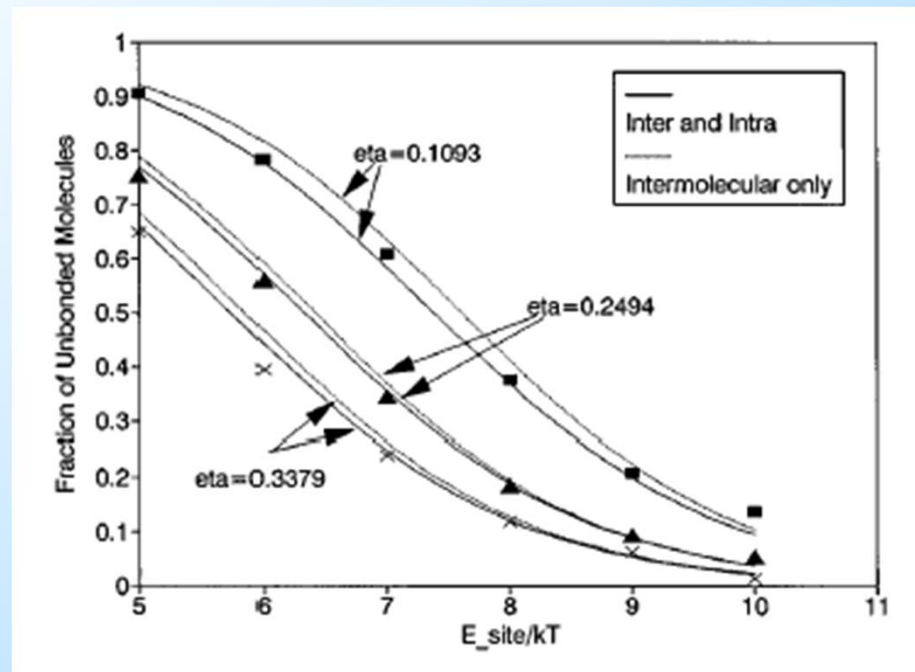


Fig. Fraction of unbonded molecules vs  $\epsilon_{\text{site}}/kT$  at three densities; symbols represent simulation results

# Results and Discussions

- As density increases,  $N_{\text{inter}}/N_{\text{intra}}$  also increases

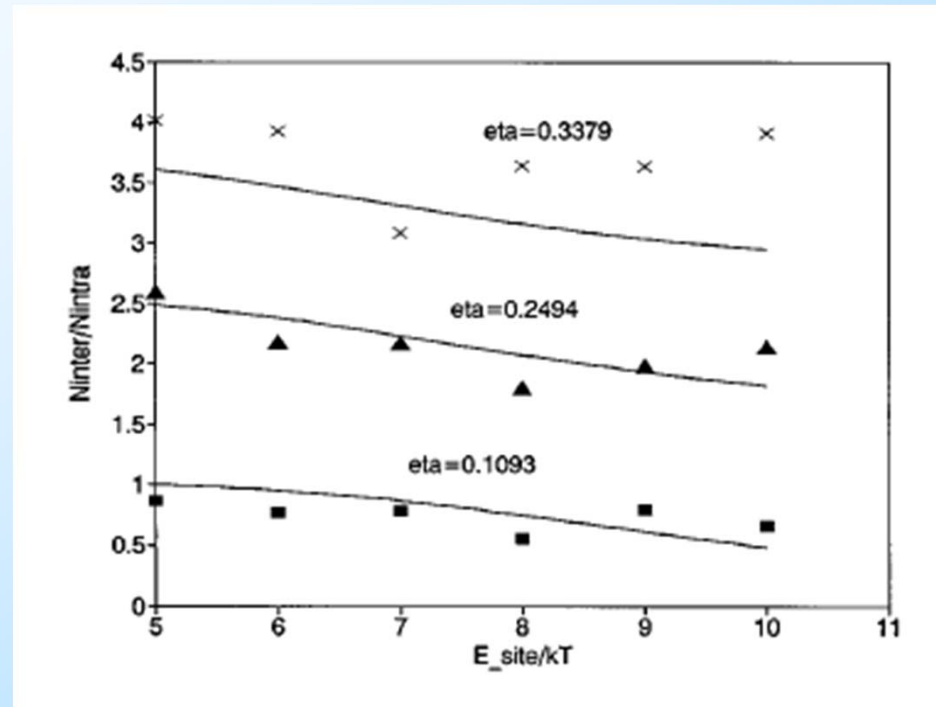


Fig. Ratio of the number of intermolecular bonds to the number of intramolecular bonds vs  $\epsilon_{\text{site}}/kT$  at three densities



# Results and Discussions

- At the highest density, the errors are increased at the high association energy.

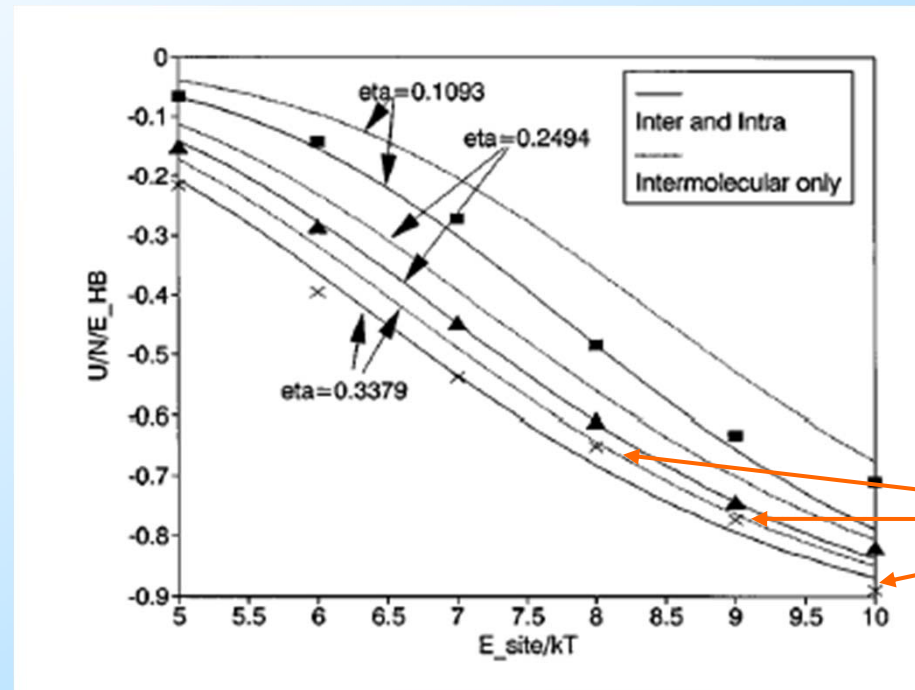


Fig. Configurational energy vs  $\epsilon_{site}/kT$  at three densities

# Results and Discussions

- The minimum in the compressibility factor
  - ◆ The competition between inter- and intramolecular association
  - ◆ At lower density, intramolecular association is dominated.

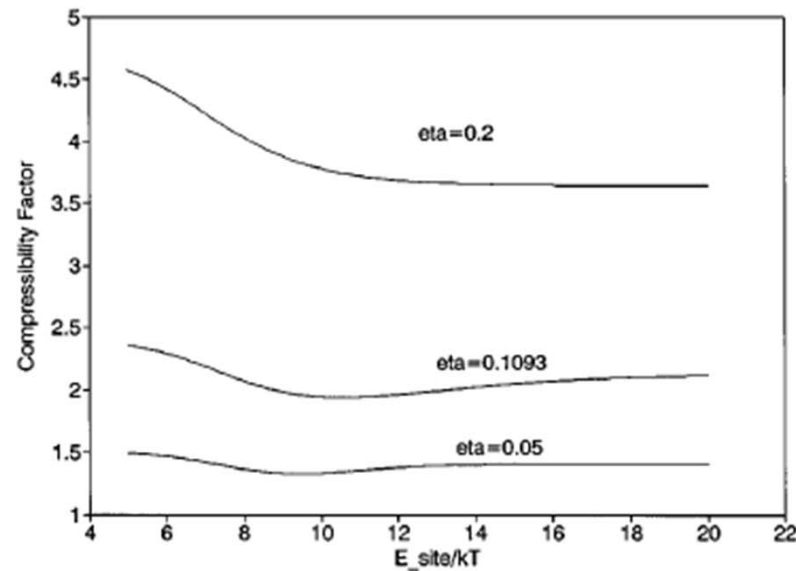


FIG. 5. Predictions from theory of compressibility factor vs  $\epsilon_{\text{site}}/kT$  at  $\eta=0.2, 0.1093,$  and  $0.05$ .

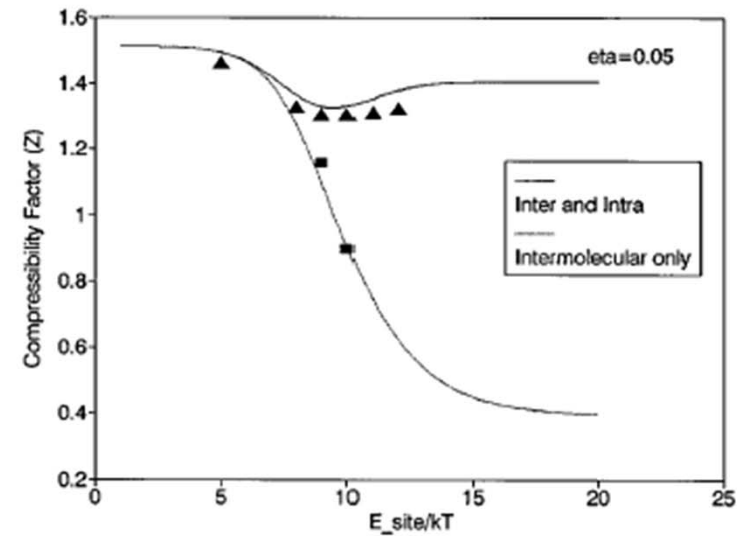


FIG. 6. Compressibility factor vs  $\epsilon_{\text{site}}/kT$  at  $\eta=0.05$ . Symbols represent simulation results, solid triangles (inter- and intramolecular association), solid squares (no intramolecular association). Curves represent predictions from theory.

# Conclusion

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- Development of a theory to explain the competition between inter- and intramolecular association
- The good agreement with Monte Carlo simulation results for most of the conditions studied.

# References

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