# CHIBE320 LECTURE X<br>
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CONTOL SYSTEMS<br>
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Fall 2021<br>
Dept. of Chemical and Biological Engineering<br>
Face University 10-1<br>
Control Control And Control A CHBE320 LECTURE X STABILITY OF CLOSED-LOOP CONTOL SYSTEMS

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Fall 2021 Dept. of Chemical and Biological Engineering Korea University  $\begin{array}{lllllllllll} \textbf{STABILITY OF CLOSED-LOOP} & & & \text{Defundis} & \text{Cokeral stability} & & \text{Cokeral stability} & & \text{Cokeral stability} & & \text{EIBO (saulif)} & & \text{EIBO (sualif)} & & \text{EIBO (sualif)} & & \text{EIBO (sualif)} & & \text{EIB$ — **Professor Dae Ryook Yang**<br>
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DEFINITION OF STABILITY<br>
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Road Map of the Lecture X

# Road Map of the Lecture X<br>
Calcolidity of closed-loop control system<br>
— Definition<br>
— Concert stability<br>
— IBBO staalility<br>
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— IBBO staalility criterion<br>
— Ibertet substitution method<br>
— Rode stability cr • Stability of closed-loop control system

- Definition
	- General stability
	- BIBO stanility
- Stability criteria
	- Routh-Hurwitz stability criterion
	- Direct substitution method
	- Root locus method
	- Bode stability criterion
	- Gain margin and Phase margin
	- Nyquist stability criterion
- Robustness

### DEFINITION OF STABILITY

- BIBO Stability
- CHBE320 Process Dynamics and Control<br>
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CHBES STABILITY<br>
 Eva unconstantion (Internation Care in the control of or all bundle output response is bounded for all bounded inputs. Otherwise it is said to be unstable." 1 + ை() = 0
- General Stability
	- denominator in the transfer function are negative or have negative real parts (OLHP). Otherwise, the system is unstable.
	- $\Rightarrow$  What is the difference between the two definitions?
	- Open-loop stable/unstable
	- Closed-loop stable/unstable
	- Characteristic equation:  $1 + G_{OL}(s) = 0$
	- Nonlinear system stability: Lyapunov and Popov stability

### • Supplements for stability

- For input-output model,
	- point, if  $u(t)=0$  for all time t implies  $v(t)$  goes to zero with time. – Same as "General stability": all poles have to be in OLHP.
	- point, if  $u(t)=0$  for all time t implies  $v(t)$  is bounded for all time.
- Finition<br>
 General stability<br>
 BIBO stanility<br>
 BIBO stanility<br>
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 Direct substitution method<br>
 Gota learned and Phase margin<br>
 Cold margin and Phase • General stability<br>• MBO stanlility<br>• Math-Hurwitz stability criterion<br>• Note locus method<br>• Hotel substitution method<br>• Hotel stability criterion<br>• Gain margin and Phase margin<br>• Nyquist stability criterion<br>bustness<br>• – Same as BIBO stability: all poles have to be in OLHP or on the imaginary axis with any poles occurring on the imaginary axis non-repeated.
	- If the imaginary pole is repeated the mode is  $t\sin(wt)$  and it is unstable.
- For state-space model,
- CHBE320 Process Dynamics and Control Korea University 10-2<br>
Correspondents and Control (AS): For a system with zero equilibrium<br>
point, if  $u(t) = 0$  for all time timplies  $y(t)$  gees to zero with time,<br>  $-\sin m$  as "General st • Even though there are unstable poles and if they are cancelled by the zeros exactly (pole-zero cancellation), the system is BIBO stable.
	- Internally AS: if  $u(t)=0$  for all time, it implies that  $x(t)$  goes to zero with time for all initial conditions  $x(0)$ .
		- Cancelled poles have to be in OLHP.

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### EXAMPLES



- 
- $-$  As  $K_c$  increases, the step response gets more oscillatory.
- If  $K_c > 12.6$ , the step response is unstable.<br>CHBE320 Process Dynamics and Control Korea University 10-5

### • Simple Example 1

Characteristic equation:  $1 + G_{OL}(s) = 1 + K_c K_v K_n/(\tau_n s + 1) = 0$ 

 $K_c K_v K_n > -1$  for stability

- When  $K_r>0$  and  $K_r>0$ , the controller should be reverse acting  $(K > 0)$  for stability.  $G_c(s) = K_c$ ,  $G_p(s) = K_p$ ,  $G_m(s) = 1$ ,  $G_p(s) = K_p/(\tau_p s + 1)$ <br>
Characteristic equation:  $1 + G_{OL}(s) = 1 + K_c K_v K_p/(\tau_p s + 1) = 0$ <br>  $\tau_p s + (1 + K_c K_v K_p) = 0 \Rightarrow s = -(1 + K_c K_v K_p)/\tau_p$ <br>  $\therefore K_c K_v K_p > -1$  for stability<br>
- When  $K_p > 0$  and  $K_v > 0$ , the controll **mple Example 1**<br>
(s) =  $K_e$ ,  $G_v(s) = K_v$ ,  $G_m(s) = 1$ ,  $G_p(s) = K_p/(r_p s + 1)$ <br>
aracteristic equation:  $1 + G_{OL}(s) = 1 + K_c K_w K_p/(r_p s + 1) = 0$ <br>  $s + (1 + K_c K_v K_p) = 0 \Rightarrow s = -(1 + K_c K_v K_p)/(r_p s + 1) = 0$ <br>  $\therefore K_c K_w K_p > -1$  for stability<br>
When  $K_p > 0$  and  $K_r >$ **Example 1**<br>  $G_v(s) = K_v,$   $G_m(s) = 1,$   $G_p(s) = K_p/(\tau_p s + 1)$ <br>
tic equation:  $1 + G_{OL}(s) = 1 + K_c K_v K_p/(\tau_p s + 1) = 0$ <br>  $K_c K_v K_p$ )=0  $\Rightarrow s = -(1 + K_c K_v K_p)/\tau_p$ <br>  $K_c K_v K_p > -1$  for stability<br>  $K_p > 0$  and  $K_v > 0$ , the controller should be reverse acti Simple Example 1<br>  $G_c(s) = K_c$ .  $G_v(s) = K_{pv}$ .  $G_m(s) = 1$ ,  $G_p(s) = K_p/(\tau_p s + 1)$ <br>
Characteristic equation:  $1 + G_{OL}(s) = 1 + K_c K_v K_p/(\tau_p s + 1) = 0$ <br>  $\tau_p s + (1 + K_c K_v K_p) = 0 \Rightarrow s = -(1 + K_c K_v K_p)/\tau_p$ <br>  $\therefore K_c K_v K_p > -1$  for stability<br>  $\therefore (K_c > 0)$  for sta **Solution Example 1**<br>  $G_c(s) = K_c$ ,  $G_v(s) = K_v$ ,  $G_m(s) = 1$ ,  $G_p(s) = K_p/(r_p s + 1)$ <br>
characteristic equation:  $1 + G_{OL}(s) = 1 + K_c K_b K_p/(r_p s + 1) = 0$ <br>  $\tau_p s + (1 + K_c K_b K_p) = 0 \Rightarrow s = -(1 + K_c K_b K_p)/\tau_p$ <br>  $\therefore K_c K_v K_p > -1$  for stability<br> **When**  $K_p > 0$  **and K Simple Example 1**<br>  $G_e(s) = K_e,$   $G_e(s) = K_w,$   $G_m(s) = 1,$   $G_p(s) = K_p/(r_p s + 1)$ <br>
Characteristic equation:  $1 + G_{0I}(s) = 1 + K_c K_p K_p/(r_p s + 1) = 0$ <br>  $\tau_p s + (1 + K_c K_v K_p) = 0 \Rightarrow s = -(1 + K_c K_v K_p)/r_p$ <br>  $\therefore K_c K_v K_p > -1$  for stability<br> **When**  $K_p > 0$  **and K\_s > imple Example 1**<br>  $\epsilon_c(s) = K_c$ .  $G_e(s) = K_w$ .  $G_m(s) = 1$ ,  $G_p(s) = K_p/(r_p s + 1)$ <br>
haracteristic equation:  $1 + G_{0L}(s) = 1 + K_c K_b K_p/(r_p s + 1) = 0$ <br>  $\gamma_b s + (1 + K_c K_b K_p) = 0 \Rightarrow s = -(1 + K_c K_b K_p)/r_p$ <br>  $\therefore K_c K_b K_p > -1$  for stability<br>  $\therefore K_c K_b K_p > -1$  for stab **Example 1**<br>  $c_r$   $G_r(s) = K_r$ .  $G_m(s) = 1$ ,  $G_p(s) = K_p/(r_p s + 1)$ <br>
sistic equation:  $1 + G_{OL}(s) = 1 + K_c K_s K_p/(r_p s + 1) = 0$ <br>  $K_c K_v K_p = 0 \Rightarrow s = -(1 + K_c K_v K_p)/r_p$ <br>  $K_c K_v K_p > -1$  for stability<br>  $K_c K_p * 0$  and  $K_c > 0$ , the controller should be reverse **Simple Example 1**<br>  $G_c(s) = K_c$   $G_c(s) = K_r$   $G_m(s) = 1$ ,  $G_p(s) = K_p/(r_p s + 1)$ <br>
Characteristic equation:  $1 + G_{OL}(s) = 1 + K_c K_c K_p/(r_p s + 1) = 0$ <br>  $r_p s + (1 + K_c K_c K_p) = 0 \Rightarrow s = -(1 + K_c K_c K_p)/(r_p$ <br>  $\therefore K_c K_r K_p > -1$  for stability.<br>
- When  $K_p > 0$  and  $K_r > 0$  $G_m(s) = 1,$   $G_p(s) = K_p/(r_p s + 1)$ <br>  $g_L(s) = 1 + K_c K_w K_p/(r_p s + 1) = 0$ <br>  $= -(1 + K_c K_w K_p)/r_p$ <br>
ability<br>
the controller should be reverse acting<br>  $s + 1$ ,  $G_m(s) = 1$ ,  $G_p(s) = 1/(5s + 1)$ <br>  $/(2s + 1)(5s + 1) = 0$ <br>  $= [-7 \pm \sqrt{49 - 40(1 + K_c)}]/20$ <br>
ty<br>
(or Kor **Simple Example 1**<br>  $c_r(s) = K_r$ ,  $c_r(s) = K_r$ ,  $c_n(s) = 1$ ,  $c_r(s) = K_p/(r_p s + 1)$ <br>
Characteristic equation:  $1 + c_{0t}(s) = 1 + K_r K_r K_p/(r_p s + 1) = 0$ <br>  $r_p s + (1 + K_r K_r K_p) = 0 \Rightarrow s = -(1 + K_r K_r K_p)/(r_p$ <br>  $\therefore K_r K_r K_p > -1$  for stability<br>  $\therefore$   $K_r K_r K_p > -1$  for s **le Example 1**<br>  $K_c$ ,  $G_c(s) = K_r$ ,  $G_n(s) = 1$ ,  $G_p(s) = K_p/(r_p s + 1)$ <br>
eristic equation:  $1 + G_{OL}(s) = 1 + K_c K_r K_p/(r_p s + 1) = 0$ <br>  $1 + K_c K_r K_p) = 0 \Rightarrow s = -(1 + K_c K_r K_p)/r_p$ <br>  $K_c K_r K_p > -1$  for stability<br>  $\mathbf{c} \mathbf{K} \mathbf{K} \mathbf{K} \mathbf{K} \mathbf{K} \mathbf{K} \mathbf{K$ determine the qualitation of the  $\alpha_{\ell}(t) = 1 + R_{\ell}R_{\ell}R_{\ell}$ ,  $\beta = 0 \Rightarrow s = -(1 + R_{\ell}R_{\ell}R_{\ell}N_{\ell})/\tau_p$ <br>  $\therefore R_{\ell}R_{\ell}R_{\ell} = 0 \Rightarrow s = -(1 + R_{\ell}R_{\ell}R_{\ell}N_{\ell})/\tau_p$ <br>  $\therefore R_{\ell}R_{\ell}R_{\ell} = 0 \Rightarrow s = -(1 + R_{\ell}R_{\ell}R_{\ell}N_{\ell})/\tau_p$ <br> **Nhen K**
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### ROUTH-HURWITZ STABILITY CRITERION

• From the characteristic equation of the form:

• Construct the Routh array



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- A necessary condition for stability: all  $a_i$ 's are positive<br>– "A necessary and sufficient condition for all roots of the characteristic equation to have negative real parts is that all of the elements in the left column of the Routh array are positive."<br>CHBE320 Process Dynamics and Control

### • Example for Routh test

- Characteristic equation
	-
- Necessary condition
	-
- $\frac{1}{a_{n-1}}$ • If any coefficient is not positive, stop and conclude the system is unstable. (at least one RHP pole, possibly more)

– Routh array



– Stable region

### • Supplements for Routh test

- It is valid only when the characteristic equation is a polynomial of s. (Time delay cannot be handled directly.)
- If the characteristic equation contains time delay, use Pade approximation to make it as a polynomial of s.
- Routh test can be used to test if the real part of all roots of characteristic equation are less than -c.
	- Original characteristic equation
- CHE3220 Process Dynamics and Control CHE3220 Process Dynamics and Control CHE3220 Process Dynamics and Control Section 2013<br>
CHE3220 Process Dynamics and Control CHE3220 Process Dynamics and Control CHE3220 Process Dynami • Modify characteristic equation and apply Routh criterion  $a_n(s+c)^n + a_{n-1}(s+c)^{n-1} + \cdots + a_1(s+c) + a_0$ **polements for Routh test**<br>
is valid only when the characteristic equation is a polynomial of s. (Time delay cannot be handled directly)<br>
of s. (Time delay cannot be handled directly)<br>
dens cannot be proposition contains  $= a'_n s^n + a'_{n-1} s^{n-1} + \dots + a'_1 s + a'_0 = 0$ **F Routh test**<br>
example handed directly.<br>
and the caudion contains time delay, use Pade<br>
it equation contains time delay, use Pade<br>
it examples in the real part of all roots of<br>
the real part of all roots of<br>
time is a po **ENDEAN SOF ROUTH ASSET (EXECUTE CONTRACT CONTRACT** 
	- $-$  The number of sign change in the 1<sup>st</sup> column of the Routh array indicates the number of poles in RHP.
	- $-$  If the two rows are proportional or any element of  $1<sup>st</sup>$  column

### • Remedy for special cases of Routh array

- Only the pivot element is zero and others are not all zero
	-
- **Example 19 For Special cases of Routh array**<br>
 Replace zero with positive small number (e), and proceed.<br>
 If there is no sign change in the 1<sup>st</sup> column, it indicates there is a<br>
pair of pure imaginary roots (marginal pair of pure imaginary roots (marginally stable ). If not, the sign change indicates the no. of RHP poles. Remedy for special cases of Routh array<br>
– Only the pivot element is zero and others are not all zero<br>
• Replace zero with positive small number  $(e)$ , and proceed.<br>
• If there is no sign change in the 1<sup>4</sup> column, it indi • Remedy for special cases of Routh array<br>
– Only the pivot element is zero and others are not all zero<br>
• Replace zero with positive small number (e), and proceed.<br>
• If there is no sign change in the 1<sup>w</sup> column, it ind

- polynomial one row above (always even-ordered polynomial). It implies the characteristic polynomial is divided exactly by the
	- Replace the row with the coefficients of the derivative (auxiliary polynomial) of the polynomial one row above and proceed.
- $x \rightarrow x$  Re symmetric about the origin (one unstable), and/or ty pairs symmetric about the origin (one unstable pair). This situation indicates at least either a pair of real roots
- Remedy for special cases of Routh array<br>
 Only the pivot element is zero and others are not all zero<br>
 Rehere zero with positive small number (e), and proceed.<br>
 If there is no sign change in the <sup>14</sup> column, it indi **Hedy for special cases of Routh array**<br>
• Replace zero with positive small number (*c*), and proceed.<br>
• Replace zero with positive small number (*c*), and proceed.<br>
• If there is no sign change in the 1<sup>*u*</sup> column, it i redy for special cases of Routh array<br>y the pivot element is zero and others are not all zero<br>Replace zero with positive small number  $(e)$ , and proceed.<br>If there is no sign change in the  $1^\circ$  column, it indicates there i **pairs of the origin (one unstable pair)** the pivot element is zero and others are not all zero explore zero with positive small number (e), and proceed.<br>
• If there is no sign change in the 1<sup>2</sup> column, it indicates ther  $\mathbb{I}^{\mathsf{m}}$  indicated that the polynomial prior to the auxiliary polynomial <sup>x</sup><br>Re has all nure imaginary roots

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### DIRECT SUBSTITUTION METHOD

- Find the value of variable that locates the closedloop poles at the imaginary axis (stability limit).  $a_n(z + c)^n + a_{n-1}(z + c)^{n-1} + \cdots + a_1(z + c) + b_0$ <br>  $= a'_n s^n + a_{n-1} s^{n-1} + \cdots + a_1' s + a_0' = 0$ <br>
The number of pign change in the IV column of the Routh<br>
Tray indicates the number of poles in RHP.<br>
The two rows are proportional or an =  $a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$ <br>
The number of sign change in the 1<sup>14</sup> column of the Routh<br>
The two rows are proportional or any element of 1<sup>14</sup> column<br>
The two rows are proportional or any element of 1<sup>14</sup> colu 2.2 (A contained a series Control of Proceedian Research (A contained by the Proceedian Research (A contained by the Parameter of Contained by the Parameter of Contained by the Parameters of the parameters of the paramet
- Example
	- **Characteristic equation:**  $1 + G_c G_p = 10s^3 + 17s^2 + 8s + 1 + K_c = 0$
	- On the imaginary axis s becomes  $i\omega$ .  $j\omega$ .

```
-10j\omega^3 - 17\omega^2 + 8j\omega + 1 + K_{cm} = (1 + K_{cm} - 17\omega^2) + j\omega(8 - 10\omega^2) = 0
```

```
∴ (1 + K_{cm} - 17\omega^2) = 0 and \omega(8 - 10\omega^2) = 0\binom{2}{1} – 0
```
– Try a test point such as  $K_z=0$ 

### ROOT LOCUS DIAGRAMS

- 1 and a ad apply beat is the proceed of  $a_1a_2 = 0$  ( $a_2 = 0$  for  $a_3 = 0$  ( $a_4 = 0$  ( $a_5 = 0$ ))<sup>12</sup>  $a_6 = 1$  ( $a_7 = 12.6$ ) ( $a_8 = 16.6$ ) and  $a_9 = 10$ <br>
and the Proceed of The Routh the straining of the column of the Ro • Diagram shows the location of closed-loop poles (roots of characteristic equation) depending on the parameter value. (Single parametric study)
	- Find the roots as a function of parameter
	- $a_{2}=0$  approaches to zeros or  $\pm\infty$ . • Each loci starts at open-loop poles and





### BODE STABILITY CRITERION

### • Bode stability criterion

- **BODE STABILITY CRITERION**<br> **CAIN MARGIN**<br>  $\alpha$  dosed-loop system is unstable if the frequency response of the<br>  $\alpha$  closed-loop ransfer function  $G_{OL} = G_s G_s G_p G_m$  has an amplitude<br>  $\alpha$  closed-loop system is unstable."<br>
cl open-loop transfer function  $G_{OL} = G_c G_v G_p G_m$  has an amplitude<br>on the property that are not the suite of the property of the property of the suite of the suit ratio greater than one at the critical frequency. Otherwise, the closed-loop system is stable."
- Applicable to open-loop stable systems with only one critical frequency
- Example:





### GAIN MARGIN (GM) AND PHASE MARGIN (PM)

• Margin: How close is a system to stability limit?



- 
- Small GM and PM: oscillatory
- If the uncertainty on process is small, tighter tuning is possible.

# EFFECT OF PID CONTROLLERS ON FREQUENCY RESPONSE

### • P

- $-$  As K<sub>c</sub> increases, AR<sub>QL</sub> increases (faster but destabilizing)
- 
- PI
	- Increase  $AR<sub>OL</sub>$  more at low freq.
	- $-$  As  $\tau_I$  decreases, AR<sub>OL</sub> increases (destabilizing)  $-\frac{1}{2}$
	- More phase lag for lower freq. (moves critical freq. toward

lower freq. => usually destabilizing)

- Increase  $AR<sub>OL</sub>$  more at high freq.
- As  $\tau_D$  increases, AR<sub>OL</sub> increases at high freq. (faster)
- More phase lead for high freq. (moves critical freq. toward higher freq. => usually stabilizing)

## NYQUIST STABILITY CRITERION

### • Nyquist stability criterion

- CHBE320 Process Dynamics and Control<br>
CHBE320 Process Dynamics and Control<br>
CHE220 Process ON<br>
CHE220 Process AR<sub>CA</sub> increases (faster but destabilizing)<br>
 As K, increases AR<sub>CA</sub> increases (faster but destabilizing)<br>
 P CHE320 PROCESS DESCRIPTION EXACT AND CONTROLLERS ON<br>  $\kappa$ , increases, AR<sub>GA</sub> increases (faster but detailed integral<br>  $\kappa$ , increases, AR<sub>GA</sub> increases (faster but detailed integral<br>  $\kappa$ , increases, AR<sub>GA</sub> increases ( "If  $N$  is the number of times that the Nyquist plot encircles the point (-1,0) in the complex plane in the clockwise direction, and P is the number of open-loop poles of  $G_{OL}(s)$  that lies in RHP, then  $Z=N+P$  is the number of unstable roots of the closed-loop characteristic equation."
	- Applicable to even unstable systems and the systems with multiple critical frequencies
	- The point (-1,0) corresponds to AR=1 and PA=-180°.
	- Negative N indicates the encirclement of (-1,0) in counterclockwise direction.

 $\Lambda$ 











## CLOSED-LOOP FREQUENCY RESPONSE

• Closed-loop amplitude ratio and phase angle



 $\mathbb{F}_{m+}$  For set point change,

- *M* should be unity as  $\omega \rightarrow 0$ . (No offset)
- $-M$  should maintain at unity up to high frequency as possible. (rapid approach to a new set point)
- $-$  A resonant peak  $(M_p)$  in  $M$  is desirable but not greater than 1.25. (large  $\omega_p$  implies faster response to a new set point)
- Large bandwidth  $(\omega_{bw})$  indicates a relatively fast response with a short rise time.

### **ROBUSTNESS**

- Definition
- CHBE320 Process Dynamics and Control  $\kappa(0.17)$ <br>  $\$ the process model, if the control system is insensitive to the uncertainties in the system and functions properly."
	- The robust control system should be, despite the certain size of uncertainty of the model,
		- Stable
		- Maintaining reasonable performance
	- Uncertainty (confidence level of the model):
		- Process gain, Time constants, Model order, etc.
		- Input, output
	- If uncertainty is high, the performance specification cannot be too tight: might cause even instability